GENERALIZED LORENTZ TRANSFORMATIONS

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This article presents the generalized Lorentz transformations of time, space, velocity and acceleration which can be applied in any inertial or non-inertial (uniform circular motion) frame.

Introduction

If we consider an inertial or non-inertial (uniform circular motion) frame S and another inertial frame Σ whose origins coincide at time zero (in both frames) then the time (t), the position ($\mathbf{r}$), the velocity ($\mathbf{v}$) and the acceleration ($\mathbf{a}$) of a (massive or non-massive) particle relative to the inertial frame Σ are:

$$t \;=\; \int_0^{\mathtt{t}} \gamma \, \mathtt{dt} \,-\, \gamma \, \frac{\vec{r} \cdot \mathbf{V}}{c^2}$$

$$\mathbf{r} \;=\; \vec{r} \,+\, \frac{\gamma^2}{\gamma + 1} \, \frac{(\, \vec{r} \cdot \mathbf{V} \,) \, \mathbf{V}}{c^2} \,-\, \gamma \, \mathbf{R} \,-\, \frac{\gamma^2}{\gamma + 1} \, \frac{(\, \mathbf{R} \times \mathbf{V} \,) \times \mathbf{V}}{c^2}$$

$$\mathbf{v} \;=\; \frac{d\mathbf{r}}{dt}$$

$$\mathbf{a} \;=\; \frac{d\mathbf{v}}{dt}$$

where ($\mathtt{t}, \vec{r}$) are the time and the position of the particle relative to the frame S, ($\mathbf{R}, \mathbf{V}, \mathbf{A}$) are the position, the velocity and the acceleration of the origin of the inertial frame Σ relative to the frame S, (c) is the speed of light in vacuum, and $\gamma \;=\; (1 - \mathbf{V} \cdot \mathbf{V} / c^2)^{-1/2}$

- $\frac{d\mathbf{r}}{dt} = \left(\frac{d\mathbf{r}}{\mathtt{dt}} + \Omega \times \mathbf{r} \right) \left(\frac{1}{dt \,/\, \mathtt{dt}} \right)$

- $\frac{d\mathbf{v}}{dt} = \left(\frac{d\mathbf{v}}{\mathtt{dt}} + \Omega \times \mathbf{v} \right) \left(\frac{1}{dt \,/\, \mathtt{dt}} \right)$

- $\Omega = \frac{\gamma^2}{\gamma + 1} \frac{(\mathbf{A} \times \mathbf{V})}{c^2}$

- $\frac{\gamma^2}{\gamma + 1} \frac{1}{c^2} = \frac{\gamma - 1}{\mathbf{V}^2} \quad (\, \mathbf{V}^2 = \mathbf{V} \cdot \mathbf{V} \,)$

- $\gamma \, \mathbf{R} + \frac{\gamma^2}{\gamma + 1} \frac{(\mathbf{R} \times \mathbf{V}) \times \mathbf{V}}{c^2} = \mathbf{R} + \frac{\gamma^2}{\gamma + 1} \frac{(\mathbf{R} \cdot \mathbf{V}) \, \mathbf{V}}{c^2}$

General Observations

If the frame S is inertial then $(\, \mathbf{A} = 0 \,)$, $(\, \mathbf{V} = \mathrm{cte} \,)$, $(\, \mathbf{R} = \mathbf{V} \, \mathtt{t} \,)$, $(\, \gamma = \mathrm{cte} \,)$ $(\, \int_0^{\mathtt{t}} \gamma \, \mathtt{dt} = \gamma \, \mathtt{t} \,)$, $(\, \mathbf{R} \times \mathbf{V} = 0 \,)$ & $(\, \Omega = 0 \,)$

If the frame S is non-inertial (uniform circular motion) then $(\, \mathbf{A} \neq 0 \,)$ $(\, \mathbf{A} \cdot \mathbf{V} = 0 \,)$, $(\, \mathbf{V} \cdot \mathbf{V} = \mathrm{cte} \,)$, $(\, \gamma = \mathrm{cte} \,)$ $(\, \int_0^{\mathtt{t}} \gamma \, \mathtt{dt} = \gamma \, \mathtt{t} \,)$ & $(\, \Omega \neq 0 \,)$

In addition, if the frame S is non-inertial (uniform circular motion) then the observer S should preferably use an origin O' such that $(\, \mathbf{R} \cdot \mathbf{V} = 0 \,)$

Bibliography

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