A new approach to Quantum Mechanics I : Overview

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Abstract

In this article, a new topological way to define first quantization procedure is overviewed. Technical ingredients and metaphysical ideas used throughout this series of works are introduced in a way as non-technical as possible for motivating both physically and mathematically interested readers. This overview contains schematic summary of part II and part III of this series.

I. Overview

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1 Introduction

This section is written colloquially for mathematically interested readers who are not familiar with physics.

1.1 Problems of first quantization

First quantization is a peculiar mathematical procedure. It has been established via many different sources of mathematics. There are quantization methods called canonical quantization, geometric quantization, deformation quantization, path integral quantization and so on. They are built upon linear algebra, functional analysis, operator algebra, measure theory, probability theory and so forth. These procedures, each depend on different background of mathematics, send the information of physical observables into Hilbert space.

However, these ad hoc procedures are not mathematically satisfying. Simply saying, the procedures are not functors, so that one can hardly expect rigorous results by studying the map. Algebraic, geometric, analytic structure of defining physical inputs are not kept by sending it into Hilbert space. In many practical cases, the existence of the theory itself is even questionable. Some mathematicians think it is black magic, when physicists show some results by studying these non-rigorously defined quantization methods. The technical crack has not been fully closed since the theory was born. This is a problem of first quantization from mathematical side.

Physical problems are deeper, even connecting to metaphysical issues. In classical mechanics, physical observables such as energy or angular momentum are described as functions or vectors over some parameters. So, in a sense, classical mechanics provides one a definite answer for any given physical questions, whenever one has the information of parameters that are needed to calculate. Hamiltonian mechanics or Lagrangian mechanics are examples of such classical theory that equivalently represent physical systems in different manners. This theoretical framework works well until one confronts problems in a very small scale.

It turns out in an atomic scale or even smaller scale, particles behave strangely. For example, some physical observables showed only in some integer times certain constants. And even in the same experimental setup, the result shows up differently. Classical methods were not efficient to contain this new phenomena of small scale physics. In the end, quantum mechanics solves these issues dramatically. Instead of considering functions over parameter space, for example phase space or configuration space, considering operators on state vectors over Hilbert space replaces classical arguments. Unlike classical mechanics, what one calculates via quantum mechanics is not a deterministic result but only a probabilistic result.

This invokes philosophical questions. Since the act of observation collapses the particle's probability amplitude, the observer cannot be independent of the physical system. It is a big difference from classical case, in which physical system is independent of observers. In classical mechanics, what is measured and which order it is done are not affecting the observables. On the other hand, in quantum mechcanics, the observer's choice can change the amplitude of observables. Technically saying, some operators in quantum mechanics are not commute to each other. This philosophical issue makes many physicists to think that there should exist something more than just technical difference between classical and quantum mechanics. Still there is no clear answer to this question.

One of big difficulties to study first quantization procedure is from the mathematical properties of Hilbert space. Unlike parameter space such as phase space, Hilbert space is an infinite dimensional inner product space. While phase space or configuration space gives physical intuition that is closely connected to spacetime, Hilbert space only provides calculational framework. Moreover, the geometric feature of Hilbert space is not something tangible to analyze. One can hardly benifit from the current developments of mathematical tools to study Hilbert space. In other words, powerful tools especially from algebraic geometry or algebraic topology are not effective to study it, not to mention differential geometry.

This series of papers suggest an approach on these problems. By inscribing quantum mechanics on algebraically even arithmetically interesting topological objects, both regaining physical intuition and regaining mathematical adaptability are pursued. New approach is from both technical and metaphysical directions. Not only unconventional mathematics is used to represent the idea, but also some basic notions of physics is reconsidered.

1.2 Layout of this series of work

'A new approach to quantum mechanics' is written in five parts. I is about overview of whole ideas, II,III are main technical contents and IV,V are philosophical and technical appendix. This series of work is not for providing full coverage of the study of quantum physics via p-adic geometry, but for initiating the study. It is separated so, in order to satisfy as much as possible both physicists who are not technically familiar with the language used throughout the series, and mathematicians who have no motivational origins from physics side. For this reason, this series contains two different technical contents.

A new approach takes both new physical intuition and new mathematical methods. Part II of this series is focused on conveying new physical intuition. While part III is more focused on applying this idea to mathematical question, which is about relating mathematics of first quantization to geometric and local Langlands connection. Since the motivation is different, both papers have different tones. Part II is written practically for physical use, containing physical toy models constructed by using new approach. While part III is written for pursuing mathematical connection between first quantizations and different ways of realizing p-adic geometry. Assuming the legitimacy of new qunatization, the application to mathematical problem is sought after by studying some special p-adic phase space.

In order to fulfill two different motivations, mathematical techniques used is differed. In the first part, theory of Berkovich analytic space is used and in the second part, theory of adic analytic space is used. In a sense, both techniques are similiar. They are devised to study geometrical nature of spaces over non-Archimedean fields, filling the gap which one confronts when studying spaces over non-Archimedean fields by naive algebraic geometric methods. Two theories have deep ramifications, so at some branchs they bring the same results. However, it is decided to use analytic nature of Berkovich space to consider the analytic aspect of newly developed quantum state space, and use topological aspect of adic space to consider the algebraic side of quantum space.

Both techniques have its own advantages and disadvantages. Berkovich space characterize naturally the analytic property of the space. It is locally Hausdorf and locally compact. For physical applications, the space can provide more effective background to physicst, since it has familiar local geometric nature which is reminiscent of complex analytic space. It is not hard to see Berkovich space supports functions and vector bundles as physically interesting space normally does. Even though it has this nice local analytic topological properties, in order to squeeze the arithmetic information from Berkovich analytic space one needs to apply more technical approachs including Grothendieck topology.

On the other hand, adic space charaterize more naturally the algebraic and arithmetic property in its topology. In algebraic sense, it contains schemes and formal schemes as subcategories. In topological sense, it's topology is defined in a more algebraic geometrical way.[1] So it provides a convenient tool to study arithmetic questions, connecting to known results of algebraic geometry. However, it takes more effort to grasp the analytic property of the space, because it is totally disconnected, in general. Even showing some adic spaces support line bundle is not trivial problem.

One notable recent progress in arithmetic topology via adic space techniques is theory of perfectoid space. It provides topological bridges between two different spaces over different non-archimedean fields. By studying this arithmetic topological bridge, there are some applications on questions of arithmetic geometry. At first sight, it seems hard to find physical implications in studying peculiar topological structure of perfectoid spaces. But its locally symmetrical structure can be used to imbue the symmetries appearing in quantum physics problems. Even for mathematical applications, the study of perfectoid quantum parameter space may provide hints to arithmetic problems.

In summary, Part II builds a quantum state space by using Lagrangian method over 1 dimensional Berkovich analytic configuration space. For motivational and technical reasons, it is restricted only to 1 dimensional toy model at this stage. Part III builds a theory analogous to Hamiltonian mechanics via perfectoid phase space. Some new definitions and interpretations are made to dissolve physical ideas into the space. Also, it is built over 1-dimensional special space called Fargues-Fontaine curve. In order to focus on constructing new first quantization model defined over p-adic geometry, other physically important conditions, such as Lorentz transformations and internal symmetries, are ignored at this step.[2]

2 Motivations

In this section, various motivations are stated in a nutshell. Rather than specifying small number of motivations and zooming in, it is intended to expose wide view. The importance of each motivations may be weighed differently for either part II or part III. Those who want close look can check the references at the end of subsection. (Note: references introduced in this section covers only a small portion.)

2.1 Motivations from physics side

Some of physical motivations are introduced in short. For those who already know the subjects, the following may remind them of the contents. For those who are not familar

[1] The topology of Berkovich space is defined in a more analytic way involving net and quasi-net.

[2] Actually, easiest way of considering those symmetries are via limiting process, limit that generates physics over $\mathbb{R}$. However, there may exist other interesting metaphysical realizations. Some of these issues will be introduced in part IV, 'DO philosophy for physics over p-adic geometry'.

with the subjects, this section only provides a little bit of taste. It should be noted there underlies deeper and wider web of stories.

2.1.1 Quantum mechanics and number theory

In 1960's, a tantalizing connection between distributions of nontrivial zeros of Riemann zeta functions and distributions of eigenvalues of random matrix was found. Riemann zeta function is closely related to prime numbers. Many number theoretic questions have connections to Riemann zeta functions. Random matrix theory was originally developed for studying energy eigenstates of nucleus. It was suggested to deal with the complexity of quantum behavior of nucleus. Nowadays, random matrix theory is a strong method to study arithmetic problems.

However, the physical motivation of why the atom's energy eigenstate has something to do with the places of prime numbers is still unknown. L-functions such as Riemann zeta functions have it's origin from arithmetic geometry. Physical spaces usually appearing in classical mechanics has no arithmetic nature. It is locally $\mathbb{R}^n$ or $\mathbb{C}^n$, so there is no room for arithmetically interesting thing happening. Quantum state space or operator space also have not been defined to be arithmetic space. One motivation from this argument is to generate quantum mechanics by anylyzing physical parameter space as certain arithmetically interesting geometric space.

If it is really the case that number theoretic knowledge is connected to certain quantum mechanical wisdom, then one can consider variations of this bond. Other L-functions with arithmetic origin can be studied for finding symmetrical properties of certain quantum state. By yielding arithmetic structure from quantum state space, this would be a possible mission.

References: [Dys62]

2.1.2 Holographic principle, AdS/CFT and entangled geometry

Holographic principle states that there exists a duality between physical theory A in the bulk and physical theory B in the boundary. It originally stems from calculations of blackhole entropy. Unexpectedly, blackhole entropy turns out to be proportional to the blackhole's surface area not to the volume. One systematic idea to perform holography is AdS/CFT correspondence. It is a string theoretic duality between supergravity and strongly coupled gauge theory. Heuristrically, it connects calculations of correlation functions of semi-classical quantum supergravity theory at the bulk of AdS space and highly quantum large N limit of SU(N) supersymmetric conformal gauge field theory at the boundary.

AdS/CFT correspondence has affected a vast area in physics. Later, calculations of quantum entaglement entropy on the part of the boundary was shown to be equal to the minimal surface area in the bulk for 1 and 2 dimensions. More specifically, on the one hand side, there is a field theoretic calculations of von Neumann entropy of quantum entanglement. On the other hand side, there is the minimal area enveloping the part in the bulk of AdS space, which can be holographically interpreted as gravitational entropy named as Bekenstein-Hawking entropy. von Neumann entropy measures how quantum particles are entangled to each other. Bekenstein-Hawking entropy is the entropy inside the bulk caused by gravity, which is proportional to the surface area. In short, two types of entropy calculations, one is originated from quantum physics while the other is from gravity, are shown to be the same for simple cases.

Since spacetime is closely related to gravity, there are attempts to consider spacetime as emergent concept arisen from quantum entaglement. And it is called entangled geometry or emergent spacetime, or even quantum geometry.

Introducing a new mathematical framework for entangled geometry can be thought to be the backbone of this series of papers, for physically interested readers. It is done by extending locally $\mathbb{R}^n$ tissue of spacetime into more sophisticated fabric with tree structure. As one toy model example, AdS/CFT type duality can be translated to quantum mechanics/gravity duality. Non-Archimedean geometry such as Berkovich space is used to represent this extended spacetime and extended parameter space.

This kind of approach was studied by [HMSS16], [GP17, GKP$^+$17, Gub17], considering quantum field theory over non-Archimedean field. Their study already shows effectiveness of non-Archimedean geometrical approach to AdS/CFT philosophy. However it is more about 'field theory' approach rather than 'quantum mechanical'. In other words, calculation is mostly restricted to semi-classical limit of quantum field theory and utilizing field theoretic properties, leaving behind tantalizing first quantization issue as is normally done in other literatures.

In this series of papers, the main objective is to redefine first quantization procedure, instead of just base changing quantum field theory to p-adic fields. The mathematics involved to analyze p-adic geometry is more technical than what is used in [HMSS16], [GP17, GKP$^+$17, Gub17], in order to pursue mathematical application which is connected to local-global Langlands bridge. In short, it is not only for showing the effectiveness of certain p-adic geometric aspect to contain the idea of AdS/CFT, but also for adding new metaphysical interpretation into the story to have geometrical theory of first quantization. This new approach may pave a road to a geometrical theory of quantum gravity.

References: [Sus95], [Hoo93], [Mal99], [RT06b, RT06a, NRT09, RT17, MTW17], [HMSS16], [GP17, GKP$^+$17, Gub17]

2.1.3 *Entropic force, information theory and gravity*

Einstein's theory of gravity is a classical theory. It is deterministic and geometrical. It's geometrical motivation is beautiful, because physical observables are seamlessly fit to the data of Riemannian geometry. There is no interpretational issues occuring. Einstein's equation states energy-momentum tensor on the one hand side is equal to the curvature tensor on the other hand side. Even though analyzing the equation deeper brings various cumbersomeness, the theory is remarkably simple and clean in a philosophical sense. However, when one considers quantum scale, the seamlessness of the theory results critical problems. It turns out that having a legitimate theory of quantum gravity is hard task.

Gravity itself is a strange force. Compared to other forces in nature, it is so weak. However, due to it's universalness, it is most important force in large scale. Though it is written beautifully by Einstein's theory, still there is large room to fill in. Mysterious sources called dark matter and dark energy are believed to consist of most part of the universe. Still there is no clear resolution for the origin of dark matter and dark energy. Gravitational force is underlying, because it governs the idea of spacetime. On the other hand, it is incomplete, because it does not match with quantum physics.

There have been studies seeing that gravity is not fundamental. In a classical mechanical sense, gravity was found to have similarities with hydrodynamics. In quatum mechanical direction, thermodynamics was found to be useful to study the property of black holes. Black hole is a singular solution of Einstein's gravity. Black hole has intimate relation to quantum physics, because it makes one to encounter very small scale physics. There are results that quatum physics affects theory of black holes. For example, considering quantum effect near event horizon, Hawking suggested that black hole radiates. Because of lack of power to handle quantumness in general relativity, thermodynamical behavior of black hole became a tool to analyze the physics in a singularity. In a similar vein, information theory is used to study the quantum behavior of black holes.

This line of thought brings up the question whether gravity is really a rudimentary force of the nature. There are attempts to consider gravity as an emergent force by considering thermodynamics or information theory as underlying fundamental principles. One idea in this direction is via entropic force. Entropy is a term used in thermodynamics and information theory. Roughly, it records the degree of disorder or uncertainty. In thermodynamics, entropy determines the flow of time. By second law of thermodynamics the sum of entropy of a system always increases. By using this argument, Erik Verlinde suggested that gravity is originated by entropic force as he calls it. According to this theory, as gravity emerges by thermodynamical principles, the notion of spacetime also should get emerged. The term emergent spacetime is coined in this sense.

However, losing control on the concept of spacetime takes too much sacrifices. Many fundamental physical wisdoms are built upon field theories. Field theory needs a canvas to draw on. The canvas should have specific properties such as smoothness and certain local symmetries. 20th century witnessed great change on the property of this canvas. For example by special relativity, the canvas becomes spacetime with Poincaré symmetry from space and time with Euclidean symmetry. And by general relativity, it became a curved spacetime written in Riemannian geometry from Euclidean geometry. Maybe it is another turning point to see the current canvas to evolve. The canvas needs to incorporate the murkiness steming from emergent spacetime. This motivates the developement of extended spacetime via p-adic geometry. Moreover, by following entropic force argument and the conjectural pFT/gravity duality[3], it motivates one to consider that quantum mechanics also might be understood from entropic argument.

References: [Bek73], [PYHP15], [Eis13], [Ver11, Ver17]

2.1.4 Hilbert space vs. p-adic geometry

Hilbert space is a main mathematical object for studying quantum mechanics. It provides a calculational background for quantum mechanical system of a single particle. Hilbert space is an infinite dimensional inner product space which is frequently represented by linear algebraic methods or functional analytic methods. The problem is that it is too big. The definition is so general that almost no particular structure can be specified in this infinite dimensional space. Working with narrower definition would help the situation, but there has not been good reasons and motivations to do that.

There are two motivations to substitute the study of p-adic geometry for the study of Hilbert space. First, it lacks physical intuitions. In quantum mechanics, physical observables are defined as eigenvalues of operators. Even position and momentum are calculated by operators acting on a state vector. Classically, position and momentum can be illustrated as vector spaces. For example, a simple phase space can be represented

[3] pFT for p-adic field theory.

as cotangent space over position vector space. In a sense, basic parameters are already there to exist in classical mechanics. Intuitionally, this seems to be natural. Since classical particle is supposed to be there, if it is known to be positioned there. So fabric of space is definite concept in classical mechanics. Which means that a randomly chosen position should already exist, no matter what the observer try to do on that parameter.

However, in quantum mechanics, particle's observables are not definitely determined. For example, a particle can be present on different positions coherently. The observable is determined only when the observation is made. Current notion of Hilbert space only provides a calculational framework. It helps to calculate how often some observables can be detected. Unlike classical phase space, it does not give any clues of what this infinite dimensional space has to do with the universe. A problem of using classical mechanics to describe quantum particle is the lack of the ability to represent the ubiquity of quantum particles. Newly defined parameter space is motivated to include the uncertainy of observables. By changing the parameter space into extended parameter space with p-adic geometric structure, a new intuition for a particle on the space will be gained.

Second, it would be nice to study quantum mechanics by algebraic or arithmetic tools. As first part of this subsection showed, it has long been anticipated that there exist some connections between quantum physics and number theory. There have been many attempts to connect the two fields. But most of them take detour. Because it is hard to find algebraic notions directly from the Hilbert space defined by general $L^2(\mathbb{R}^3)$. It is motivated by this situation to replace state vector over Hilbert space with certain fucntions or sheaves over extended parameter spaces, which will be defined as topological representation of certain p-adic geometric phase space.

In part II, certain set of harmonic functions over Berkovich projective line will be considered as the representation of quantum state of a particle. Then the wave function realization of the particle's positional observable is rederived from it. The main difference is that there will be no 'quantization procedure', which is about sending the information of a particle into Hilbert space. The quantum particle's characteristics, even it's probabilistic behavior, will be encapsulated in the topological analysis of the harmonic functions over configuration space, which is defined by Berkovich projective line.

In part III, homological algebraic study of newly defined quantum phase space is followed. As the above arguments suggest the new approach is about making first quantization as geometric and topological analysis of a given p-adic phase space or configuration space. Simply saying, current quantization procedure is about sending the information from symplectic manifold, which is locally trivial $\mathbb{R}^{2n}$, to Hilbert space. Reducing this non-rigorous act, new quantization strategy is to read quantum mechanical observables directly from a p-adic quantum phase space. This means that the p-adic quantum phase space is locally very non-trivial object such as moduli stacks of vector bundles. Mathematical legitimacy of this construction will be proposed by studying derived category of certain p-adic phase space compared to that of deformation quantization of Posisson manifolds. In other words, homological algebraic equivalence between new p-adic topological quantization and deformation quantization is suggested.

Also mathematical application of new categorical unification of quantizations is pursued. First, comparisons between the symmetry of physical notions of the extended parameter space and the topological symmetry inherent in the p-adic geometrical structure are enunciated. Then by assuming the equivalence of two quantization methods over different base fields, one via complexification of phase space and the other via perfectoidification of p-adic phase space, the connection between local Langlands and geometric Langlands is contemplated. It is suggested how to put quantum cohomology, equivalently Floer homology, appearing in complexified side into the shcolze's universal picture of p-adic cohomololgy. Quantum cohomology of 1-d complexified symplectic manifold are compared to étale cohomology and crystalline cohomology of Fargues-Fontaine curve.

References: [BR], [Sch17a, Sch17b]

2.1.5 Large extra dimensional model and hierarchy problem of gravity

Gravity is extremely weak compared to other forces in the nature. This is called hierarchy problem. One idea to resolve the unbalance between gravity and other forces is large extra dimensional model. Roughly, the 4 dimensional brane is considered as a boundary of 5 dimensional brane. The reason why gravity gets so weak on the 4D spacetime is explained as gravity is universal over the entire 5D space. As a result, it gets dilute over the boundary while other forces only pervade on the 4D spacetime.

The relationship between this model with the extended parameter space may open a new mathematical approach to a theory of quantum gravity. It seems there is an analogy between the extra dimension and the newly inserted points of extended parameter space. Throughout this series of papers, the extended parameter space will be classified into two subspaces. One is 'Idea space' which is composed of points other than classical points, while the other is 'Reality space' which is composed of classical points. By doing this, 'Idea space' can do somewhat similar role as the extra dimension does in the argument of large extra dimensional model.[4]

References: [RS99]

2.2 Motivations from mathematics side

This subsection assumes some familiarity with mathematics. It is written less colloquially. Mathematical notions used throughout this subsection are not defined rigorously. It is intended to show some future routes from physics to mathematics, and vice versa. The focus is to show, schematically, how quantum physics and the mathematics of p-adic geometry interact each other. Part III of this series of work is devised to pursue this direction.

2.2.1 SUSY Yang-Mills, geometric Langlands and local Langlands

In 1970's, Montonen and Olive found a peculiar duality between two different gauge field theories. At first, it was found as electric-magnetic duality, then later, conjectured to more general cases. The duality has exact solutions in 4 dimensional N=4 super Yang-Mills theory. In this case, one gauge theory with gauge group G and coupling constant g is equivalent to another theory with gauge group ${}^{L}G$ and coupling $1/g$.[5] Physically, this duality is usuful for studying strong coupling constant problems. By duality, one can save perterbative methods. Mathematically, interestingly Langlands dual group shows up. Langlands dual group essentially shows up in the study of representation theory of Galois groups. This adds another suspicion about the connection between number theory

[4] In other words, points of Berkovich space other than type I can be thought of as the extra dimensions.

[5] For simply laced Lie group case.

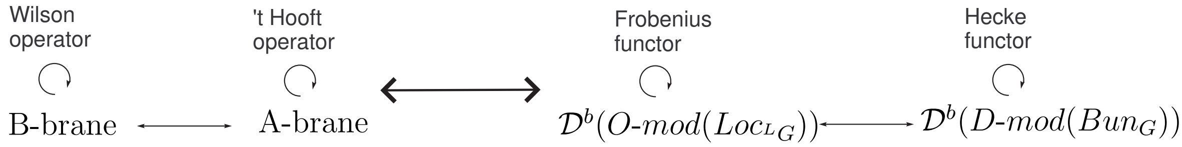

Fig. 1: A schematic depiction of correspondence between S-duality and geometric Langlands suggested by [KW06].

and quantum physics. However, it took some time to find tangible connections to this direction.

In 2000's, Kapustin and Witten suggested a connection between S-duality and geometric Langlands correspondence. S-duality is string theoretic duality. Via correspondence between certain SUSY gauge theories and certain super string theories, MO duality can be thought of as one of S-duality. Roughly, S-daulity can be defined by two different sigma models. One of sigma model is called A-model, while the other is called B-model. Both sigma models have different target spaces. The equivalence of theories via these two different sigma models is called S-duality. For example, mirror symmetry is a S-duality between A-model whose target space has certain symplectic structure and B-model whose target space has certain complex structure. In order to show mathematical connection, Kapustin-Witten defined categories called A-Brane and B-brane. Each branes contain the theory of sigma models with specific boundary conditions over target spaces. In the end, S-duality can be characterized as equivalence of categories between A-brane and B-brane. Two different gauge theories are defined as functors on this categories. (See left hand side of fig.1)

Geometric Langlands is conjectural categorical equivalence between two derived categories of dual moduli spaces. On the one hand side, there is quasi-coherent sheaf over moduli stack of local system of ^LG over a curve and on the other hand side, there is perverse sheaf over moduli stack of G-bundles over a curve. Following the study of relationship of D-modules to perverse sheaves, the correspondence can be concisely depicted as a diagram on the right hand side of Fig.2.

Even though geometric Langlands correspondence is a geometric analogue[6] of original Langlands program, there are some gaps between the two. Geometric Langlands is built on a more general categorical language while classical Langlands can be succinctly written by equivalence of L-functions. Moreover, generically Langlands program is studied for arithmetic questions. A representation of Galois group is to be compared to dual automorphic representations, and vice versa. Galois representations is calculated by étale cohomology. In geometric Langlands, this arithmetic side is replaced by fundamental group. As a result, geometric Langlands itself does not directly uncover any arithmetic questions.[7]

Recently, there are progresses in Langlands correspondence over non-archimedean local fields.[8] It turns out studying representations over local fields is more approachable than global number fields. Exact solutioin for GL(2) case is known for both equal and mixed characteristic non-Archimedean fields. For it's geometrical reliablity, equal characteristic

[6]It is a Langlands correspondence over $\mathbb{C}$.

[7]However, it is still powerful tool for it's geometrical and categorical nature. For example, celebrated Ngo's proof of fundamental lemma [][Ngo] relys on geometric techniques.

[8]Let's call this type of Langlands correspondence as local Langlands from now on.

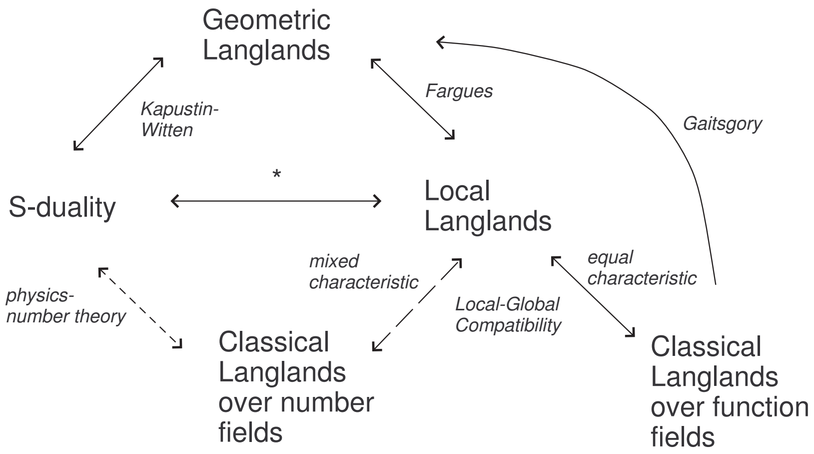

Fig. 2: A big picture. All arrows are conjectural except for the one via equal characteristic. Dotted lines represent mysteriousness and difficulty.

case was solved earlier. And then, mixed characteristic case was solved by using newly developed techniques. One of proofs uses Berkovich analytic space. Another recent proof, using a method called perfectoid space, is notable. It connects both equal and mixed characteristic cases topologically. As a result, one can utilize geometrical methods more systematically for arithmetic object.

As one of applications, currently local Langlands correspondence is built in a flavor of geometric Langlands. By using diamonds, which is a technical gadget developed for encapsulating local arithmetic topological infromation via perfectoid space, the local Langlands correspondences can be geometrized. This new technology may be able to open a new path between quantum mechanics and number theory. As Fig.2 depicts, there are studies for connecting these ideas. The path $*$ of Fig.2 is a motivation for part III of this series of work. However, S-duality argument of KW on the physics side will be replaced by new first quantization method developed throughout this series. In order to make sense of this conversion, it is required to contemplate the relationship between quantization and branes.

References: [MO77], [KW06, GW08b, Wit17], [HT01], [Far16], [Gai16a, Gai16b], [Fre07, Fre09], [HT07]

2.2.2 Quantization via complex analytic continuation of phase space and it's p-adic analogues

In [GW09], quantization process is reproduced by using string theory. Roughly, it is constructed by a string whose ends connects between two specifically defined sub-branes of A-brane, which is complex analytification of symplectic manifold. In [Wit10], path integral quatization is constructed over complex analytification of symplectic manifold. In both studies, complex analytification of symplectic manifold opens new insight on quantization. This motivates for one to think about p-adic analytification of phase space and it's quantization.[9] There will arise new calculational insights and new physical interpretations.

[9] Phase space here is defined over non-archimedean fields. Giving physical meaning to this p-adic phase space is done partly throughout this series and more in 'DO philosophy for physics over p-adic geometry'.

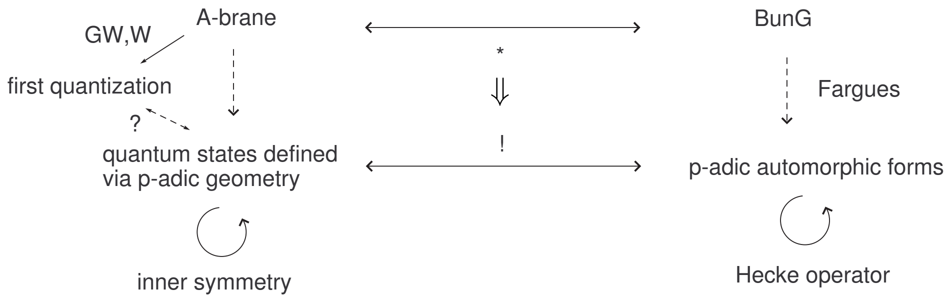

Fig. 3: A very heuristic picture of only one side of Kapustin-Witten ($\star$) and new quantization approach-local langlands (!). GW is for Gukov-Witten and W is for Witten.

This is one motivation for developing p-adic analytic phase space and it's quantization.

Deeper motivation is from the argument of former subsection. By following the big picture (Fig.2) description, one can question the connection ($\star$). Unlike geometric Langlands correspondence, local Langlands can be established rather directly by speaking of equivalence of representation theory. So on the left hand side, one also wants representation theoretic framework. Since A-brane, which corresponds to Bun_G side of geometric Langlands, can give rise to a quantization process, one can consider a connection between representation theory of quantization and automorphic side of representation of G, for example. In other words, instead of using string theory technology, representation theoretic aspect of quantization is going to be directly related to the representation theory of local Langlands correspondence. In the part III of this series of papers, the correspondence (!) in Fig.3 is studied by augmenting new physical interpretation.

There seems to arise further mathematical insights from this point of view. In [Wit10], topology plays an important role to distinguish physical paths from path integrals over complexification of symplectic manifold. It is done by using topological technique called Floer homology. In p-adic analogue, topological argument is also significant. Paths space in p-adic analogues also form some topological structure. And it can be analyzed by étale cohomology or crystalline cohomology and etc. By assuming the equivalence of first quatization between via complex analytification method and via p-adic analytic method,[10] one can presume a connection between two different topological arguments that are even over different base field, p-adic and complex. In part III, it will be followed to study comparisons between two different topological stuructures emerging in the name of quantization.

This can be noted in even bolder language. One notable application of perfectoid space is to establish comparison theorems between different cohomological theories over p-adic geometry. [Sch12] Perfectoid space provides the topological framework on which p-adic moduli space problems are analyzed. In a sense, the idea is closely related to the study of p-adic motive. Instead of establishing abstract category of motives and its functors, perfectoid space technology helps understanding each cohomology theories rather concretely by studying the local profinite topology of perfectoid moduli spaces. [Sch17b]

For now, one should consider it just as mathematical object which is created by base change. Then, p-adic analytification means Berkovichfication, or perfectoidification of the phase space over a chosen non-archimedean fields.

[10] In other words, assuming (?) in Fig.3 is natural isomorphism. Or assuming there exists category called 'universal quantization', which is the source of other equivalent quantization methods.

In fact, this is not the end of the story. Due to tilting equivalence of perfecoid space, geometric methods such as local shtukas are available in studying local Langlands. Study of Fargues, Fargues-Scholze indicates the existence of geometrization of local Langlands.

One of mathematical applications by considering perfectoid phase space is to add another cohomology theory, which is originated quantum physics, into the non-Archimedean motivic dictionary. In part III, Quantum cohomology and equivalently Floer homology defined via complexification of symplectic manifold will be compared to étale and crystalline cohomology of p-adic analytic phase space. By following big picture figure 2, this new cohomological comparison can be considered as an evidence of the existence of geometric-local Langlands bridge. Moreover, totally different method for measuring topology, which is originated from Morse theory, can be utilized to study arithmetic symmetry. Maybe, it might be possible to study arithmetic symmetry via symmetry existing in nature which may even be tested in an observatory.

In summary, having a p-adic geometric methods of first quantization may affect the study of motive. The conjectural existence of universal theory of first quantization is to the existence of universal cohomology theory. Following this slogan, one can consider p-adic geometrical structure as a framework which fullfils both arithmetic and quantum mechanical motivations. Furthermore, geometric Langlands - local Langlands connection can be studied by two different first quantization techniques, one over complex analytification of symplectic manifold and the other over p-adic analytification of p-adic phase space. The existence of the bridge between p-adic geometric realm and quantum physics realm may lead to the physics-number theory connection ultimately, as the study of p-adic geometry gives clues to the study of $\operatorname{spec}\mathbb{Z} \times \operatorname{spec}\mathbb{Z}$ world.

References: [GW09], [Wit10], [Far16], [Sch17a]

2.2.3 *Different quantization methods and motivic p-adic geometry*

There are different sorts of quantization procedures in quantum mechanics. They differ by mathematical tools used to represent. And they differ by their physical origin, which is whether from Hamiltonian mechanics or Lagrangian mechanics. The focus of part II of this series is to redefine path integral formalism from classical Lagrangian mechanics over certain p-adic configuration space. One reason why path integral formalism is chosen to be reproduced first is to show physical motivation more clearly.

This new path integral quantization via p-adic geometric configuration space is rather free from definability issue, since it is defined directly by analysis of certain topological space omitting the step that sending information to Hilbert space. Moreover, the measure problem of path integral formalism can be reduced in a new path integral definition over p-adic geometric configuration space. It can be shown that path sums over Berkovich projective line is constructible due to it's peculiar tree structure. Conjecturally, this property works in more general p-adic geometric configuration space, so that pathology in the definition of path integral quantization would be naturally resolved.

There are other quantization formalisms. Especially, deformation quantization of Poisson manifold and canonical quantization via Fukaya category are considered in part III. The legitimacy of studying new quantization scheme via p-adic geometry will be followed by showing topological comparisons to these two categorical quantization technics. For example, deformation quantization will be interpreted by representing quantum particle by $\mathbb{A}_{inf}$ topology over certain p-adic geometry. Here the Moyal product of observables is derived from certain product rule over $W(\mathcal{O}_C)$, which is a ring of Witt vector of some non-Archimedean integral ring $\mathcal{O}_C$. A mathematical definition of deformation quantization will be extracted by p-adic topological methods.

The unification of deformation quantization of Poisson manifold and canonical quantization via Fukaya category is suggested by Kontsevich. The argument is based on Riemann-Hilbert correspondence. New definition of first quantization via perfectoid phase space will provide another vantage point to this unification of quantization picture. The existence of universal quantization is to the existence of universal topological comparisons over $\operatorname{spec}\mathbb{Z}_p \times \operatorname{spec}\mathbb{Z}_p$. Boldly saying, the argument is about connecting universal quantization scheme to the idea of universal motive.

From purely mathematical point of view, quantum cohomology of certain special complexified symplectic manifold is compared to other cohomology theories over certain p-adic geometry. The big picture here is to think quantum cohomology over complex symplectic manifold as a part of p-adic universal cohomology at $p \to \infty$. This argument extends Scholze's idea of universal cohomology of p-adic geometry to include quantum cohomology of certain complex symplectic manifolds, which is defined from purely physical motivation. Following this argument, the origin of resurgence can be explained by the limit process from ∞ to finite p.

References: [Kon], [Sch17a]

2.2.4 p-adic GAGA and representations of quantum particles

One physically notable thing in the approach of part II and III is that quantization procedure is defined topologically.[11] The origin of different qunatization procedures will be presumed as different ways to analyze topology of certain p-adic geometric space. This way of viewing quantization in an universal way can be connected to Kontsevich's idea on unification of two quatizations, one from deformation quantization and the other from Fukaya category. He suggested homological algebraic equivalence between two different quantizations, by following Riemann-Hilbert correspondence. In part III, p-adic quantization will be added into this dictionary.

One mathematical point of view on the existence of universalness in first quantizations stems from geometrization of local Langlands correspondences. By assuming the existence of geometric-local Langlands formalization, new definition of first quantization via p-adic topology will be traced down to Kapustin-Witten correspondence.

The other vantage point would be Scholze's suggestion on universal p-adic cohomology theory via diamonds. In [Sch17a], he suggested alternative viewpoint on motive, by concretely establishing equivalences between cohomology theories over p-adic fields. The quantum connection between deformation quantization and p-adic quantization can be considered as putting in category of deformation quantization or equivalently category of Fukaya category at the place of $p \to \infty$ in the figure 3 of [Sch17a]. As the cohomology theories are equivalently defined near the place (p, p), duality in complex symplectic manifolds will be understood as topological equivalence at (∞, ∞) in Scholze's picture.

Mathematical punch line of quantization over p-adic analytic phase space is representing quantum particle's state in $H^0(\widetilde{M_p})$.[12] Here $\widetilde{M_p}$ is one of p-adic analytifications of

[11] Unlike the term topological quantum field theory indicates topological reduction of original quantum field theory, topological analysis in a new approach to p-adic quantization is original as it is. In other words, the peculiar topological space is considered as physical space itself, not as topological reduction.

[12] Physical punch line is to represent quantum particle as topological hole over p-adic geometric local

phase space M. p-adic GAGA result says topology of X^{al} and topology of X^{an} is the same. One can think quantum state via $H^0(\widetilde{M_p}^{an})$ and also via $H^0(\widetilde{M_p}^{al})$. One by analytic space will bring a representation that is reminiscent of wavefunctions of classical quantization. While the other algebraic realization of the space will bring a linear algebraic representations that is connected to arithmetic representations. To show physical motivation rather clear, part II only deals with analytic structure of p-adic space. Main task in that paper is to rederive wavefunction realization from p-adic quantum state space.

In III, arithmetic and topological aspect of this new quantization technique is studied. In order to utilize recent developement on p-adic topology, the language is changed from Berkovich analytic space to adic analytic space. The topological structure of Idea space is inscribed on the topology of perfectoid space. By doing this, the Fourier transform that changes position and momentum observables is replaced by tilting that send topology of K to topology of $K^\flat$. The unitary representation of symmetry group of quantum state will be studied from the perspective of p-adic automorphic representations.

The arguments involved in perfecoid space is totally topological. This framework is chosen for studying topological and symmetrical aspect of physical theory. It needs to be emphasized that the perfecoid space provide a space with infinitely fractalized topological structure, but without intuitional analytic metric structure. This means one may lose calculational power when working over perfectoid ground. A long term objective in this direction is to gain topological equivalence relation between representations of arithmetic origin and representations arising from new quantization method.[13] By following this philosophy, maybe some properties of arithmetic representations can be experimented via quantum state of particles.

References: [Kon01, Kon03, Kon09], [Sch17a]

2.2.5 Fourier transformation vs. tilting

In perfectoid space, tilting is a process that transforms one topological space into another. perfectoid space is a space that tilting does not change the topological structure. In the second part of this work, phase space is defined as $X^{perf} \times_k X^{perf,\flat}$. Equivalently, Fargues-Fontaine curve is the main mathematical object. The representation theory of inner symmetry of the phase space will be studied by analyzing the topology of this space. Physically, in analogy with classical phase space one coordinate can represent position and the other one for momentum. Mathematically, base change of phase space into Witt ring will be studied. As if complex representation, or Fock-Bargmann representation, of quantization can give rise a framework that treats position and momentum operators in a single body, this representation over Witt ring can give a intriguing medium that contains both position and momentum information.

Especially, when the chosen non-Archimedean field is perfectoid field, the tilting process transforms one to the other in topologically equivalent way. In quantum mechanics, Fourier transformation connects position and momentum operators in duality. As it will be introduced in next section, physical notions such as position and momentum will be defined topologically. As a result, tilting in perfectoid spaces replaces the role of Fourier transformation in the study of Hamiltonian mechanics over perfectoid phase space and it's quantum mechanical interpretation. References: [Sch11b]

tissue of spacetime.

[13] More technically saying, the existence of quantum motive and it's universality.

3 Overview

This section is composed of two subsections. Conceptual overview subsection will convey the main idea without using technical language. It is written for providing philosophical background. More mechanical picture of what this philosophical approach is about will be dealt in part IV.[14] Technical overview introduces the mathematical set-ups that will be played out throughout the series. It is not supposed to provide full details, but only a schematic sketch. The mathematics used to build the framework of this series may not be familiar to physicists. Since there is no physicist's version on the theory of p-adic goemety, most of notations and arguments used to build up physical notions in part III will rely on the raw form of p-adic geometry by mathematicians. Again more elaborate introduction to the mathematical language used throughout the series will be dealt in part V, 'p-adic geometry for physicists'.[15]

3.1 Conceptual overview

There is a big gap in the notion of position space between gravitationally defined position and quantum mechanically defined position.[16] Gravitationally defined space is locally $\mathbb{R}^3$. It is close to our intuition because areas or volumes that are defined by locally $\mathbb{R}^2$ and $\mathbb{R}^3$ spaces are familiar concepts in daily life. It is very natural to think that, even though some volume of space does not contain any contents in it, it should be locally $\mathbb{R}^3$.

A breakthrough on the concept of space was via Einstein's relativity. By including time into a spacetime, it becomes locally $\mathbb{R}^{3,1}$ or, by Wick rotation, $\mathbb{R}^4$. General theory of relativity revolutionized the concept of space and time by replacing it's geometry into Riemannian 4-manifold with locally $\mathbb{R}^{3,1}$ structure. This local topology is nice enough to give a curvature in the language of differential geometry.

In general relativity, energy and momentum are determined by the curvature of Riemannnian spacetime. The concept of curvature, in the sense of differential geometry, works properly with locally smooth $\mathbb{R}^{n,1}$ spacetime. So the local smoothness is assumed in gravitationally defined spacetime. In this framework, locally $\mathbb{R}^n$ space provides the information of position in n real numbers. Philosophically, this means that there should exist physical points of any n-dimensional vector over real numbers. Mathematically saying, position of space in gravitational scope is defined by compact and smooth subsets that are locally homeomorphic to $\mathbb{R}^n$.

For example, in 3 dimensional gravitational space, if one picks a real number vector by $(3.141592\cdots, 0, 0)$, then there should exist that physical position in the space. Since classical trajectory of particle's motion is smooth, it is not unnatural to imagine. If one particle starts at $(3, 0, 0)$ and ends up at $(4, 0, 0)$, then it is obvious to think that the particle has passed the point $(3.141592\cdots, 0, 0)$. So one can determine $(3.141592\cdots, 0, 0)$ really exists there.

Quantum mechanically, this conception on position of space does not work. Simply

[14]There may exist various mataphysical ways to see physics over this new approach. This supplement might suggest only one specific way that motivated the author.

[15]The term 'p-adic geometry' is not in the sense of adic space. It is used in a more general sense. Throughout these works, 'p-adic geometry' means algebraic geometry over non-Archimedean fields, including rigid space, Berkovich's theory and adic space.

[16]More generally, the gap is about any physical observables observed in both quantum and classical way. Let's concentrate on position parameter for convenience.

by Heisenberg uncertainty principle, determining a particle's position more precise results increase of uncertainty of particle's momentum. This takes more energy as one tries to confine a particle within a space more accurately. It takes infinite amount of energy to pin a quantum particle down.

For example, one can say a quantum particle is in a 1-dimensional infinite potential box [0,1]. Right notion of position of the particle is that the particle exists inside the box with probability 1. Assume that one tries to shrink the box to specify the position of the particle from [0,1] to [0.3,0.4]. Now right notion of position of the particle is that the particle stays in [0.3,0,4] with probaility 1. Then again from [0.3,0,4] to [0.31,0.32] and so on. Each procedure requires more energy into it for the uncertainty of the particle's momentum increasing.[17] In the end, it requires infinite amount of energy to determine a point exists at $0.3141592\cdots$. Even the entire energy of the universe is finite. So it is not meaningful to say quantum particle is at certain point such as $0.3141592\cdots$.[18]

Considering trajectory of the motion of a quantum particle also does not guaratee the physical meaning of any chosen point of $\mathbb{R}^3$. Unlike classical particle, quantum particle does not always take straight line or geodesic line between the initial point and the end point. So, a particle starting at $(3,0,0)$ and end up at $(4,0,0)$ does not guarantee that it has passed a line that contains point $(3.141592\cdots,0,0)$. Mathematically, quantum position is not exactly represented by a point over locally smooth and compact space, such as $\mathbb{R}^n$.

Actually the idea that a quantum particle takes smooth path over locally $\mathbb{R}^n$ space is troublesome. Following Feynman's path integral interpretation, a quantum particle behaves as if it navigates all the paths that connects the initial point and the end point. This idea is mathematically ill-defined in general, because the measure of uncountably infinite path sums is not free from divergent issue rigorously. Physicists somehow use it to get meaningful results by exquisite process which cuts off infinities. However, non-existence of rigorous definition of path integral in general is one of obstacles from unifying other physics with quantum mechanics.[19]

There are more problems of seeing quantum world from the scope of locally $\mathbb{R}^n$ fabric of spacetime. First, non-locality problem arises. As Eistein Podolsky Rosen claimed, the information between two quantum entangled particles can exceed the speed of light. It is later confirmed experimentally. Also there is discreteness problem. There is a phenomenon called quantum jump that a particle's state can discretely change from one to the other. This is also experimentally observed effect. Non-local and discrete nature of quantum particle is not effectively described by using locally smooth $\mathbb{R}^{3,1}$ space. In order to keep the intuition of quantum particle cruising through spacetime, physicists explain that the fabric of spacetime is to be nontrivial in Planck scale. So in Planck scale, quantum particle is fluctuating randomly over this ad hoc nontrivial local tissue of space. Mathematically, locally smooth $\mathbb{R}^n$ geometry is powerless to depict these quantum nature, but only invoking

[17]Note that this quantum mechanical model is a single particle system with non-interactive and non-relativistic property. Throughout this paper, quantum mechanical model in consideration is such restricted model.

[18]Actually, it is done so in real life. When it is said a quantum particle is at x, then it should be read as the particle is at x with physical error. More specifically, it means a particle is situated at $[x-\delta, x+\delta]$ with probability 1 where δ is negligible in physical sense. Already because of experimental reason, 2δ can not be smaller than the wavelength of observing beams.

[19]This measure problem of path integral can be resolved in the new path integral interpretation over extended parameter space.

more interpretational issues.

One possible way to fix this discrepancy is to reconsider the notion of spacetime. As it is stated in the previous motivation section, there are studies to see the spacetime as an emergent concept stemming from quantum entanglements. For example, in condensed matter physics, AdS/CFT type correspondence is applied to solve questions in quantum systems, by using a method called tensor network. On the other hand, some entangled system of many particles in a lattice can produce an intriguing geometric web, which can be considered as an incarnation of physical space in the bulk. Also in information theory, quantum error correction code is suggested to be a generator of emerging spacetime.

This series of papers approach these issues from a reconsideration of quantum position space or quantum parameter space in general. Reading the information of quantum observables is attempted by studying peculiar geometrical objects, as if gravitational observables are read by the curvature of Riemannian geometry. In order to reconcile this concept of quantum position observable with gravitational space, spacetime is replaced with a geometric structure called 'extended spacetime' which has locally non-trivial topology.[20] Instead of discarding the notion of spacetime and considering it as emergent concept, extended spacetime is physical space which is devised to combine quantum particle's positional uncertainty within it's local topological structure. The classical spacetime is a certain limit of this new spacetime. As a result, the gap between the two different notion of position spaces in physics, one in gravity and another in quantum mechanics, is reconciled in extended spacetime. In order to represent new points of extended space, a terminology 'Idea space' needs to be introduced.

3.1.1 Idea space

Classical space representing position observable has locally $\mathbb{R}^3$ topology. Information of position is contained in 3 dimensional vector space over $\mathbb{R}$. As it was pointed out in the previous subsection, quantum mechanical position of a quantum particle is defined non-classically. Main difference is that quantum particle does not have definite place like the classical notion of spacetime. A quantum particle's position is represented by a wavefunction. One can only calculate probability density of a quantum particle's position. Until wavefunction is collapsed by any observation, a particle can exist here and there with such and such probability density. First step is to recover the definiteness of position in quantum mechanics.

In order to deal with quantum nature of position in small scale, classical notion of spacetime will be replaced by a space called 'extended spacetime'. The points of this extended spacetime will be composed of set of balls, which are represented by (c, r).

$$(c, r) \text{ represents (center,radius) for } c \in \text{some number system}, \quad r \in \mathbb{R}_{\geq 0}$$

For example, when $c \in \mathbb{R}^n$ and $r = 0$, it is just $\mathbb{R}^n$ vector space. When $c \in \mathbb{Q}_p$ or $c \in \mathbb{F}_p((T))$ and the distance is defined by nontrivial absolute values, then a peculiar geometric space with non-trivial topology arises. A quantum particle's position will have definite place over this space. Roughly saying, the minimum volume of space, in which a quantum particle stays with probability 1, is defined to be the definite position of a particle. For example, consider $c \in \mathbb{R}^2$. Then (c,r) represents one of circles of Fig.4

[20]More general term is 'extended parameter space'.

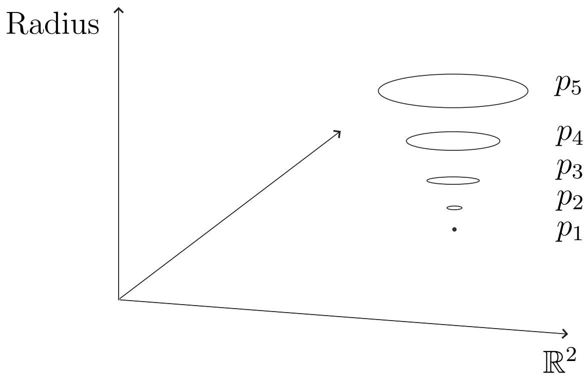

Fig. 4: $p_i = (c, r_i)$ where $r_i < r_j$ for $i < j$

From the figure, familiar $\mathbb{R}^2$ space is recovered by setting $r = 0$. Classical particle has it's position observable over $r = 0$. The points that quantum particle's definite position will be defined are above $r = 0$. Let's call the space of sets of balls with radius 0 as 'Reality space' and radius other than 0 as 'Idea space'.[21] The term 'extended' will be used relying on this argument.[22] It includes new points in 'Idea space', while classical position, not including time, is represented by 'Reality space'.

Roughly, quantum particle's position is represented by a point over 'Idea space'. Unlike wavefunction argument in quantum mechanics, particle will have definite position. Wavefunction will be rederived as a consequence of projecting particle down to 'Reality space'. The parameter r will have important physical meaning. It represents the uncertainty of position of a particle. This is closely connected to the entropic argument. By the entropic force argument, it acts as if time flows. In other words, any free quantum particle represented by points of Idea space will increase it's r coordinate, due to second law of thermodynamics. The role of time will be interpreted in this manner.

The example above does not yield locally nontrivial topological space. It is homeomorphic to $\mathbb{R}^3$. One of reasons that extended position space is introduced is to combine nontrivial connectedness of quantum scale spacetime. By changing number system $\mathbb{R}$ to non-Archimedean fields, the Idea space can have more complicated structure.

There is another reason to think about using number system other than real numbers. As it was discussed in motivation section, number system $\mathbb{R}$ is not effective in representing position of quantum particle beyond Planck scale. One may consider non-Archimedean local fields as underlying number system for describing quantum scale geometry of spacetime. Unlike Archimedean Local field such as $\mathbb{R}$ or $\mathbb{C}$, non-Archimedean local field has distinct way of measuring distances. The fact that points are measured in non-canonical way means that one has to modify the locality on the small scale fabric of spacetime. Also it has (pseudo)uniformizer. By giving the (pseudo)uniformizer special physical meaning such as Planck length, some non-Archimedean number system can effectively represent certain quantum system.[23] In this regard, Idea space with non-Archimedean fields is

[21] The term 'Idea' is from Plato's philosophy.

[22] In order to amplify the existence of non-classical physical points, the term 'extended' is chosen.

[23] In $\mathbb{R}$, Planck constant is not special number, but only a randomely looking number which is computed by dimensional analysis of other physical constants. If one chooses a norm of (pseudo)uniformizer as Planck constant, then it becomes special number because non-Archimedean fields has the unique (pseudo)uniformizer.

studied in the following series of papers.

Algebraic geometry over non-Archimedean fields has been developed by many figures. Compared to algbraic geometry over global field, it has some topological pathologies at first. Tate defined rigid analytic space overcoming some of problems. Raynaud defined it with the language of formal scheme, amalgamating it with the language of scheme. Berkovich studied it by considering set of semi-norms instead of maximal ideals. By doing it, analytic nature of the geometry becomes easier to handle. Huber studied it in most general sense. By defining category of adic space, formal schemes and algebraic geometry over non-Archimedean fields can be considered in a single frame.

Berkovich space will be used to read the analytic nature of Idea space in part II. In part III, perfectoid space which is defined via adic space will be used to read topological and symmetrical nature of extended phase space. In II, propagators will be notated via heat kernel over $\mathbb{P}_1^{Berk}$. Path integral method will be explained via stochastic process over p-adic geometry. In III, mathematical evidence of this new approach will be followed. Main result is a cohomological comparison between quantum cohomology defined over 1-d complex symplectic manifolds and crystalline cohomology defined over Fargues-Fontaine curve. By doing this, arithmetic geometry can be studied via Morse theory. Also physics of inner symmetry can be considered via arithmetic symmetry.

3.1.2 Time vs. uncertainty

The role of time is special in quantum mechanics. Time is not considered as an observable. Instead, it provides smooth and definite background parameter over which path integral or derivative is defined. In other words, in first quantization there is an asymmetry between space and time. In second quantization or quantum field theory, this is resolved in very non-trivial ways by considering operator valued fields over (1,3) coordinates with Lorentz transformation. In quantum field theory, spacetime works as background parameter space supporting fields which is creation and annihilation operator valued distributions. However, the calculation performed in QFT ends up being quantum mechanical. Even though the notion of spacetime is used, quantum observables do not change. So, still time is not observable but position is. As a result, there exists controversy in combining the notions of space and time in quantization procedure.

The time in first quantization will be replaced by the radius coordinate of Idea space. The radius will be interpreted as parameter for uncertainty. And entropy will be defined over this parameter of radius. Upto second law of thermodynamics, any isolated quantum particle will experience increase of radius. It can be said with the notion of time that a particle tends to go to the future direction. Philosophically, the act of observation is decreasing the entropy of a particle so that the particle approaches to Reality space. The probability interpretation will be reproduced by considering Laplacian of state function over Idea space. In a limit this procedure is giving wave-functional probability measure over the Reality space.

The asymmetry between the notion of space and time in first quantization will be explained in p-adic geometric quantization approach by defining position and uncertainty differently over the Idea space. First quantization of position observable is defined as projection procedure from Idea space to Reality space. The philosophy of emergent spacetime says time is also an emergent concept. It will be interpreted as entropic direction over Idea space. Interestingly, this entropic flow of a particle was shown in [Ver11], yielding a new interpretation on the gravity. Combining these two entropic interpretation over time from two different background of fundamental physics could provide a new scheme to build a quantum gravity theory.

In Berkovich space setting, time parameter is considered over $\mathbb{R}^1$ which has trivial topology. However, in a more general adic setting, this parameter can be considered as abelian group with higher rank. The uncertainty in this setup is written by a parameter with non-trivial topology. This technical tweak requires unconventional view point on the notion of time. It seems the role of time in adic setting should be focused as index set for certain stochastic process. Moreover, there needs an explanation for particle's dynamical system over the Idea space. In order to pursue deeper understanding on this new approach to quantum mechanics, it is important that one also has to encounter philosophical or methaphysical questions.

Metaphysics of this new approach on quantum physics via p-adic geometry is followed by many unconventional rethinking about even most fundamental physical notions. Not only the notion of time is something radical as it is suggested partly in this subsection, but also the notion of position as physical observable is something more than points over vector space. Because points of Berkovich space is not just points over a number system, but topologically defined objects. These issues will be shown in 'DO philosophy for physics over p-adic geometry' and 'p-adic geometry for physicists'. Technical parts II,III are more focused on constructing a mathematical model of first quantization in simple conditions, reducing metaphysical remarks.[24]

3.1.3 Defining physical notions by topological arguments

The new approach is not only about base change of coefficients, but more about altering physical notions. For example, classical physical observable is defined as a point over a set, but in new approach physical observable is defined as topological set over topological space. And quantization procedure will be defined as a procedure to give a probability distribution on the subset of topological set. In this framework, quantum to classical transition can be thought as a limit from topological sets to points. By adopting this world-view, algebraic geometric techniques are legitimately used to analyze the local tissue of spacetime.

First quantization methods via Berkovich space realization of p-adic geometry already provide intriguing connection between physics and p-adic algebraic geometry. A little bit deeper connection between quantum physics and arithmetic geometry will be envisaged via Huber's setup of p-adic geometry. Part III of this series of papers will deal with this connection by constructing geometric quantization via perfectoid space.

Because of technical details, two quantization constructions by Berkovich analytic space and adic analytic space are not in a parallel. It seems that one quantization procedure is easier to handle in one case while the other is easier in other case. Studying the gap would fertilize the understanding on p-adic geometry.

The objective of this series of papers is to initiate developing a new mathematical language from p-adic algebraic geometry and p-adic topology to understand first quantization. In II, they are done by configuration space over $\mathbb{P}^1_{Berk}$. While in III, it is done over Fargues-Fontain curve. It turns out that it is not equivalent task to build up quantizations

[24]Even though, new metaphysics involved in builing this new approach has stilmulated the author, that way of seeing the phenomena would not be universal and absolute. In that reason, metaphysical remark is sorted out as much as possible in order to invoke more various ways of understanding the physic that underlies.

in both set-ups. Constructing a first quantization framework in two different gadgets of p-adic geometry is motivated by physical and mathematical adaptability.

There is no exact mathematical equivalence relations between different first quantization methods defined over $\mathbb{R}$. Because the procedures that send those informations inscribed in various mathematical ways to Hilbert space are not rigorously defined at first. So there is no way to compare those scpecific quantization methods each other. However, in a physical criterion, these methods are all treated to be equivalent. Even though not all details are trivially legitimate process in mathematical sense, physicists' effective calcuations indicate equivalent results. And surprisingly, observations indicate there is no obstruction in considering these methods equivalent.

One of advantages of this new approach is to build a mathematically legitimate theory of first quantization. Classical first quantization's problem arise when one sends those informations constructed via geometric, analytic or linear algebraic methods to infinite dimensional Hilbert space. Because of this process, the procedure becomes non-geometric, non-algebraic and non-analytic. The new interpretation on quantum mechanics will revise this procedure and make whole procudure as geometrical. Quantum particle will be considered as topological tweak over the local tissue of spacetime, which is represented as certain p-adic geometric space. Like Einstein's gravity theory lets equivalently geometry of spacetime with energy and momentum of matters, quantum particle's charateristics will be read as certain topological informations.

By following this philosophy, diverse ways to read the topology of p-adic geometry can be considered different ways to read quantum particle's characteristics. In this sense, the idea of motive and the equivalence of first quantizations can be related. Even though new technology is mathematically well-defined, those first quantization defined over $\mathbb{R}$ so far are not. So one cannot establish exact universalness of first quantization directly. Only partly for special cases, comparison theorem will be established in part III. For now, it is only from physical or empirical assumption to believe the existence of general universal quantization that even include newly defined first quantization method. This assumption is important, because it is the source to believe the existence of connection between the topology of certain p-adic geometry and the quantum topology of certain complexified symplectic manifolds. Mathematically, it is connected to the idea of universal p-adic cohomologies and the existence of correspondence between geometric and local Langlands correspondence.

3.2 Technical overview

In physics, classical observables are represented by real numbers. Physical concepts such as mass, energy, momentum, area, volume, etc are smoothly representable and closed under addition and closed under constant multiplication. Real number system seemed to fit to calculate these physical observables. Since real number has nice analytic property, calculus was developed and found to be useful to study classical mechanics. However, in quantum scale physics, observables turn out to be represented in a quanta. This discrete nature of observable space prohibits one from naively utilizing classical differential geometric approachs, for analytic property is not trivial in such a quantized parameter space. This is one of reasons why it is hard to connect geometric intuition gained by studying classical mechanics to quantum mechanics.

In order to regain geometrical intuition of Planck scale spacetime, new microscope is

	General Relativity via Riemannian geometry	**A first quantization via p-adic geometry**
Physical observables	Energy-momentum tensors	quantum particle's charateristics such as position, energy level, spins, etc.
Mathematical descriptions	curvature tensors of Riemannian geometry	topological informations of p-adic geometry

Table 1: An oversimplifying analogy to the theory of general relativity

used to look at it. Schematically, it is approached by using mathematical setup as follows.

- Local non-Archimedean base fields instead of real number field for parameters representing local spacetime.
- Algebraic geometric and algebraic topological tools for analyzing geometrical structure of quantized parameter space.
- P-adic analytification instead of complex analytic continuation.
- P-adic representation for representations of quantum states. (in III)
- Arithmetic symmetry for gauge symmetry. (in III)

This is a basis of mathematical approachs taken throughout this series of articles. In order to build geometry of analytic and topological structure over non-Archimedean base fields, Berkovich's method and Huber's method are used. Both methods are employed for two track approach. Originally, perfectoid space was first considered as a potential topological background which bridges the quantum gauge symmetry to local Langlands correspondence. By dressing phase space in perfectoid topological structure, first quantization procedure is reinterpreted as an activity of reading stochastic behavior of topological bubble over p-adic geometry. It is augmented by topological comparisons between quantum cohomology and p-adic Hodge theory. This is a main content of part III of this work.

However, the construction of perfectoid space relys on heavy algebraic topological arguments. So, physical intuition is hardly connected to this peculiar locally symmetric topological space. In order to give better physical intuition, Berkovich analytic space is used to construct a geometrical theory of first quantization.[25] It is done by studying harmonic functions over Idea space and by studying it's Laplacian and measure theory. This mathematical procedure may provide a little bit better intuition for physicists. Also Berkovich analytic space has some property which is familiar from complex analytic space. For example, dynamics over Berkovich space has similarity with dynamics over complex analytic space. So, in this toy model construction step, it would be nice to have a theory over Berkovich projective line. This is a main content of part II.

To physicists, what this construction is about is analogous to Einstein's theory of gravity. An analogy to geometrical description of gravity theory is drawn in the table 1.

[25] It would be possible to construct such a geometrical theory over certain nice analytic adic space. However, since adic space is constructed in a flavor of algebraic geometry, it seems to be ineffective for physical uses.

via Hilbert space	p-adic geometry
quantum state $\varphi \in L^2(X)$	idea state $\phi \in H^0(X_{II}^{Berk})$
inner product $< \varphi_1, \varphi_2 >_{Hilb}$	Gromov product $< \phi_1, \phi_2 >_{Grom}$
eigen vectors and eigen values $\alpha_1 \vert e_1 > + \alpha_2 \vert e_2 > + \dots$	type I points and norms of type I points $(x_1, \vert x_1 \vert), (x_2, \vert x_2 \vert), \dots$
operators $\mathcal{O}\varphi = \psi$	projectors $\mathcal{P}\phi_{II} = \phi_I$

Table 2: Schematic comparisons between Hilbert space analysis and new approach. In this table X is position parameter space. Subscripts I,II mean type I,II points of Berkovich space. Only Berkovich analytification is considered in this table.

However, the analogy is read very carefully. One most notable thing is unlike Riemannian geometric space, the spaces of adic spaces are very topological objects rather than geometrical. So the physical informations of a quantum particle will be considered as topological tweak of background local tissue of adic spacetime. The intuitions grown from gravitational geometry is not helpful on getting the intuiton of p-adic geometry.

This may raise another metaphysical issues. Whether considering quantum particle's property as topological information is only an approximation or something natural in the universe is interesting question to ask. A bit of idea on the natural side will be followed in 'DO philosophy of physics over p-adic geometry'.

Current definition of first quantization has bizarre steps to follow. Simply put, first quantization has two steps. First, prequantum setup is the step one prepares physical information in an apt form. Second, send prequantum information into Hilbert space. What makes first quantization hard to define and analyze is second step. While the information defined over finite dimensional physical parameter space gets sent to infinite dimensional Hilbert space, one loses control of mathematical legitimacy in many aspects.

New approach in this series will make first quantization in an one step process as if theory of gravity is. As the slogan of Einstein's gravity theory is 'geometrical curvature of Riemannian manifold is defined (and defines) by energy-momentum.', the slogan of this new approach will be 'topological tweak of p-adic geometry is defined (and defines) by the characteristics of quantum particle.' As a result, the analysis of quantum state over Hilbert space will be replaced by the analysis of topological hole over p-adic geometry. The schematic comparisons between first quantization via Hilbert space and via p-adic geometry is in the table 2.

As first quantization is topologically defined, different sort of quantization methods will be interpreted as different ways to read topological data. For example, deformation quantization method will turn out to be equivalent to the new quantization approach over the ring of Witt vectors which is connected to q-deformation cohomology over p-adic geometry. In a similar vein, quantum cohomology and equivalently Floer homology defined over complexified symplectic manifolds will be compared to crystalline cohomology and equivalently deRham cohomology of Fargues-Fontain phase space. This study may invoke tantalizing ideas of connecting topology from p-adics to complex world and vice versa.

3.2.1 Overview on a first quantization over $\mathbb{P}^1_{Berk}$

Before starting to overview the content of part II, let's give some vantage points that may help physically interested reader understand what will be studied. They are already mentioned in physical motivation section. For clarity, let's pick three simplest aspects.

- From new meta physical point of view.

  This vantage point is the essential component in both part II and party III. Defining physical observable as a ball over p-adic geometry. And consider observation procedure as reducing the radius of ball. More concretely, define 'quantum observable' as type II point of Berkovich space. Define 'real observable' as the norm of type I point which is projected from the type II point. Probability density is defined by the distance called Gromov product.

- From Gubser ét al.

  p-adic AdS/CFT was studied in [GKP+17, Gub17]. Some of propagators and three, four point functions are calculated over Bruhat-Tits tree. From this vantage point, part II is providing propagators over Berkovich projective line. Instead of deriving p-adic AdS/CFT from this calculation, the propagator calculation will be used to define first quantization.

- From Witten

  Path integral quantization over complex analytification of symplectic manifold was considered in [Wit10]. Floer homology was used to sort out real and auxiliary paths over the space. From this vantage point, part II is providing path integral analogue of p-adic analytification of certain p-adic phase space, which will act as if symplectic manifold does in quantum mechanics over real number. q-deformation cohomology or $\mathbb{A}_{inf}$-cohomology will be used to read physical paths.

Let's take a look at the technical overview of part II. As indicated above, more mathematically involved motivation will be pursued in part III. Part II will pursue rather physical motivation. The main objective is to build a relatively comprehensible mathematical model of first quantization. In order to do that Berkovich projective line is used to describe position observable of configuration space. A family of functions over $\mathbb{P}^1_{Berk}$, which will be defined as $CPA(\mathbb{P}^1_{Berk})$,[26] will act as configuration space for a quantum particle. Path integral quantization over configuration space will be reproduced by Lagrangian mechanics over $\mathbb{P}^1_{Berk}$. Laplacians of harmonic functions as $CPA(\mathbb{P}^1_{Berk})$ are the source of measure on Reality space, which is the type I points of Berkovich space.

A little bit more concrete steps of first quantization via $\mathbb{P}^1_{Berk}$ is as follows. First, configuration space over algebraically closed non-Archimedean fields is defined. It will be function space over $\mathbb{P}^1_{Berk}$, specifically continuous piecewise affine functions $CPA(\mathbb{P}^1_{Berk})$ in short. Quantum states will be considered to be encoded in this type of functions. So quantum observables is definitely defined over it. Classical quantum mechanical interpretation will be rederived by probability measures of this function space.

In summary, in part II

- Configuration space over a completion of algebraically closed non-Archimedean field $\mathbb{C}_p$ is considered. Define it as $Conf(X_{\mathbb{C}_p})$.[27]

[26] CPA is for continuous piecewise affine. It is a simple exmaple of harmonic functions.

[27] Algebraic extensions are not important in part II. Only algebraically closed non-Archimedean field $\mathbb{C}_p$ will be in consideration. The symmetry from algebraic extension of non-archimedean fields will be considered in part III. It is important issue there, because it connects to arithmetic symmetry.

- P-adic analytification of $X_{\mathbb{C}_p}$, which is Berkovichfication in this paper, is defined. Let's call it as X_{Berk}.
- Lagrangian mechanics is established over $Conf(X_{Berk})$. Configuration space is represented by $H^0(X_{Berk})$. Specifically, it is represented by certain harmonic functions over X_{Berk}. Let's call it 'Idea state' of a particle.
- Quantum mechanical wavefunction representation of position is rederived by projection from 'Idea state' to 'Reality space'. It will be interpreted as stochastic process through p-adic topological connected paths.
- The notion of energy level is interpreted as the number of holes of disc which defines position parameter from Idea state.
- Some examples such as particle in a infinite potential well, and a particle in a free potential are described.

Mathematical references which provide technical background of this line of argument is [BR]. All of the above construction is done locally over 1 dimensional Berkovich affine line case. Examples are basic and simplest cases. Basically, the objective of part II is to initiate constructing a geometrical framework on which calculable first quantization is defined. As a first step, the notion of position space for a single quantum particle is established. Even though this method provides familiar framework that is analogous to classical Lagrangian methods, the whole story of developing first quantizations by geometric and topological gadgets of p-adic geometry can be nurtured by more sophisticated language. It will be done over perfectoid phase space in part III.

3.2.2 Overview on a first quantization over C^{FF}

Before starting to overview the content of part III, let's give three vantage points that can help mathematically interested reader understand what will be studied.

- Geometric Langlands and S-duality vs. geometrization of local Langlands and first quantization.

  Follow the arrows in big picture fig.1.

- From Kontsevich's unification of quantization

  Add constructible sheaf originated from p-adic geometry into the unification picture.

- From Scholze's motivic p-adic cohomology

  Add quantum cohomology (or equivalently Floer homology) over 1-dimensional complex symplectic manifold into the picture.

In part III, the objective is slightly more technical. Assuming first quantization can be defined via p-adic geometry, deformation quantization is reproduced by studying convolution product of algebra over perfectoid phase space. For interpreting physics from p-adic geometry, there will arise alternative topological technology to replace symplectic manifold. Phase space will be described by Fargues-Fontaine curve. Local shtukas and algebraically Breuil-Kisin modules will be used to represent quantum particles. Moyal product of deformation quantization over complex symplectic manifold will be compared to convolution product of Witt vectors.

Also the topological comparisons between quantum cohomology and Floer homology of complex symplectic manifold and crystalline and étale cohomology of perfectoid space is considered. This structural comparison would provide mathematical evidence to define quantization as a topological characteristics of p-adic geometry. Also in a mathemtaical direction, it produces unified perspective on (complex) Hodge theory and p-adic Hodge theory.

The advantage of constructing first quantization more generally by using perfectoid spaces is mathematical at first. Some general features of physical theorems, such as topological equivalence of position and momentum observables, are immediate results from perfectoid setup. And as the first motivation of this paper indicated, the mysterious connection between quantum physics and number theory could be studied over this framework.[28] It's power of bridging topology of two different realm of arithmetic geometry turns out to be useful in many applications. By adding quantization interpretation into this context, the relationship between quantum physics and number theory can be glimpsed.

One may ask whether this specific construction of first quantization via perfectoid space, which is defined by a mathematical language that physicists would not be familiar with, is essential for physical use too. The answer seems to be yes. There are physical and even meta-physical differences in seeing 'local tissue of spacetime' in perfectoid space realization of p-adic geometry rather than Berkovich space realization. Basically, there exists mathematical gap which brings different sorts of mathemtatical applicability. One of examples is the applicability of the theory of diamonds to analyze local Langlands as geometric Langlands analogy.

Not only for it's mathematical applicability, but perfectoid setting seems to be more natural to encode all the physical data into the topology of local p-adic spacetime. As the slogan said, the topology of certain p-adic geometric tissue of spacetime is considered to contain the whole informations of particle itself. However, in Berkovich setting of part II, Idea state is defined over type II points in the form of H^0, containing only position and momentum parameters. Because of purity theorem there exists no other topological data inside p-adic phase space than 0-dimensional one. To fill the gap type V points seems to have a role to encode more informations in it.

More specifically, first quantization is defined over 'local tissue of spacetime' which is represented by p-adic geometric technical language. The reason why defining physical space is called 'local tissue of spacetime' is for it's non-global structure. Because of local nature of p-adic geometry, it does not have higher dimensional topology. So there is no global symmetry exists in this quantization scheme. However, gravity and gauge theory is defined as global symmetry of underlying geometrical structure. So there needs some kind of ways to patch this 'local tissues', only with local symmetry, and emerge global structure with global symmetry. It seems the study of shriek map, which is a compact support embedding from any p-adic topological space to other, via étale cohomology of

[28]Perfectoid space does not directly shed light on questions of number theory. However, Scholze suggested a scenario to extend toward $\mathrm{spec}\mathbb{Z} \times \mathrm{spec}\mathbb{Z}$ world. [Sch17a] Best hope in future direction is to make studying both the mathematical problem of making local argument global and the physical problem of patching local tissue of spacetime into global spacetime in parallel.

diamonds has some hint in this direction.

There even arises philosophical turning point to the notion of time in adic setting. In Berkovich setup, time is still parametrized as semi-norm which is represented as $\mathbb{R}^1_{\geq 0}$. In adic setup it is parametrized by more general valuations which is totally ordered abelian group. It normally is represented $\mathbb{R}^1_{\geq 0} \times \gamma^{\mathbb{Z}}$. So it has higher rank points other than Berkovich's theory defines. The notion of time in this case should be redefined. In general, it needs to be considered as index set for stochastic model over p-adic geometry. More about these kind of issues will be come again in part IV, 'DO philosophy for physics over p-adic geometry'.

In summary, in 'A new approach to quantum mechanics III' the following is a schematic procedure to produce quantum phase space over p-adic geometry.

- Curve over Witt ring of non-Archimedean field or Fargues-Fontaine curve is considered as 1 dimensional phase space.

  $(M \otimes W(K), \tau^*)$ will replace $(M_{\mathbb{C}}, \Omega)$. Here, τ is tilting, Ω is complexified symplectic structure.

- Position and momentum are represented by perfectoid field K and $K^\flat$.

- Tilting equivalence will be read as position-momentum equivalence via Fourier transformations.

- First quantization will be defined over this phase space. Operator algebras of Hilbert space will be replaced by projector algebras defined in part II.

- Topological comparison

  Derived category of deformation quantization over certain 1-dimensioal Poincaré manifold and derived category of p-adic cohomology over Fargues-Fontaine curve is compared.

3.2.3 Overview on part IV and V

In part III, there is no suggestive picture on the dynamics of a particle over perfectoid set-up. For one to have physical meaning of playing with this non-trivial highly symmetrical topological space, dynamics of a quantum particle should not be conceived as classical smooth dynamics. It seems to require rather peculiar point of view on the dynamics of particles on such perfectoid spaces. This philosophical argument will be followed in the name of 'DO philosophy' in part IV. Also, part II and part III establishes first quantization only over 1-dimensional p-adic space. Some perspectives for extending this idea to more general cases are introduced.

Part IV is an essay on the new metaphysics which stimulated the author to develope the idea. The world-view proposed in IV may not be affirmative to others. Hopefully, it triggers more philosophical thoughts about amalgamating the web of ideas in quantum physics and the stochastic process of particles over p-adic geometry.

An introduction to mathematical tools used throughout the series is included in part V. The criterion is to provide an basic introduction to the mathematical language of p-adic geometry for physicists who are assumed to be not familiar with technics of algebraic geometry.

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