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SINGLE VALUED NEUTROSOPHIC EXPONENTIAL SIMILARITY MEASURE FOR MEDICAL DIAGNOSIS AND MULTI ATTRIBUTE DECISION MAKING

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Abstract

Neutrosophic set (NS) is very useful to express incomplete, uncertainty, and inconsistent information in a more general way. In the modern medical technologies, each element can be expressed as NS having different truth – membership, indeterminacy – membership, and falsity – membership degrees. Thus, the similarity measures for NS are important tool to deal with the decision making problems with neutrosophic informations. However, to overcome some drawbacks of existing similarity measures, this paper focus on introducing Single Valued Neutrosophic Exponential Similarity Measure (SVNESM) and weighted SVNESM. Then, we compare the proposed SVNESM with the similarity measures available in the literature and show their efficiency using some numerical examples for overcoming the drawbacks. Finally the similarity measure is applied for a medical diagnosis problem and Multi Attribute Decision Making (MADM) problem.

Keywords Neutrosophic Set (NS), Single Valued Neutrosophic Set (SVNS), Similarity measure (SM), Single Valued Neutrosophic Exponential Similarity Measure (SVNESM), Medical Diagnosis, Multi Attribute Decision Making (MADM)

1. Introduction

Smarandache [10] introduced the concept of neutrosophic set (NS) to deal with imprecise, indeterminate and inconsistent data. Indeterminacy plays an important role in many real world decision making problems. In NS, truth-membership, indeterminacy-membership and falsity-membership are independent. The concept of NSs generalizes, the fuzzy set introduced by Zadeh [18] in 1965 and the intuitionistic fuzzy set proposed by Atanassov [1] in 1986. From philosophical point of view truth-membership, indeterminacy-membership and falsity-membership of the NS assume the value from real standard or non-standard subsets of $]^-0,1^+[$. Realizing the difficulty in applying the neutrosophic sets in realistic problems, Wang [11] introduced the concept of a single valued neutrosophic set (SVNS), which is a subclass of the neutrosophic set.

Neutrosophic set have been applied in the field of medical diagnosis [4], decision making problems [6, 13, 14] etc. Because of the increased volume of information available in medical diagnosis, it contains a lot of incomplete, uncertainty, and inconsistent information to physicians. In the multi attribute decision making (MADM) problems, similarity measure approach can be used in ranking the alternatives and determining the best among them in neutrosophic form [6, 7, 13, 14].

2. Literature Review

Let $A = \{\langle x_j, T_A(x_j), I_A(x_j), F_A(x_j)\rangle \mid x_j \in X\}$ and $B = \{\langle x_j, T_B(x_j), I_B(x_j), F_B(x_j)\rangle \mid x_j \in X\}$ be two SVNSs in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. Then, Ye [15] , Ye [16] and Ye [17] presented the Jaccard, Dice, cosine, Improved Cosine and Tangent similarity measures between SVNSs A and B in vector space, respectively, as follows:

Jaccard Similarity measure [15]

$$S_J(A,B) = \frac{1}{n}\sum_{j=1}^{n} \frac{T_A(x_j)T_B(x_j) + I_A(x_j)I_B(x_j) + F_A(x_j)F_B(x_j)}{\left[\left(T_A^2(x_j) + I_A^2(x_j) + F_A^2(x_j)\right) + \left(T_B^2(x_j) + I_B^2(x_j) + F_B^2(x_j)\right) - \left(T_A(x_j)T_B(x_j) + I_A(x_j)I_B(x_j) + F_A(x_j)F_B(x_j)\right)\right]} \quad (1)$$

Dice Similarity measure [15]

$$S_D(A,B)=\frac{1}{n}\sum_{j=1}^{n}\frac{2\left(T_A(x_j)T_B(x_j)+I_A(x_j)I_B(x_j)+F_A(x_j)F_B(x_j)\right)}{\left(T_A^2(x_j)+I_A^2(x_j)+F_A^2(x_j)\right)+\left(T_B^2(x_j)+I_B^2(x_j)+F_B^2(x_j)\right)}$$

(2)

Cosine Similarity measure [15]

$$S_C(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{T_A(x_j)T_B(x_j)+I_A(x_j)I_B(x_j)+F_A(x_j)F_B(x_j)}{\sqrt{\left(T_A^2(x_j)+I_A^2(x_j)+F_A^2(x_j)\right)}\sqrt{\left(T_B^2(x_j)+I_B^2(x_j)+F_B^2(x_j)\right)}}$$

(3)

Improved Cosine Similarity measure [16]

$$SC_1(A,B)=\frac{1}{n}\sum_{j=1}^{n}\cos\left[\frac{\pi\left(\left|T_A(x_j)-T_B(x_j)\right|\vee\left|I_A(x_j)-I_B(x_j)\right|\vee\left|F_A(x_j)-F_B(x_j)\right|\right)}{2}\right]$$

(4)

$$SC_2(A,B)=\frac{1}{n}\sum_{j=1}^{n}\cos\left[\frac{\pi\left(\left|T_A(x_j)-T_B(x_j)\right|+\left|I_A(x_j)-I_B(x_j)\right|+\left|F_A(x_j)-F_B(x_j)\right|\right)}{6}\right]$$

(5)

Tangent Similarity measure [17]

$$T_1(A,B)=1-\frac{1}{n}\sum_{j=1}^{n}\tan\left[\frac{\pi}{4}\max\left(\left|T_A(x_j)-T_B(x_j)\right|,\ \left|I_A(x_j)-I_B(x_j)\right|,\ \left|F_A(x_j)-F_B(x_j)\right|\right)\right]$$

(6)

$$T_2(A,B)=1-\frac{1}{n}\sum_{j=1}^{n}\tan\left[\frac{\pi}{12}\left(\left|T_A(x_j)-T_B(x_j)\right|+\left|I_A(x_j)-I_B(x_j)\right|+\left|F_A(x_j)-F_B(x_j)\right|\right)\right]$$

(7)

3.2 Drawbacks of the existing similarity measure

Some drawbacks of Eqs. (1) to (7) as follows:

(i) For two SVNSs A and B, if $T_A(x_j)=I_A(x_j)=F_A(x_j)=0$ and/or $T_B(x_j)=I_B(x_j)=F_B(x_j)=0$ for any x_j in X (j=1, 2, …, n), Eqs. (1), (2), and (3) is undefined or unmeaningful.

(ii) If $T_A(x_j)=2T_B(x_j)$, $I_A(x_j)=2I_B(x_j)$, and $F_A(x_j)=2F_B(x_j)$ or $2T_A(x_j)=T_B(x_j)$, $2I_A(x_j)=I_B(x_j)$ and $2F_A(x_j)=F_B(x_j)$ for any x_j in X (j=1, 2, … , n), applying Eq. (3), we have

$$S_C(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{2T_A(x_j)T_B(x_j)+2I_A(x_j)I_B(x_j)+2F_A(x_j)F_B(x_j)}{2\sqrt{\left(T_A^2(x_j)+I_A^2(x_j)+F_A^2(x_j)\right)}\sqrt{\left(T_B^2(x_j)+I_B^2(x_j)+F_B^2(x_j)\right)}}=\frac{1}{n}\sum_{i=1}^{n}\frac{T_A^2(x_j)+I_A^2(x_j)+F_A^2(x_j)}{T_A^2(x_j)+I_A^2(x_j)+F_A^2(x_j)}=1$$

Since $A\neq B$, $S_C(A,B)=1$. This means that it fails to satisfy property (S3) in Definition 2.

(iii) For the SVNSs $A=\{\langle x,0.2,0.3,0.4\rangle\}$, $B=\{\langle x,0.2,0.3,0.4\rangle\}$, $C=\{\langle x,0.4,0.2,0.6\rangle\}$ and $D=\{\langle x,0.2,0.1,0.3\rangle\}$. By applying Eq. (4), we get $SC_1(A,B)=SC_1(C,D)=1$. In this case, improved cosine similarity measure produce an unreasonable phenomenon and also not carry out the recognition for the two SVNSs C and D, as well fails to satisfy property (S3) in Definition 2.

(iv) For the SVNSs $A=\{\langle x,0,1,1\rangle\}$, $B=\{\langle x,0,0,0\rangle\}$ and $C=\{\langle x,1,0,0\rangle\}$; $T_1(B,C)=T_2(B,C)=0$, $T_1(A,B)=0,\ T_2(A,B)=0.7321$, $T_1(A,C)=0,\ T_2(A,C)=0.4223$ implies in Eq. (6) $T_2(A,B)$ carry out higher similarity value and produces an unreasonable phenomenon for the similarity measures of two SVNSs A and B. Hence the similarity measures in Eq. (6) are not suitable for handling pattern recognition and medical diagnosis problems.

3. Comparison of the similarity measures

In this section, to examine the performance of the proposed similarity measure on precision and discriminatory ability, a comparative study is conducted on different sets used in the literature. Let A, B and C be three neutrosophic sets in the universe of discourse. Table 1 is used to compare the proposed similarity measure S_{NS} with the existing similarity measures[15 - 17]. In table 1, five different patterns, case 1 to case 5 are taken and the result obtained by [15 - 17] and S_{NS} are listed. It is seen that the proposed similarity measure S_{NS} can overcome the drawbacks of getting the unreasonable results of the existing similarity measures $S_J(A,B)$ [15], $S_D(A,B)$ [15], $S_C(A,B)$ [15], $C_1(A,B)$ [16], $C_2(A,B)$ [16], $T_1(A,B)$ [17] and $T_2(A,B)$ [17].

From Table 1, S_C [15] cannot carry out the recognition between case 1 and case 5, also produces the unreasonable and undefined phenomenon for case 5 and case 4. Then, we can see that the similarity measures S_J , S_D , C_1 and T_1 cannot carry out the recognition between case 3 and case 4, while the similarity measures C_2 and T_2 can carry out all recognitions. Hence the similarity measure C_2 and T_2 demonstrate stronger discrimination among them and are superior to other similarity measures, while the cosine similarity measure S_C in vector space demonstrates bad discrimination among them and is inferior to other similarity measures. Even though C_2 and T_2 reveal stronger discrimination, it gives the higher similarity value for all the cases discussed in Table 1. Hence the similarity measure C_2 and T_2 are not suitable for handling pattern recognition and medical diagnosis problems. The proposed measure overcomes the shortcomings of the existing measure by defining the similarity measure using the exponential operation. Therefore the similarity measure S_{NS} will be applied to medical diagnosis problems.

Table 1: Similarity measure values of Eqs. (1)-(7), (19)

	Case 1	**Case 2**	**Case 3**	**Case 4**	**Case 5**
A	**<x,0.2,0.3,0.4>**	**<x,0.3,0.2,0.4>**	**<x,1,0,0>**	**<x,1,0,0>**	**<x,0.4,0.2,0.6>**
B	**<x,0.2,0.3,0.4>**	**<x,0.4,0.2,0.3>**	**<x,0,1,1>**	**<x,0,0,0>**	**<x,0.2,0.1,0.3>**
$S_J(A,B)$ [15]	1	0.9333	0	0	0.6667
$S_D(A,B)$ [15]	1	0.9655	0	0	0.8000
$S_C(A,B)$ [15]	1	0.9655	0	Null	1
$C_1(A,B)$ [16]	1	0.9877	0	0	0.8910
$C_2(A,B)$ [16]	1	0.9945	0	0.8660	0.9511
$T_1(A,B)$ [17]	1	0.9213	0	0	0.7599
$T_2(A,B)$ [17]	1	0.9476	0	0.7321	0.8416
$S_{NS}(A,B)$	1	0.8305	0	0.3508	0.5549

4. Applications

In this section, the proposed SVNESM is applied to medical diagnosis problems and multi attribute decision making problems as discussed below.

6.1 Medical Diagnosis using Neutrosophic exponential operation

Let us consider the medical diagnosis problem adopted from [17]. Assume that the set of diagnosis $Q = \{Q_1\,(Viral\ Fever),\ Q_2\,(Malaria),\ Q_3\,(Typhoid),\ Q_4\,(gastritis),\ Q_5\,(Stenocardia)\}$ and a set of symptoms $S = \{S_1\,(Fever),\ S_2\,(Headache), S_3(Stomach\ pain), S_4(Cough), S_5(Chest\ pain)\}$.

Then the attribute values of the considered diseases are represented by SVNSs, shown in Table 2.

In the medical diagnosis, assume that we take a sample from a patient P_1 with all the symptoms, which is represented by the following NSs information:

$$P_1\,(Patient) = \{\langle S_1,\ 0.8,\ 0.2,\ 0.1\rangle, \langle S_2,\ 0.6,\ 0.3,\ 0.1\rangle, \langle S_3,\ 0.2,\ 0.1,\ 0.8\rangle,\ \langle S_4,\ 0.6,\ 0.5,\ 0.1\rangle,\ \langle S_5,\ 0.1,\ 0.4,\ 0.6\rangle\}$$

.

For comparison, we can utilize the existing similarity measure [15 - 17] for handling the diagnosis problem. Using Eqs. (1) to (7) and (19), we can obtain the results of the three similarity measures between the patient P_1 and considered disease $Q_i,\ i = 1,\ 2, 3, 4, 5$ as shown in Table 3.

Table 2: Attribute values of the considered diseases represented by NSs

	S_1(Fever)	S_2(Headache)	S_3(Stomachpain)	S_4(Cough)	S_5(Chestpain)
Q_1(Viral fever)	<s_1, 0.4, 0.6, 0>	<s_2, 0.3, 0.2, 0.5>	<s_3, 0.1, 0.3, 0.7>	<s_4, 0.4, 0.3, 0.3>	<s_5, 0.1, 0.2, 0.7>
Q_2(Malaria)	<s_1, 0.7, 0.3, 0>	<s_2, 0.2, 0.2, 0.6>	<s_3, 0, 0.1, 0.9>	<s_4, 0.7, 0.3, 0>	<s_5, 0.1, 0.1, 0.8>
Q_3(Typhoid)	<s_1, 0.3, 0.4, 0.3>	<s_2, 0.6, 0.3, 0.1>	<s_3, 0.2, 0.1, 0.7>	<s_4, 0.2, 0.2, 0.6>	<s_5, 0.1, 0, 0.9>
Q_4(Gastritis)	<s_1, 0.1, 0.2, 0.7>	<s_2, 0.2, 0.4, 0.4>	<s_3, 0.8, 0.2, 0>	<s_4, 0.2, 0.1, 0.7>	<s_5, 0.2, 0.1, 0.7>
Q_5(Stenocardia)	<s_1, 0.1, 0.1, 0.8>	<s_2, 0, 0.2, 0.8>	<s_3, 0.2, 0, 0.8>	<s_4, 0.2, 0, 0.8>	<s_5, 0.8, 0.1, 0.1>

Table 3: Similarity measure values for NSs information

	Viral Fever (Q_1)	Malaria (Q_2)	Typhoid (Q_3)	Gastritis (Q_4)	Stenocardia (Q_5)
$S_J(A,B)$ [15]	0.7395	**0.7922**	0.7090	0.3854	0.3279
$S_D(A,B)$ [15]	0.8398	**0.8635**	0.8029	0.5131	0.4230

$S_C(A,B)$ [15]	0.8505	**0.8661**	0.8185	0.5148	0.4244
$C_1(A,B)$ [16]	0.8942	**0.8976**	0.8422	0.6102	0.5607
$C_2(A,B)$ [16]	0.9443	**0.9571**	0.9264	0.8214	0.7650
$T_1(A,B)$ [17]	0.8852	**0.8867**	0.8399	0.6035	0.5467
$T_2(A,B)$ [17]	0.9333	**0.9425**	0.9102	0.8024	0.7549
$S_{NS}(A,B)$	0.5665	**0.6316**	0.6154	0.3356	0.3235

From Table 3, the largest similarity values indicate the proper diagnosis. Hence the patient P_1 suffers from malaria. Evidently, the medical diagnosis using various similarity measures indicate the same diagnosis results and reveal the efficiency of these diagnosis.

6.2 Multi Attribute Decision Making Problems

For convenient comparison, an illustrative example about the selection problem of investment alternatives adopted from [6] is provided to demonstrate the applications and effectiveness of the proposed SVNESM.

An investment company needs to invest a sum of money to the best industry. Then, four possible alternatives are considered as four potential industries: (i) P_1 - a car company; (ii) P_2 - a food company; (iii) P_3 - a computer company; (iv) P_4 - an arms company. The four possible alternatives in the decision-making process, must satisfy the requirements of the three attributes: (i) R_1 is the risk; (ii) R_2 is the growth; (iii) R_3 is the environment, where the attributes R_1 and R_2 are benefit types and the attribute R_3 is a cost type. Assume that the weighting vector of the attributes is given by w= (0.35, 0.25, 0.4). By the statistical analysis and the evaluation of investment data regarding the four possible alternatives of P_i *(i=1, 2, 3, 4)* over the three attributes of R_j *(j=1, 2, 3)*, we can establish the following decision matrix [6]:

$$M\left(p_{ij}\right)_{4\times3}=\begin{bmatrix} <0.4,0.2,0.3> & <0.4,0.1,0.2> & <0.7,0.2,0.4> \\ <0.6,0.1,0.2> & <0.6,0.1,0.2> & <0.3,0.5,0.8> \\ <0.3,0.2,0.3> & <0.5,0.2,0.3> & <0.4,0.2,0.7> \\ <0.7,0.1,0.2> & <0.6,0.1,0.1> & <0.6,0.3,0.8> \end{bmatrix}$$

Then, we use Equation (19) with neutrosophic information, which is described by the following procedures:

Step1: Establish an ideal solution (an ideal alternative) $P^* = \{p_1^*, p_2^*, ..., p_n^*\}$ expressed by the ideal NS $P^* = \{< 0.7, 0.1, 0.2 >, < 0.6, 0.1, 0.1 >, < 0.3, 0.5, 0.8 >\}$ corresponding to the benefit types and cost types of attributes.

Step 2: Calculated the weighted SVNESM between the alternative P_1 and the ideal solution P^* by using Equation (8)

The weighted SVNESM between each alternative P_i (i=1, 2, 3, 4) and the ideal solution P^* can be given as the following values of $S_{NS}(P_1, P_1^*) = 0.4936$, $S_{NS}(P_2, P_2^*) = 0.9416$, $S_{NS}(P_3, P_3^*) = 0.6181$, $S_{NS}(P_4, P_4^*) = 0.8722$, the ranking order of the alternatives is $P_2 > P_4 > P_3 > P_1$ and the best one is P_2. These results are the same as in [6]. Hence, the proposed neutrosophic decision-making method based on the exponential function illustrates its feasibility and effectiveness. Compared with existing decision-making methods based on the NSs, the proposed decision-making method based on the Neutrosophic exponential similarity measure shows that it is simpler to employ than existing neutrosophic decision-making methods in [6, 7, 9] under neutrosophic environments because the decision-making method proposed in this paper implies its simple algorithms and decision steps in the neutrosophic decision-making problems.

5. Conclusions

This paper presented a new similarity measure of SVNESM based on the exponential function. Then, by considering the importance of each element, the weighted similarity measures of SVNESM are introduced. Several novel measures are available in the literature to access the similarity measures of NSs but the proposed measure correlates better than the other measures. The proposed similarity measure effectively deals with some demanding situations by satisfying the basic properties, as well overcome the drawbacks of the existing similarity measures. In this study, the proposed similarity measure can be effectively used in real applications of medical diagnosis and decision making.

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