A Review on Entanglement and Maxwell-Dirac Isomorphism

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Abstract

In RG forum, one senior professor of physics posted a project called: "Future science and technology." As a response, one of us (VC) wrote in reply: "I think one of future science's tasks is to discover the link between entanglement and classical electromagnetic theory. This is to fulfill Einstein's position that present QM theory is incomplete, a new one must be found. We are on a way to that goal." Therefore, in this paper we will discuss how entanglement can be explained in terms of Maxwell-Dirac isomorphism. This short review may be considered as Einstein's dream of completing QM in a classical picture.

Keywords: quantum entanglement, quantum metaphysics, realism interpretation, Maxwell-Dirac isomorphism.

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Introduction

In its simplest form the quantum theory's features can be reduced to : (a) wave function description of microscopic entities and (b) entanglement. Entanglement is a key property that makes quantum information theory different from its classical counterpart.[14]

But what is entanglement? Wootter gives one of clearest description:[13]

> "In both classical mechanics and quantum mechanics, one can define a pure state to be a state that is as completely specifed as the theory allows. In classical mechanics a pure state might be represented by a point in phase space. In quantum mechanics it is a vector in a complex vector space. Perhaps the most remarkable feature of quantum mechanics, a feature that clearly distinguishes it from classical physics, is this: for any composite system, there exist pure states of the system in which the parts of the system do not have pure states of their own. Such states are called entangled."

According to Scolarici and Solombrino [5]:

> The essential difference in the concept of state in classical and quantum mechanics is clearly pointed out by the phenomenon of entanglement, which may occur whenever the product states of a compound quantum system are superposed. Entangled states play a key role in all controversial features of QM; moreover, the recent developments in quantum information theory have shown that entanglement can be considered a concrete physical resource that it is important to identify, quantify and classify.

Nonetheless, they concluded: "our research has pointed out a puzzling situation, in which the same state of a physical system is entangled in CQM, while it seems to be separable in QQM."

While entanglement is usually considered as purely quantum effect, it by no means excludes possibility to describe it in a classical way.

In this regards, from history of QM we learn that there were many efforts to describe QM features in more or less classical picture. For example: Einstein in 1927 presented his version of hidden variable theory of QM, starting from Schrödinger's picture, which seems to influence his later insistence that "*God does not play dice*" philosophy.[6][7]

Efforts have also been made to extend QM to QQM (quaternionic Quantum Mechanics), for instance by Stephen Adler from IAS.[8]

But in recent decades, another route began to appear, what may be called as Maxwell-Dirac isomorphism route, where it can be shown that there is close link between Maxwell equations of classical electromagnetism and Dirac equations of electron. Intuitively, this may suggest that there is one-to-one correspondence between electromagnetic wave and quantum wave function. But can it offer a classical description of entanglement?

This problem will be explored in the next sections.

A few alternatives of realistic Maxwell-Dirac isomorphism

There are some papers in literature which concerned with the formal connection between classical electrodynamics and wave mechanics, especially there are some existing proofs on Maxwell-Dirac isomorphism. Here the author will review two derivations of Maxwell-Dirac isomorphism i.e. by Hans Sallhofer and Volodimir Simulik. In the last section we will also discuss a third option, i.e. by exploring Maxwell-Dirac isomorphism through quaternionic language.

a. Sallhofer's method

Summing up from one of Sallhofer's papers[1], he says that under the sufficiently general assumption of periodic time dependence the following connection exists between source-free electrodynamics and wave mechanics:

$$\sigma \cdot \begin{bmatrix} rotE + \frac{\mu}{c}\frac{\partial}{\partial t}H = 0 \\ rotH - \frac{\varepsilon}{c}\frac{\partial}{\partial t}H = 0 \\ div\varepsilon E = 0 \\ div\mu H = 0 \end{bmatrix}_{divE=0} \equiv \left[(\gamma \cdot \nabla + \gamma^{(4)}\partial_4)\Psi = 0\right] \tag{1}$$

In words: Multiplication of source-free electrodynamics by the Pauli-vector yields wave mechanics.[1] In simple terms, this result can be written as follows:

$$P \cdot M = D\,, \tag{2}$$

Where:

P = Pauli vector,

M = Maxwell equations,

D = Dirac equations.

We can also say: Wave mechanics is a solution-transform of electrodynamics. Here one has to bear in mind that the well-known circulatory structure of the wave functions, manifest in Dirac's hydrogen solution, is not introduced just by the Pauli-vector.[1]

b. Simulik's method

Simulik described another derivation of Maxwell-Dirac isomorphism. In one of his papers[2], he wrote a theorem suggesting that the Maxwell equations of source-free electrodynamics which can be written as follows:

$$\begin{aligned} rotE + \frac{\mu}{c}\frac{\partial}{\partial t}H &= 0 \\ rotH - \frac{\varepsilon}{c}\frac{\partial}{\partial t}H &= 0 \\ divE &= 0 \\ divH &= 0 \end{aligned} \tag{3}$$

Are equivalent to the Dirac-like equation [2]:

$$\left[\gamma.\nabla - \begin{pmatrix} \varepsilon 1 & 0 \\ 0 & \mu 1 \end{pmatrix} \frac{1}{c}\frac{\partial}{\partial t}\right]\Psi^{c1} = 1, \tag{4}$$

Where in the usual representation

$$\gamma = \begin{pmatrix} 0 & \sigma \\ \sigma & 0 \end{pmatrix}, \tag{5}$$

And σ are the well-known Pauli matrices.

c. Maxwell-Dirac isomorphism through Quaternionic language

Textbook quantum theory is based on complex numbers of the form $a_0+a_1 i$, with i the imaginary unit $i^2 = -1$. It has long been known that an alternative quantum mechanics can be based on the quaternion or hyper-complex numbers of the form $a_0 + a_1 i + a_2 j + a_3 k$, with i,j,k three non-commuting imaginary units.[8]

On the other hand, recognizing that the Maxwell's equations were originally formulated in terms of quaternionic language, some authors investigate formal correspondence between Maxwell and Dirac equations. To name a few who worked on this problem: Kravchenko and Arbab. These authors have arrived to a similar conclusion, although with a different procedures based on Gersten decomposition of Dirac equation.[4]

This MD isomorphism can also be extended further to classical description of boson mass which was usually called Higgs boson[3], so it may be a simpler option compared to scale symmetry theory.

Quaternionic QM and Entanglement

Having convinced ourselves that Maxwell-Dirac isomorphism has sufficient reasoning to consider seriously in order to come up with realistic interpretation of quantum wave function, now let us discuss QQM and entanglement.

Singh & Prabakaran are motivated to examine the geometry of a two qubit quantum state using the formalism of the Hopf map. However, when addressing multiple qubit states, one needs to carefully consider the issue of quantum entanglement. The "quaternions" again come in handy in studying the two qubit state. [10]

In his exposition of Quaternionic Quantum Mechanics, J.P. Singh concluded that [9]:

> "Having established the compatibility of the Hopf fibration representation with the conventional theory for unentangled states, let us, now, address the issue of measurability of entanglement in this formalism. In the context, "Wootters' Concurrence" and the related "Entanglement of Formation" constitute well accepted measures of entanglement, particularly so, for pure states. …
>
> It follows that any real linear combination of the "magic basis" would result in a fully entangled state with unit concurrence. Conversely, any completely entangled state can be written as a linear combination in the "magic basis" with real components, up to an overall phase factor. In fact, these properties are not unique to a state description in the "magic basis" and hold in any other basis that is obtained from the "magic basis" by an orthogonal transformation…"

Singh & Prabakaran also suggest that this quaternionic QM may be useful for exploring quaternionic computing.[10]

In a rather different way, Najarbashi et al. explored quaternionic Möbius transformations, which can be useful in theoretical physics such as quaternionic quantum mechanics, quantum conformal field theory and quaternionic computations [11]. They found that “As in the case of two-qubits, both octonionic stereographic projection and Möbius transformation are entanglement sensitive.”

Concluding remarks

Despite its enormous practical success, many physicists and philosophers alike agree that the quantum theory is so full of contradictions and paradoxes which are difficult to solve consistently. Even after 90 years, the experts themselves still do not all agree what to make of it. In this paper, we review the most puzzling feature of QM, i.e. entanglement.

In the meantime, the problem of the formal connection between electrodynamics and wave mechanics has attracted the attention of a number of authors, especially there are some existing proofs on Maxwell-Dirac isomorphism. Here the author reviews two derivations of Maxwell-Dirac isomorphism i.e. by Hans Sallhofer and Volodimir Simulik and also quaternion language.

While this paper is not conclusive to answer the question: can Maxwell-Dirac isomorphism especially its quaternionic formulation offer a classical description of entanglement?, we have explored some recent discussions on this topic, e.g. from Hopf map and quaternionic Möbius transformations.

This paper was inspired by an old question: Is there a consistent and realistic description of wave function, both classically and quantum mechanically?

It can be expected that the above discussions will shed some lights on such an old problem especially in the context of physical meaning of quantum wave function. This is reserved for further investigations.

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