

Heisenberg Quantum Probabilities

God Does Not Throw Dice at the Planck Scale, but Below!

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Abstract

In this paper we suggest that working with the Planck mass and its link to other particles in a simple way, it possible to “convert” the Heisenberg uncertainty principle into a very simple quantum probabilistic model. We further combine this with key elements from special relativity theory and get an interesting quantum relativistic probability theory. Some of the key points presented here could help to eliminate negative and above unity (pseudo) probabilities that often are used in standard quantum mechanics. These fake probabilities may be rooted in a failure to understand the Heisenberg principle fully in relation to the Planck mass. When properly understood, the Heisenberg principle seems to give a probabilistic range of quantum probabilities that is sound. There are no instantaneous probabilities and the maximum probability is always unity. In our formulation, the Planck mass particle is always related to a probability of one. Thus, we have certainty at the Planck scale for the Planck mass particle, or for particles accelerated to reach Planck energy.

Key words: Heisenberg’s uncertainty principle, quantum probabilities, relativistic probabilities, from uncertainty to certainty.

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1 From Heisenberg's Uncertainty Principle to Quantum Probabilities

Heisenberg's uncertainty principle [1] is given by

$$\Delta p \Delta x \geq \hbar \quad (1)$$

assume now that p is the Planck momentum, $p = m_p c$. This is actually the only momentum that is directly linked to the speed c . For all non-Planck mass elementary particles, we have a momentum related to a velocity $v < c$. The velocity v is not certain and can take a large range of values, so the momentum cannot be certain for non-Planck mass particles. Inputting the Planck momentum in the equation for Heisenberg's uncertainty principle, we get

$$m_p c \Delta x \geq \hbar \quad (2)$$

Since the Planck momentum always is $m_p c$, we will claim that the inequality should be exchanged for an equality in this special case. If so, we must have $\Delta x = l_p$, where l_p is the Planck length first introduced by Max Planck in 1899 [2, 3]. In other words, we claim there is a certainty principle for the Planck mass

$$m_p c l_p = \hbar \quad (3)$$

This we can also write as

$$m_p = \frac{\hbar}{l_p} \frac{1}{c} \quad (4)$$

which is a known expression for the Planck mass. We also have

$$\begin{aligned} \Delta p c \Delta x &\geq \hbar c \\ \Delta p c \frac{\Delta x}{c} &\geq \hbar \\ \Delta E \frac{\Delta x}{c} &\geq \hbar \end{aligned} \quad (5)$$

Again, in the special case of the Planck mass we have a momentum of $p = m_p c$, so we will claim there is no uncertainty in the Planck momentum and therefore we have

$$m_p c^2 \frac{\Delta x}{c} = \hbar \quad (6)$$

Since $m_p = \frac{\hbar}{l_p} \frac{1}{c}$, the relationship above can only hold true if $\Delta x = l_p$, and $\frac{l_p}{c}$ is the Planck time. The fact that this is related to the Planck time is significant. However, this certainty in the Planck mass, the Planck momentum, and even the Planck mass particle only exists for one Planck second. In other words, we offer a hypothesis that there is a certainty principle that lasts for one Planck second, which is related to the Planck mass particle, the Planck momentum, and the Planck energy. We will see how this plays an important role in taking us from uncertainty in momentum and position in Heisenberg's uncertainty principle for all non-Planck masses towards a simple probabilistic quantum theory.

It is well-known that any subatomic mass can be written as

$$m = \frac{\hbar}{\bar{\lambda}} \frac{1}{c} \quad (7)$$

where $\bar{\lambda}$ is the reduced Compton wavelength of the particle in question. For example, for an electron we have

$$m_e = \frac{\hbar}{\lambda_e} \frac{1}{c} \approx 9.10938 \times 10^{-31} \text{ kg} \quad (8)$$

We also show that the mass of any particle can be described as a function of the Planck mass by the following formula

$$m = m_p \frac{l_p}{\bar{\lambda}} \quad (9)$$

this simple relationship was possibly first pointed out by Hoyle, Burbidge, and Narlikar in 1994; see [4]. Based on this, the electron mass can also be written as

$$m_e = \frac{\hbar}{l_p} \frac{1}{c} \frac{l_p}{\lambda_e} = m_p \frac{l_p}{\lambda_e} \quad (10)$$

$$\begin{aligned}
P &= \frac{l_p}{\bar{\lambda} \sqrt{1 - \frac{v_{max}^2}{c^2}}} \\
P &= \frac{l_p}{\bar{\lambda} \sqrt{1 - \frac{\left(c \sqrt{1 - \frac{l_p^2}{\lambda^2}}\right)^2}{c^2}}} \\
P &= \frac{l_p}{\bar{\lambda} \sqrt{1 - \frac{c^2 \left(1 - \frac{l_p^2}{\lambda^2}\right)}{c^2}}} \\
P &= \frac{l_p}{\bar{\lambda} \sqrt{1 - 1 + \frac{l_p^2}{\lambda^2}}} \\
P &= \frac{l_p \bar{\lambda}}{\lambda l_p} = 1
\end{aligned} \tag{19}$$

Thus Haug's maximum velocity very elegantly leads to a maximum quantum probability of one. This means we get a boundary condition on the quantum probability for each elementary particle for each Planck second of

$$\frac{l_p}{\lambda} \leq P \leq 1 \tag{20}$$

Still, the relativistic quantum probability range will be different for each elementary particle. The maximum relativistic mass a particle can take is directly linked to its maximum velocity and thereby to its maximum probability of one. The maximum relativistic mass for any particle is the Planck mass multiplied by the maximum relativistic probability, which is one, and not surprisingly we get

$$\text{Expected relativistic maximum mass electron} = m_p \frac{l_p}{\bar{\lambda}_e \sqrt{1 - \frac{v_{max}^2}{c^2}}} = m_p \times 1 = m_p \tag{21}$$

How should we interpret this? It means at its maximum velocity any subatomic particle becomes a Planck mass, when relying on quantum probabilities. This also means that the original Heisenberg uncertainty principle collapses and becomes the certainty principle at the Planck scale. In addition, the Lorentz symmetry is broken at the Planck scale.

The Planck mass particle is a particularly interesting case; its reduced Compton wavelength is $\bar{\lambda} = l_p$, which gives a probability range for the Planck mass particle of

$$\begin{aligned}
\frac{l_p}{l_p} &\leq P_p \leq 1 \\
1 &\leq P_p \leq 1
\end{aligned} \tag{22}$$

This can only be true if the Planck particle quantum probability always is $P_p = 1$. This naturally means there is no uncertainty for the Planck mass particle. One could criticize this approach and say this is not so strange, since we have defined the Planck momentum as being certain from the beginning. However, there is more to it and this shows that the theory is consistent. Further, as pointed out by Haug in several earlier papers, the maximum velocity for the Planck mass particle is also very unique; it is

$$v_{max} = c \sqrt{1 - \frac{l_p^2}{l_p^2}} = 0 \tag{23}$$

and this makes the relativistic quantum probability of the Planck mass particle consistent; it is

$$P_p = \frac{l_p}{l_p \sqrt{1 - \frac{v_{max}^2}{c^2}}} = \frac{l_p}{l_p \sqrt{1 - \frac{0^2}{c^2}}} = \frac{l_p}{l_p} = 1 \tag{24}$$

Our interpretation here is that the Planck mass particle is the very collision point of the light particles making up each elementary particle. This also explain why a Planck mass particle can have momentum of $m_p c$. Naturally, the Planck mass particle must follow relativistic rules, so we must have

$$p_p = \frac{mc}{\sqrt{1 - \frac{v^2}{c^2}}} = mc \tag{25}$$

which can only happen if $v = 0$. Our interpretation is that the Planck mass particle bursts into energy within one Planck second and the entire momentum is used internally to achieve that transformation. We also get a hint about the lifetime of a Planck particle from the Planck acceleration, $a_p = \frac{c^2}{l_p} \approx 5.56092 \times 10^{51} \text{ m/s}^2$. The Planck acceleration is assumed to be the maximum possible acceleration by several physicists; see [10, 11], for example. The velocity of a particle that undergoes Planck acceleration will actually reach the speed of light within one Planck second: $a_p t_p = \frac{c^2}{l_p} \frac{l_p}{c} = c$. However, we know that nothing with rest-mass can travel at the speed of light, so no “normal” particle can undergo Planck acceleration if the shortest possible acceleration time interval is the Planck second. The solution is simple. The Planck acceleration is an internal acceleration inside the Planck particle that within one Planck second turns the Planck mass particle into pure energy.

Further, the Planck mass particle is the same from every reference frame, across all reference frames. It clearly breaks Lorentz symmetry, something that several quantum gravity theories also predict [12], among them a newly-introduced quantum gravity theory derived from the Heisenberg principle that relies on the quantum probability approach presented here.

3 The Link to Heisenberg Quantum Gravity

Our approach to quantum probabilities explains why McCulloch [13] has been able to derive Newtonian gravity from Heisenberg’s uncertainty principle. Initially this seems impossible and inconsistent, as Newtonian gravity is intended for the gravity of objects at the macroscopic and cosmic scale and Heisenberg’s uncertainty principle was developed to understand uncertainty at the atomic and subatomic scale. The key is that McCulloch utilizes Planck masses in his theory. Even if we are dealing with protons, the sum of probabilities when we add up $N = \frac{\bar{\lambda}_p}{l_p}$ number of protons, then the uncertainty is zero, because the partial probabilities exactly add up to one. Assume, for simplicity’s sake, that cosmological objects were made of only protons; each proton has a quantum probability of $P_p = \frac{l_p}{\bar{\lambda}_p}$ and an expected mass, as observed in one Planck second, of

$$E[m] = m_p \frac{l_p}{\bar{\lambda}_p} \approx 1.673 \times 10^{-27} \text{ kg} \quad (26)$$

which is the well-known proton mass. However, this is an expiation that relies on a very low probability factor. In other words, the uncertainty is large at the Planck time scale, and is in line with what the Heisenberg uncertainty principle tells us. However, if we now add up a large number of protons, we can also add up the probabilities. Assume we add up $N = \frac{\bar{\lambda}_p}{l_p}$ protons, this give a probability of

$$P = \sum_{i=1}^N P_p = \sum_{i=1}^N \frac{l_p}{\bar{\lambda}_p} = \frac{\bar{\lambda}_p}{l_p} \frac{l_p}{\bar{\lambda}_p} = 1 \quad (27)$$

It is no coincidence that $N = \frac{\bar{\lambda}_p}{l_p}$ protons also gives us the Planck mass

$$m = \sum_{i=1}^N E[m] = \sum_{i=1}^N m_p \frac{l_p}{\bar{\lambda}_p} = m_p \quad (28)$$

Whenever we add up $N = \frac{\bar{\lambda}_p}{l_p}$ subatomic particle of the same type, that each have a reduced Compton wavelength of $\bar{\lambda}$, we end up with the Planck mass and a probability of one. A single subatomic particle can have a quantum probability of one when reaching its maximum velocity, or we can get a probability of one by working with amount of matter exactly divisible by the Planck mass.

McCulloch bases his gravity derivation from Heisenberg’s uncertainty principle on masses that are only divisible by whole Planck masses, and thereby he is working in the very limit of the Heisenberg principle, where it switches to certainty because the probabilities add up to one; see also [14]. When we deal with a mass that not is fully divisible by a Planck mass, we end up with a deterministic part and a probabilistic part. In actuality, even the first part is derived as being probabilistic, but since all of the partial probabilities add up to one, it switches to a deterministic world. Haug [15] has recently extended McCulloch’s Heisenberg gravitation derivation to hold for masses smaller than the Planck mass and are not divisible by the Planck mass. This could be the quantum gravity theory that we have been seeking for ages, where the quantum world and the macroscopic world are united in one model.

4 Probabilistic Summary

In this short section we will summarize some of our key findings in tables. Table 1 shows the range for the relativistic quantum probability for an electron, a proton, and a Planck mass particle.

| | Probability range | Probability range |
|-------------------------------------|---|---|
| Electron quantum probability | $\frac{l_p}{\lambda_e} \leq P_e \leq 1$ | $4.18532 \times 10^{-23} \leq P_e \leq 1$ |
| Proton quantum probability | $\frac{l_p}{\lambda_P} \leq P_P \leq 1$ | $7.6849 \times 10^{-20} \leq P_P \leq 1$ |
| Planck particle quantum probability | $\frac{l_p}{l_p} \leq P_p \leq 1$ | $P_p = 1$ |

Table 1: The table show the range for the relativistic quantum probability for an electron, a proton, and a Planck mass particle.

Table 2 shows the standard relativistic mass as well as the probabilistic approach; they are consistent. Be aware that there must be a maximum velocity limit on anything with mass; this will be equal to Haug's maximum velocity.

| | Standard approach | Probabilistic approach observation in one Planck second |
|----------------------|---|--|
| Electron mass | $m = \frac{m_e}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{h}{\lambda_e} \frac{1}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$ | $E[m] = m_p P_e = m_p \frac{l_p}{\lambda_e \sqrt{1 - \frac{v^2}{c^2}}} \geq m_e$ |
| Proton mass | $m = \frac{m_P}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\frac{h}{\lambda_P} \frac{1}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$ | $E[m] = m_p P_P = m_p \frac{l_p}{\lambda_P \sqrt{1 - \frac{v^2}{c^2}}} \geq m_P$ |
| Planck mass particle | $m_p = \frac{m_p}{\sqrt{1 - \frac{0^2}{c^2}}} = m_p$ | $E[m] = m_p P_p = m_p \frac{l_p}{l_p \sqrt{1 - \frac{0^2}{c^2}}} = m_p$ |

Table 2: This table shows the standard relativistic mass as well as the probabilistic approach. Be aware of the notation difference between the Planck mass m_p and the proton rest mass m_P .

Table 3 shows the relativistic mass when a particle is traveling at its maximum velocity. This will always correspond to a relativistic mass equal to the Planck mass, and a quantum probability of one. Be aware that the particle when reaching this velocity, which is above what can be achieved at LHC, likely will burst into energy within one Planck second. So the certainty we predict can only last for one Planck second when we are dealing with single particles.

| | Standard approach | Probabilistic approach observation in one Planck second |
|----------------------|--|--|
| Electron mass | $m = \frac{m_e}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} = m_p$ | $E[m] = m_p P_e = m_p \frac{l_p}{\lambda_e \sqrt{1 - \frac{v_{max}^2}{c^2}}} = m_p \times 1 = m_p$ |
| Proton mass | $m = \frac{m_P}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} = m_p$ | $E[m] = m_p P_P = m_p \frac{l_p}{\lambda_P \sqrt{1 - \frac{v_{max}^2}{c^2}}} = m_p \times 1 = m_p$ |
| Planck mass particle | $m_p = \frac{m_p}{\sqrt{1 - \frac{v_{max}^2}{c^2}}} = m_p$ | $E[m] = m_p P_p = m_p \frac{l_p}{l_p \sqrt{1 - \frac{v_{max}^2}{c^2}}} = m_p \times 1 = m_p$ |

Table 3: This table shows the standard relativistic mass as well as the probabilistic approach at the maximum velocity only.

5 How Not-Well-Specified Models Can Lead to Fake Probabilities

From the derivations above we can also gain insight into what types of model specifications can give fake probabilities. With fake probabilities, we are thinking of probabilities that are not allowed according to this set-up, and also probabilities that are not even allowed in standard probability theory, such as probabilities above unity and negative probabilities. We have the following relativistic probability

$$P = \frac{l_p}{\lambda \sqrt{1 - \frac{v^2}{c^2}}} \quad (29)$$

Only the reduced Compton wavelength and v are variables in the sense that they can take different values, so whatever values they do take in practice (and good theory) are directly linked to what are allowed probabilistically. Going outside the physical limits would mean allowing for fake probabilities. Models allowing

$v > c\sqrt{1 - \frac{l_p^2}{\lambda^2}}$ will lead to fake probabilities above unity. In addition, formulations that allow a reduced Compton wavelength shorter than the Planck length will lead to above-unity probabilities. A shorter reduced Compton wavelength than the Planck length can be introduced indirectly in the model by allowing particles with mass higher than the Planck mass, or time intervals shorter than the Planck length.

This indicates that the Heisenberg uncertainty principle is limited to maximum Planck masses. This is the upper limit where the uncertainty principle becomes the certainty principle. Further, it is not applicable for time intervals shorter than the Planck time, or for velocities higher than Haug's maximum velocity. As standard physics normally does not take these limits into account, we have a quantum theory that often relies on fake probabilities above unity that then must be fixed by introducing negative probabilities and such things as renormalization, that we will discuss briefly in the next section.

Table 4 summarizes the criteria that will likely lead to fake probabilities in the quantum world. All of these instances are also connected. For example, a relativistic particle energy larger than the Planck energy means that the velocity rule is broken. This also means the reduced Compton wavelength in the particle has undergone a length contraction, so it is shorter than the Planck length. Thus, by breaking one of these rules, one has automatically broken all of them. We can naturally have a total mass larger than the Planck energy in a collection of many particles, but then deterministic effects have taken over and we are no longer ruled by quantum probabilities. However, it is more complex than this when dealing with large amounts of particles simultaneously; this is something we will get back to in a later version of this paper or in another paper.

| Unit | Model allowing | Leads to |
|--------------------------------------|---|--------------------|
| Reduced Compton wavelength : | $\lambda < l_p$ | Fake probabilities |
| Particle mass : | $m > m_p$ | Fake probabilities |
| Particle momentum : | $m > m_p c$ | Fake probabilities |
| Particle energy : | $E > E_p$ | Fake probabilities |
| Particle force : | $F > F_p$ | Fake probabilities |
| Observational time interval : | $t < t_p$ | Fake probabilities |
| Particle (with rest-mass) velocity : | $v > c\sqrt{1 - \frac{l_p^2}{\lambda^2}}$ | Fake probabilities |

Table 4: This table shows in what type of theoretical particle situations that will lead to fake probabilities (that is in this case probabilities above unity).

6 Something Is Rotten in Standard Probabilistic Quantum Mechanics

We will claim there is something rotten in modern quantum mechanics, despite its extreme success in predicting what we observe in experiments. The Wigner [16] quasi-probability distribution, which is well-known, can take on negative values and values above unity. Even if these negative probabilities are never observed in practice, they play a central role in quantum mechanics to get the math to fit observations. Renormalization is another ad-hoc method that is closely connected to negative probabilities and has been used repeatedly for years; over time it has become an acceptable procedure that few physicists may question these days.

However, one prominent critic of renormalization was Richard Feynman [18]. Clearly, he had a central role in the development of quantum electrodynamics, and yet he claimed

The shell game that we play ... is technically called 'renormalization'. But no matter how clever the word, it is still what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate. – Richard Feynman, 1985

In 1987, Feynman [17] again commented on renormalization

Some twenty years ago one problem we theoretical physicists had was that if we combined the principles of quantum mechanics and those of relativity plus certain tacit assumptions, we seemed only able to produce theories (the quantum field theories), which gave infinity for the answer to certain questions. These infinities are kept in abeyance (and now possibly eliminated altogether) by the awkward process of renormalization. – Richard Feynman, 1987

This statement is found in an article where Feynman then looks into the possibility of using negative probabilities to solve the renormalization problem in another way. We claim that both methods are simply bad fudges

and even if they can be used to give the correct output of the model, they strongly indicate that the model is incomplete and ill-specified.

In 1956, Wolfgang Pauli pointed out directly how renormalization can lead to negative probabilities – something he also called ghost probabilities, [19]. Also, Dirac [20] had interesting discussions concerning how negative probabilities show up in quantum mechanics. Negative probabilities have also been evaluated in other fields such, including quantitative finance [21], where negative probabilities always seems to indicate that the model is incomplete. In finance, we basically know what is going on as we can observe, for example, the stock price. In quantitative finance, when negative probabilities show up in the model, it is an incomplete model that can have a state space that does not fully capture the reality. In quantum physics, it is naturally much harder to know what is going on in the subatomic world, and one can mistakenly assume that the subatomic reality is well-represented by such negative and above-unity probabilities. However, we disagree and again, we claim that if negative probabilities and above unity probabilities are needed in the model, then it is a strong indication that model is incomplete and mis-specified relative to what it is trying to describe.

We strongly suspect that the negative and above-unity probabilities, as well as renormalization in standard quantum mechanics, could be related to the fact that modern physics does not have an exact maximum velocity limit on anything with rest-mass. There is only the limit $v < c$, which means that one can get basically as close to infinite kinetic energies and infinite relativistic masses as one wants. No matter how close the speed v is to the speed of light, one can always get a little closer. If the probabilistic approach presented here is correct, this means that standard quantum mechanics likely has embedded within it quantum probabilities above unity – something that is absurd. The flawed ad-hoc adjustment for this is to add a probability function that allows negative probabilities to get the sum of probabilities in order now to not break other basic probabilistic rules. Two fake probabilities: negative and above-unity may give a correct prediction, but they can also lead to spooky interpretations. Further, the assumption of instantaneous probabilities could also lead to fake probabilities. If the shortest possible time interval is one Planck second, then the ultimate quantum probabilities should be linked up to that observational time window as we have done here. The Planck length and the Planck second are closely connected, and so are the probabilistic interpretations presented in this paper.

7 Conclusion

In this paper we have suggested that $\frac{l_p}{\lambda}$ can be interpreted as quantum probability related to the rest-mass of each particle. Further, we claim the relativistic version of this quantum probability simply is $\frac{l_p}{\lambda\sqrt{1-\frac{v^2}{c^2}}}$. This can

only hold true if elementary particles are limited by Haug’s suggested maximum velocity for elementary particles.

This speed limit is $v_{max} = c\sqrt{1 - \frac{l_p^2}{\lambda^2}}$, which actually corresponds to a maximum velocity that Haug has recently derived from the Heisenberg principle when simply assuming that the Planck length is the shortest length one can measure. Standard physics does not have such a speed limit, but simply maintains that $v < c$, and this may be a reason for the reliance on fake probabilities such as negative and above-unity quasi-probabilities. By studying this paper and other recent and related papers by Haug, it is possible that we may be able to solve some of the big challenges in quantum mechanics in a new and more elegant way, which also reduces the amount of spooky interpretations that exist in today’s quantum mechanics.

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