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Strong Degrees in Single Valued Neutrosophic Graphs

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***Abstract*—The concept of single valued neutrosophic graphs (SVNGs) generalizes the concept of fuzzy graphs and intuitionistic fuzzy graphs. The purpose of this research paper is to define different types of strong degrees in SVNGs and introduce novel concepts, such as the vertex truth-membership, vertex indeterminacy-membership and falsity-membership sequence in SVNG with proof and numerical illustrations.**

Keywords—Single valued neutrosophic graph (SVNG); neutrosophic set; sequence; strong degree

I. INTRODUCTION

In [1], [3] Smarandache explored the notion of neutrosophic sets (NS in short) as a powerful tool which extends the concepts of crisp set, fuzzy sets and intuitionistic fuzzy sets [2]-[6]. This concept deals with uncertain, incomplete and indeterminate information that exist in real world. The concept of NS sets associate to each element of the set a degree of membership $T_A(x)$,a degree of indeterminacy $I_A(x)$ and a degree of falsity $F_A(x)$, in which each membership degree is a real standard or non-standard subset of the nonstandard unit $]^-0, 1^+[$. Smaranadache [1], [2] and Wang [7] defined the concept of single valued neutrosophic sets (SVNS), an instance of NS, to deal with real application. In [8], the readers can found a rich literature on SVNS.

In more recent times, combining the concepts of NSs, interval valued neutrosophic sets (IVNSs) and bipolar neutrosophic sets with graph theory, *Broumi et al.* introduced various types of neutrosophic graphs including single valued neutrosophic graphs (SVNGs for short) [9], [11], [14], interval valued neutrosophic graphs [13], [18], [20], bipolar neutrosophic graphs [10], [12], all these graphs are studied deeply. Later on, the same authors presented some papers for solving the shortest path problem on a network having single valued neutrosophic edges length [17], interval valued neutrosophic edge length [32], bipolar neutrosophic edge length [21], trapezoidal neutrosophic numbers [15], SV-trapezoidal neutrosophic numbers [16], triangular fuzzy neutrosophic [19].Our approach of neutrosophic graphs are different from that of Akram et al. [26]-[28] since while Akram considers, for the neutrosophic environment (<=, <=, >=) we do (<=, >=, >=) which is better, since while T is a positive quality, I, F are considered negative qualities. Akram et al. include "I" as a positive quality together with "T". So our papers improve Akram et al.'s papers. After that, several authors are focused on the study of SVNGs and many extensions of SVNGs have been developed. Hamidi and Borumand Saeid [25] defined the notion of accessible-SVNGs and apply it social networks. In [24], Mehra and Manjeet defined the notion of single valued neutrosophic signed graphs. Hassan et al. [30] proposed some kinds of bipolar neutrosophic graphs. Naz et al. [23] studied some basic operations on SVNGs and introduced vertex degree of these operations for SVNGs and provided an application of single valued neutrosophic digraph (SVNDG) in travel time. Ashraf et al. [22] defined new classes of SVNGs and studied some of its important properties. They solved a multi-attribute decision making problem using a SVNDG. Mullai [31] solved the spanning tree problem in bipolar neutrosophic environment and gave a numerical example.

Motivated by the Karunambigai work's [29].The concept of strong degree of intuitionistic fuzzy graphs is extended to strong degree of SVNGs

This paper has been organized in five sections. In Section 2, we firstly review some basic concepts related to neutrosophic set, single valued neutrosophic sets and SVNGs. In Section 3, different strong degree of SVNGs are proposed and studied with proof and example. In Section 4, the concepts of vertex truth-membership, vertex indeterminacy-

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membership and vertex falsity- membership is discussed. Lastly, Section 5 concludes the paper.

II. Preliminareis and Defintions

In the following, we briefly describe some basic concepts related to neutrosophic sets, single valued neutrosophic sets and SVNGs.

Definition 2.1 [1] Given the universal set ζ. A neutrosophic set A on ζ is characterized by a truth membership function T_A ,an indeterminacy membership function I_A and falsity membership function F_A, where $T_A, I_A, F_A: \zeta \rightarrow]^{-}0,1^{+}[$. For all x∈ ζ, x=(x,$T_A(x), I_A(x), F_A(x)$) ∈ A is neutrosophic element of A.

The neutrosophic set can be written in the following form:

$$A = \{<x: T_A(x), I_A(x), F_A(x)>, x\in \zeta\} \quad (1)$$

with the condition

$$^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+} \quad (2)$$

Definition 2.2 [7] Given the universal set ζ. A single valued neutrosophic set A on ζ is characterized by a truth membership function T_A , an indeterminacy membership function I_A and falsity membership function F_A , where T_A , I_A , F_a : $\zeta \rightarrow [0,1]$. For all x ∈ ζ , x=(x, $T_A(x)$, $I_A(x), F_A(x)$) ∈ A is a single valued neutrosophic element of A.

The single valued neutrosophic set can be written in the following form:

$$A = \{<x: T_A(x), I_A(x), F_A(x)>, x\in \zeta\} \quad (3)$$

with the condition

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3 \quad (4)$$

Definition 2.3 [14] ASVN-graph G is of the form G=(A,B) where A

1. A={$v_1, v_2, \ldots v_n$ }Such that the functions T_A: A→ [0 ,1], $I_A : A \rightarrow [0, 1]$, F_A :A→[0, 1]denote the truth-membership function, an indeterminacy-membership function and falsity-membership function of the element $v_i \in A$ respectively and

$0\leq t_A(v_i)+i_A(v_i)+f_A(v_i) \leq 3 \forall v_i \in A$

i=1,2,…,n.

2.B={(v_i, v_j); $(v_i, v_j) \in A \times A$} and the function T_B:B→ [0,1],

I_B:B →[0,1] , F_B:B→ [0, 1] are defined by

$$T_B(v_i, v_j) \leq \min(T_A(v_i), T_A(v_j)) \quad (5)$$

$$I_B(v_i, v_j) \geq \max(I_A(v_i), I_A(v_j)) \quad (6)$$

$$F_B(v_i, v_j) \geq \max(F_A(v_i), F_A(v_j)) \quad (7)$$

Where T_B , I_B , F_B denotes the truth-membership function, indeterminacy membership function and falsity membership function of the edge (v_i, v_j)∈ B respectively where

$$0\leq T_B(v_i, v_j)+ I_B(v_i, v_j) +F_B(v_i, v_j) \leq 3 \quad (8)$$

$\forall (v_i, v_j) \in B$, i, j∈{1,2,…,n}

A is called the vertex set of G and B is the edge set of G.

The following Fig. 1 represented a graphical representation of single valued neutrosophic graph.

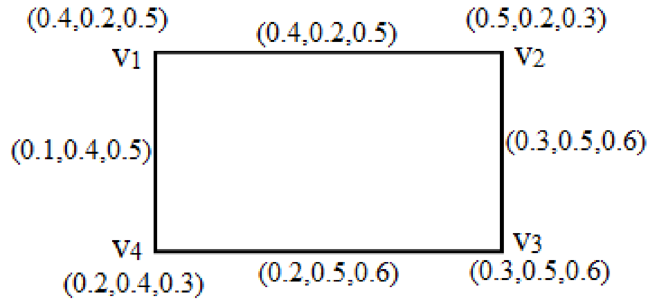

Fig. 1. Single valued neutrosophic graph.

III. Strong Degree in Single Valuedneutrosophic Graph

The following section introduces new concepts and proves their properties.

Definition 3.1 Given the SVN-graph G= (V, E). The T-strong degree of a vertex $v_i \in V$ is defined as $d_{s(T)}(v_i) = \sum_{e_{ij}\in E} T_{ij}$, e_{ij} are strong edges incident at v_i.

Definition 3.2 Given the SVN-graph G=(V, E).The I-strong degree of a vertex $v_i \in V$ is defined as $d_{s(I)}(v_i) = \sum_{e_{ij}\in E} I_{ij}$, e_{ij} are strong edges incident at v_i.

Definition 3.3 Given the SVN-graph G=(V, E).The F-strong degree of a vertex $v_i \in V$ is defined as $d_{s(F)}(v_i) = \sum_{e_{ij}\in E} F_{ij}$, e_{ij} are strong edges incident at v_i.

Definition3.4 Let G = (V,E) be SVNG. The strong degree of a vertex $v_i \in V$ is as follow

$$d_s(v_i) = \left[\sum_{e_{ij}\in E} T_{ij}, \sum_{e_{ij}\in E} I_{ij}, \sum_{e_{ij}\in E} F_{ij}\right]$$, where e_{ij} are strong edge incident at v_i.

Definition 3.5 Let G=(V, E) be a SVNG. The minimum strong degree of G is defined as

$$\delta_s(\mathrm{G}) = (\delta_{s(T)}(\mathrm{G}), \delta_{s(I)}(\mathrm{G}), \delta_{s(F)}(\mathrm{G})$$, where

$\delta_{s(\mathrm{T})}(\mathrm{G}) = \wedge\{d_{s(\mathrm{T})}(\mathrm{v}_i) / \mathrm{v}_i \in V\}$ is the minimum T-strong degree of G.

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$\delta_{s(I)}(G)=\wedge\{d_{s(\mathrm{I})}(\mathrm{v}_i)/\mathrm{v}_i\in V\}$ is the minimum I-strong degree of G.

$\delta_{s(F)}(G)=\wedge\{d_{s(F)}(\mathrm{v}_i)/\mathrm{v}_i\in V\}$ is the minimum F-Strong degree of G.

Definition 3.6 Given the SVN-graph G=(V, E). The maximum strong degree of G is defined as

$\Delta_s(G)=(\Delta_{s(\mathrm{T})}(\mathrm{G}),\Delta_{s(\mathrm{I})}(\mathrm{G}),\Delta_{s(\mathrm{F})}(\mathrm{G}))$, where

$\Delta_{s(T)}(G)=\vee\{d_{s(\mathrm{T})}(\mathrm{v}_i)/\mathrm{v}_i\in V\}$ is the maximum T-strong degree of G.

$\Delta_{s(I)}(G)=\vee\{d_{s(I)}(\mathrm{v}_i)/\mathrm{v}_i\in V\}$ is the maximum I-strong degree of G.

$\Delta_{s(\mathrm{F})}(\mathrm{G})=\vee\{d_{s(\mathrm{F})}(\mathrm{v}_i)/\mathrm{v}_i\in V\}$ is the maximum F-Strong degree of G.

Definition 3.7 Let G be a SVNG, the T-total strong degree of a vertex $v_i\in V$ in G is defined as $td_{s(\mathrm{T})}(v_i)=d_{s(\mathrm{T})}(v_i)+T_i$,

Definition 3.8 Let G be a SVNG, the I-total strong degree of a vertex in G is defined as $v_i\in V$ $td_{s(\mathrm{I})}(v_i)=d_{s(\mathrm{I})}(v_i)+I_i$,

Definition 3.9 Let G be a SVNG, the F-total strong degree of a vertex $v_i\in V$ in G is defined $td_{s(F)}(v_i)=d_{s(F)}(v_i)+F_i$,

Definition 3.10 Let G be a SVNG, the total strong degree of a vertex $v_i\in V$ in G is defined as

$$td_s(v_i)=\left[td_{s(T)}(v_i),td_{s(I)}(v_i),td_{s(F)}(v_i)\right]$$

Definition 3.11 Given the SVN-graph G=(V, E). The minimum total strong degree of G is defined as

$\delta_{ts}(G)=(\delta_{ts(T)}(G),\delta_{ts(I)}(G),\delta_{ts(F)}(G))$, where

$\delta_{ts(T)}(G)=\wedge\{d_{ts(T)}(\mathrm{v}_i)/\mathrm{v}_i\in V\}$ is the minimum T-total strong degree of G.

$\delta_{ts(I)}(G)=\wedge\{d_{ts(\mathrm{I})}(\mathrm{v}_i)/\mathrm{v}_i\in V\}$ is the minimum I-total strong degree of G.

$\delta_{ts(F)}(G)=\wedge\{d_{ts(F)}(\mathrm{v}_i)/\mathrm{v}_i\in V\}$ is the minimum F-total strong degree of G.

Definition 3.12 Given the SVN-graph G = (V, E). The maximum total strong degree of G is defined as:

$\Delta_{ts}(G)=(\Delta_{ts(T)}(G),\Delta_{ts(I)}(\mathrm{G}),\Delta_{ts(F)}(G))$, where

$\Delta_{ts(T)}(\mathrm{G})=\vee\{d_{ts(\mathrm{T})}(\mathrm{v}_i)/\mathrm{v}_i\in V\}$ is the maximum T-total strong degree of G.

$\Delta_{ts(I)}(G)=\vee\{d_{ts(I)}(\mathrm{v}_i)/\mathrm{v}_i\in V\}$ is the maximum I-total strong degree of G.

$\Delta_{ts(\mathrm{F})}(\mathrm{G})=\vee\{d_{ts(\mathrm{F})}(\mathrm{v}_i)/\mathrm{v}_i\in V\}$ is the maximum F-total strong degree of G.

Definition 3.13 Given the SVN-graph G=(V,E). The T-strong size of a SVNG is defined as

$S_{s(T)}(G)=\sum_{v_i\neq v_j}T_{ij}$ where T_{ij} is the membership of strong edge $e_{ij}\in E$.

Definition 3.14 Given the SVN-graph G=(V, E). The I-strong size of a SVNG is defined as

$S_{s(I)}(G)=\sum_{v_i\neq v_j}I_{ij}$ where I_{ij} is the indeterminacy-membership of strong edge $e_{ij}\in E$.

Definition 3.15 Given the SVN-graph G=(V, E). The F-strong size of a SVNG is defined as

$S_{s(F)}(G)=\sum_{v_i\neq v_j}F_{ij}$ where F_{ij} is the non-membership of strong edge $e_{ij}\in E$.

Definition 3.16 Given the SVN-graph G=(V, E). The strong size of a SVNG is defined as

$$S_s(G)=\left[S_{s(\mathrm{T})}(\mathrm{G}),S_{s(\mathrm{I})}(\mathrm{G}),S_{s(F)}(\mathrm{G})\right]$$

Definition 3.17 Given the SVN-graph G=(V,E). The T-strong order of a SVNG is defined as

$O_{s(T)}(G)=\sum_{v_i\in V}T_i$ where v_i is the strong vertex in G.

Definition 3.18 Given the SVN-graph G=(V, E). The I-strong order of a SVNG is defined as

$O_{s(\mathrm{I})}(G)=\sum_{v_i\in V}I_i$ where v_i is the strong vertex in G.

Definition 3.19 Given the SVN-graph G=(V, E).The F-strong order of a SVNG is defined as

$O_{s(\mathrm{F})}(G)=\sum_{v_i\in V}F_i$ where v_i is the strong vertex in G.

Definition 3.20 Given the SVN-graph G=(V, E). The strong order of a SVNG is defined as

$$O_s(G)=\left[O_{s(T)}(G),O_{s(I)}(G),O_{s(F)}(G)\right]$$

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Definition 3.21 Let G be a SVNG. If $d_{s(T)}(v_i)=k_1$, $d_{s(I)}(v_i)=k_2$ and , $d_{s(F)}(v_i)=k_3$ for all $v_i \in V$, then the SVNG is called as (k_1, k_2, k_3) - strong constant SVNG (or) Strong constant SVNG of degree (k_1, k_2, k_3).

Definition 3.22 Let G be a SVNG. If $td_{s(T)}(v_i)=r_1$, $td_{s(I)}(v_i)=r_2$ and , $d_{s(F)}(v_i)=r_3$ for all $v_i \in V$, then the SVNG is called as (r_1, r_2, r_3) – totally strong constant SVNG (or) totally strong constant SVNG of degree (r_1, r_2, r_3).

Proposition 3.23 In a SVNG, G

$2\, S_{s(T)}(G)=\sum_{i=1}^{n} d_{s(T)}(v_i)$, $2\, S_{s(I)}(G)=\sum_{i=1}^{n} d_{s(I)}(v_i)$ and

$2\, S_{s(F)}(G)=\sum_{i=1}^{n} d_{s(F)}(v_i)$

Proposition 3.24 In a connected SVNG,

1) $d_{S(T)}(v_i) \leq d_{Ti}$, $d_{S(I)}(v_i) \leq d_{Ii}$ and $d_{S(F)}(v_i) \leq d_{Fi}$
2) $td_{S(T)}(v_i) \leq td_{Ti}$, $td_{S(I)}(v_i) \leq td_{Ii}$ and $td_{S(F)}(v_i) \leq td_{Fi}$.

Proposition 3.25 Let G be a SVNG where crisp graph G^* is an odd cycle. Then G is strong constant if f $< T_{ij}, I_{ij}, F_{ij} >$ is constant function for every $e_{ij} \in$ E.

Proposition 3.26 Let G be a SVNG where crisp graph G^* is an even cycle. Then G is strong constant if f $< T_{ij}, I_{ij}, F_{ij} >$ is constant function or alternate edges have same true membership, indeterminate membership and false membership for every $e_{ij} \in$ E.

Remark 3.27 The above proposition 3.25 and proposition 3.26 hold for totally strong constant SVNG, if $< T_i, I_i, F_i >$ is a constant function.

Remark 3.28 A complete SVNG need not be a strong constant SVNG and totally strong constant SVNG.

Remark 3.29 A strong SVNG need not be a strong constant SVNG and totally strong constant SVNG.

Remark 3.30 For a strong vertex $v_i \in$ V,

1) $d_T(v_i) = d_{sT}(v_i)$, $d_I(v_i) = d_{sI}(v_i)$ and $d_F(v_i) = d_{sF}(v_i)$
2) $td_T(v_i) = td_{sT}(v_i)$, $td_I(v_i) = td_{sI}(v_i)$ and $td_F(v_i) = td_{sF}(v_i)$

Theorem 3.31 Let G be a complete SVNG with V = $\{v_1, v_2, \ldots, v_n\}$ such that $T_1 \leq T_2 \leq T_3 \leq \ldots \leq T_n$, $I_1 \geq I_2 \geq I_3 \geq \ldots \geq I_n$ and $F_1 \geq F_2 \geq F_3 \geq \ldots \geq F_n$ Then

1) T_{1j} is minimum edge truth membership, I_{1j} is the maximum edge indeterminacy membership and F_{1j} is the maximum edge falsity membership of e_{ij} emits from v_1 for all j = 2,3,4,…, n.

2) T_{in} is maximum edge truth membership, I_{in} is the minimum edge indeterminacy membership and F_{in} is the minimum edge falsity membership of among all edges from emits from v_i to v_n for all i = 1, 2,3,4,…, n-1.

3) $td_T(v_1) = \delta_{td_T}(G)$ = n.T$_1$, $td_I(v_1) = \Delta_{td_I}(G)$ = n.I$_1$ and $td_F(v_1) = \Delta_{td_F}(G)$ = n.F$_1$.

4) $td_T(v_n) = \Delta_{td_T}(G) = \sum_{i=1}^{n} T_i$, $td_I(v_n) = \delta_{td_I}(G) = \sum_{i=1}^{n} I_i$, and $td_F(v_n) = \delta_{td_F}(G) = \sum_{i=1}^{n} F_i$.

Proof: Throughout the proof, suppose that $T_1 \leq T_2 \leq T_3 \leq \ldots \leq T_n$, $I_1 \geq I_2 \geq I_3 \geq \ldots \geq I_n$ and $F_1 \geq F_2 \geq F_3 \geq \ldots \geq F_n$.

1) To prove that T_{1j} is minimum edge truth membership, I_{1j} is the maximum edge indeterminacy membership and F_{1j} is the maximum edge falsity membership of e_{ij} emits from $v_1 \forall$j=2,3,….,n. Assume the contrary i.e. e_{1l} is not an edge of minimum true membership, maximum indeterminate membership and maximum false membership emits from v_l. Also let e_{kl} , 2 ≤ k ≤ n,k≠ 1 be an edge with minimum true membership,, maximum indeterminate membership and maximum false membership emits from v_k.

Being a complete SVNG,

T_{1l}= min { T_1, T_l } , I_{1l}= max { I_1, I_l } and F_{1l}= max { F_1, F_l }

Then T_{kl}= min { T_k, T_l } , I_{kl}= max { I_k, I_l } and

F_{kl}= max { F_k, F_l }

Since $T_{kl} < T_{1l} \Rightarrow$ min { T_k, T_l }< min { T_1, T_l }

Thus either $T_k < T_1$ or $T_l < T_1$.

Also since $I_{kl} > I_{1l} \Rightarrow$ max { I_k, I_l } > max { I_1, I_l }, so either $I_k > I_1$ or $I_l > I_1$.

Since l, k ≠ 1, this is contradiction to our vertex assumption that T_1 is the unique minimum vertex true membership, I_1 is the maximum vertex indeterminate membership and F_1 is the maximum vertex false membership.

Hence T_{1j} is minimum edge true membership, I_{1j} is the maximum edge indeterminate membership and F_{1j} is the maximum edge false membership of e_{ij} emits from v_1 to v_j for all j = 2, 3, 4,…, n.

2) On the contrary, assume let e_{kn} is not an edge with maximum true membership, minimum indeterminate membership and minimum false membership emits from v_k for 1 ≤ k ≤ n-1. On the other hand, let e_{kr} be an edge with maximum true membership, minimum indeterminate membership and minimum false membership emits from v_r from 1 ≤ r ≤ n-1, k ≠ r.

Then $T_{kr} > T_{kn} \Rightarrow$ min { T_k, T_r } > min{ T_k, T_n } = T_k , so $T_r > T_k$,

$I_{kr} < I_{kn} \Rightarrow$max{$I_k, I_r$ } <max{ I_k, I_n } = I_k , so $I_r < I_k$ and

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Similarly $F_{kr} < F_{kn} \Rightarrow \max\ \{ F_k, F_r \} < \max\ \{ F_k, F_n \} = F_k,\ \Rightarrow F_r < F_k$

So $T_{kr} = T_k = T_{kn}$, $I_{kr} = I_k = I_{kn}$ and $F_{kr} = F_k = F_{kn}$, which is a contradiction. Hence e_{kn} is an edge with maximum true membership, minimum indeterminate membership and minimum false membership among all edges emits from v_k to v_n.

3) Now

$td_T(v_1) = d_T(v_1) + T_1$

$= \sum_{e_{ij}\in E} T_{1j} + T_1 = \sum_{j=2}^{n} T_{1j} + T_1$

$= (n-1).T_1 + T_1 = nT_1 - T_1 + T_1 = nT_1$,

$td_I(v_1) = d_I(v_1) + I_1$

$= \sum_{e_{ij}\in E} I_{1j} + I_1 = \sum_{j=2}^{n} I_{1j} + I_1$

$= (n-1).I_1 + I_1 = nI_1 - I_1 + I_1 = nI_1$ and

Similarly,

$td_F(v_1) = d_F(v_1) + F_1$

$= \sum_{e_{ij}\in E} F_{1j} + F_1 = \sum_{j=2}^{n} F_{1j} + F_1$

$= (n-1).F_1 + F_1 = nF_1 - F_1 + F_1 = nF_1$

Suppose that $td_T(v_1) \neq \delta_{td_T}(G)$ and let v_k , k ≠ 1 be a vertex in G with minimum T- total degree.

Then,

$td_T(v_1) > td_T(v_k)$

$$\Rightarrow \sum_{i=2}^{n} T_{1i} + T_1 > \sum_{k\neq 1, k\neq j} T_{kj} + T_k$$

$$\Rightarrow \sum_{i=2}^{n} T_1 \wedge T_i + T_1 > \sum_{k\neq 1, k\neq j} T_k \wedge T_j + T_k$$

Since $T_1 \wedge T_i = T_1$ for i = 1, 2, 3, …, n and for all other indices j, $T_k \wedge T_j > T_1$, it follow that

$(n-1).T_1 + T_1 > \sum_{k\neq 1, k\neq j} T_k \wedge T_j + T_k > (n-1).T_1 + T_1$

Hence, $td_T(v_1) > td_T(v_1)$, a contradiction.

Therefore, $td_T(v_1) = \delta_{td_T}(G)$.

Suppose that $td_I(v_1) \neq \Delta_{td_I}(G)$ and let v_k , k ≠ 1 be a vertex in G with maximum I- total degree.

Then,

$td_I(v_1) < td_I(v_k)$

$$\Rightarrow \sum_{i=2}^{n} I_{1i} + I_1 < \sum_{k\neq 1, k\neq j} I_{kj} + I_k$$

$$\Rightarrow \sum_{i=2}^{n} I_1 \vee I_i + I_1 < \sum_{k\neq 1, k\neq j} I_k \vee I_j + I_k$$

Since $I_1 \vee I_i = I_1$ for i = 1, 2, 3, …, n and for all other indices j, $I_k \vee I_j < I_1$, it follow that

$(n-1).I_1 + I_1 < \sum_{k\neq 1, k\neq j} I_k \vee I_j + I_k < (n-1).I_1 + I_1$

So that $td_I(v_1) < td_I(v_1)$, a contradiction.

Therefore, $td_I(v_1) = \Delta_{td_I}(G)$.

Also, Suppose that $td_F(v_1) \neq \Delta_{td_F}(G)$ and let v_k , k ≠ 1 be a vertex in G with maximum F- total degree.

Then

$td_F(v_1) < td_F(v_k)$

$$\Rightarrow \sum_{i=2}^{n} F_{1i} + F_1 < \sum_{k\neq 1, k\neq j} F_{kj} + F_k$$

$$\Rightarrow \sum_{i=2}^{n} F_1 \vee F_i + F_1 < \sum_{k\neq 1, k\neq j} F_k \vee F_j + F_k$$

Since $F_1 \vee F_i = F_1$ for i = 1, 2, 3, …, n and for all other indices j, $F_k \vee F_j < F_1$, it follow that

$(n-1).F_1 + F_1 < \sum_{k\neq 1, k\neq j} F_k \vee F_j + F_k < (n-1).F_1 + F_1$

So that $td_F(v_1) < td_F(v_1)$, a contradiction .

Therefore, $td_F(v_1) = \Delta_{td_F}(G)$.

Hence,

$td_T(v_1) = \delta_{td_T}(G) = n.T_1$,

$td_I(v_1) = \Delta_{td_I}(G) = n.I_1$ and

$td_F(v_1) = \Delta_{td_F}(G) = n.F_1$.

4) Since, $T_n > T_i$, $I_n < I_i$ and $F_n < F_i$, i = 1, 2, 3, …, n-1 and G is complete

$T_{ni} = T_n \wedge T_i = T_i$, $I_{ni} = I_n \vee I_i = I_i$ and $F_{ni} = F_n \vee F_i = F_i$.

Hence, $td_T(v_n) = \sum_{i=1}^{n-1} T_{ni} + T_n$

$= \sum_{i=1}^{n-1} (T_n \wedge T_i) + T_n = \sum_{i=1}^{n-1} T_i + T_n$

$= \sum_{i=1}^{n} T_i$,

$td_I(v_n) = \sum_{i=1}^{n-1} I_{ni} + I_n$

$= \sum_{i=1}^{n-1} (I_n \vee I_i) + I_n = \sum_{i=1}^{n-1} I_i + I_n$

$= \sum_{i=1}^{n} I_i$

And $td_F(v_n) = \sum_{i=1}^{n-1} F_{ni} + F_n$

$= \sum_{i=1}^{n-1} (F_n \vee F_i) + F_n = \sum_{i=1}^{n-1} F_i + F_n$

$= \sum_{i=1}^{n} F_i$.

Suppose that $td_T(v_n) \neq \Delta_{td_T}(G)$. Let v_l , 1 ≤ l ≤ n-1 be a vertex in G such that $td_T(v_l) = \Delta_{td_T}(G)$ and

$td_T(v_n) < td_T(v_l)$. In addition,

$td_T(v_l) = [\ \sum_{i=1}^{l-1} T_{il} + \sum_{i=l+1}^{n-1} T_{il} + T_{nl}\ ] + T_l$

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$\leq [\sum_{i=1}^{l-1} T_i$ + (n-l)$T_l + T_l$] + T_l

$\leq \sum_{i=1}^{n-1} T_i + T_l$

$\leq \sum_{i=1}^{n} T_i$ = t d_T (v_n). Thus t d_T (v_n) $\geq$ t d_T (v_l) , contradiction. So, t$d_T(v_n)$ = Δ_{td_T}(G) = $\sum_{i=1}^{n} T_i$.

Suppose that t$d_I(v_n)$ $\neq \delta_{td_I}$(G). Let v_l , 1 ≤ l ≤ n-1 be a vertex in G such that t$d_I(v_l)$ = δ_{td_I}(G) and t$d_I(v_n) >$ t$d_I v_l)$. In addition,

t$d_I(v_l)$ = [$\sum_{i=1}^{l-1} I_{il} + \sum_{i=l+1}^{n-1} I_{il} + I_{nl}$] + I_l

$\geq [\sum_{i=1}^{l-1} I_i$ + (n-l)$I_l + I_l$] + I_l

$\geq \sum_{i=1}^{n-1} I_i + I_l$

$\geq \sum_{i=1}^{n} I_i$ = t$d_I(v_n)$. Thus t$d_I(v_n) \leq$ t$d_I(v_l)$, contradiction. So, t$d_I(v_n)$ = δ_{td_I}(G) = $\sum_{i=1}^{n} I_i$.

Also, suppose that t$d_F(v_n)$ $\neq \delta_{td_F}$(G). Let v_l , 1 ≤ l ≤ n-1 be a vertex in G such that t$d_F(v_l)$ = δ_{td_F}(G) and t$d_F(v_n) >$ t$d_F v_l)$. In addition,

t$d_F(v_l)$ = [$\sum_{i=1}^{l-1} F_{il} + \sum_{i=l+1}^{n-1} F_{il} + F_{nl}$] + F_l

$\geq [\sum_{i=1}^{l-1} F_i$ + (n-l)$F_l + F_l$] + F_l

$\geq \sum_{i=1}^{n-1} F_i + F_l$

$\geq \sum_{i=1}^{n} F_i$ = t d_F (v_n). Thus td_F (v_n) $\leq$ t d_F (v_l) , contradiction. So, $td_F(v_n)$ = δ_{td_F}(G) = $\sum_{i=1}^{n} F_i$.

Hence the lemma is proved.

Remark 3.32 In a complete SVNG G,

1) There exists at least one pair of verticesv_i and v_j such that d_{T_i}= d_{T_j}= Δ_T(G) , d_{I_i}= d_{I_i} = δ_I(G) and d_{F_i}= d_{F_i} = δ_F(G),
2) t$d_T(v_i)$=$O_T(G) = \Delta_{td_T}(G)$, t$d_I(v_i)$= $O_I(G) = \delta_{td_I}$(G) and $td_F(v_i)$=O_F (G)=δ_{td_F}(G)for a vertex $v_i \in$ V,
3) $\sum_{i=1}^{n} td_T(v_i)$ = 2 S_T (G)+ O_T (G) , $\sum_{i=1}^{n}$ t$d_I(v_i)$ = $2S_I(G) + O_I(G)$ and $\sum_{i=1}^{n}$ t$d_F(v_i)$ = 2S_F(G) + O_F(G).

IV. Vertex Truth Membership , Vertex Indterminacy Membership and Vertex Falsity Memebership Sequence in SVNG

In this section, vertex truth membership, vertex indeterminacy membership and vertex falsity membership sequences are defined in SVNGs.

Definition 4.1 Given a SVN-graph G with $|V|$ = n. The vertex truth membership sequence of G is defined to be $\{x_i\}_{i=1}^{n}$ with $x_1 \leq x_2 \leq x_3 \leq \ldots \leq x_n$ wherex_i , 0< $x_i \leq$ 1, is the truth membership value of the vertex v_i when vertices are arranged so that their truth membership values are non-decreasing.

Particular, x_1 is smallest vertex truth membership value and x_n is largest vertex truth membership value in G.

Note 4.2 If vertex truth membership sequence x_i is repeated more than once in G, say r ≠ 1 times, then it is denoted by ${x_i}^r$ in the sequence.

Example 4.3 In Fig. 2 the vertex truth membership sequence of G is {0.1, 0.1, 0.3, 0.3, 0.4, 0.8 } or { 0.1^2, 0.3^2 , 0.4, 0.8 }.

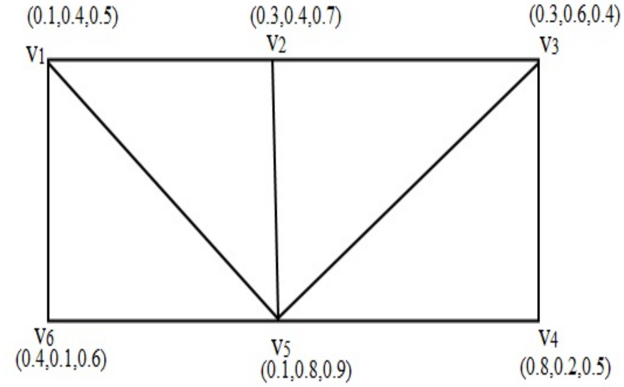

Fig. 2. Vertex truth membership sequence.

Definition 4.4 Let G be a SVNG with $|V|$ = n. The vertex indeterminacy membership sequence of G is defined to be $\{y_i\}_{i=1}^{n}$ with $y_1 \leq y_2 \leq y_3 \leq \ldots \leq y_n$where y_i , 0< $y_i \leq$ 1, is the indeterminacy membership value of the vertex v_i when vertices are arranged so that their indeterminacy membership values are non- increasing.

Particular, y_1 is largest vertex indeterminacy membership value and y_n is smallest vertex indeterminacy membership value in G.

Note 4.5 If vertex indeterminacy membership sequence y_i is repeated more than once in G, say r ≠ 1 times, then it is denoted by ${y_i}^r$ in the sequence.

Example 4.6 In Fig. 3 the vertex indeterminacy membership sequence of G is {0.7, 0.6, 0.6, 0.5, 0.4, 0.4 } or { 0.7, 0.6^2, 0.5,0.4^2 }.

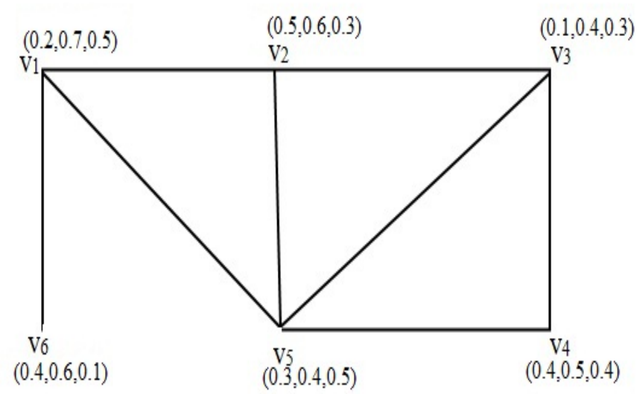

Fig. 3. Vertex indeterminacy membership sequence.

Definition 4.7 Let G be a SVNG with $|V|$ = n. The vertex falsity membership sequence of G is defined to be $\{z_i\}_{i=1}^{n}$ with $z_1 \leq z_2 \leq z_3 \leq \ldots \leq z_n$ where z_i , 0< $z_i \leq$ 1, is the falsity membership value of the vertex v_i when vertices are arranged so that their falsity membership values are non- increasing. Particular, z_1 is largest vertex falsi Y membership value and z_n is smallest vertex falsity membership value in G.

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Note 4.8 If vertex falsity membership sequence z_i is repeated more than once in G, say r ≠ 1 times, then it is denoted by ${z_i}^r$ in the sequence.

Example 4.9 In Fig. 4 the vertex falsity membership sequence of G is {0.8, 0.8, 0.7, 0.6, 0.6, 0.5} or $\{0.8^2$, 0.7,0.6^2, 0.5$\}$.

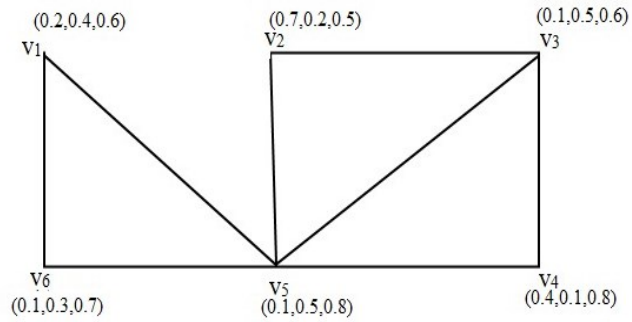

Fig. 4. Vertex falsity membership sequence.

Definition 4.10 If a SVNG with $|V|$ = n has vertex truth membership sequence $\{x_i\}_{i=1}^n$, vertex indeterminacy membership sequence$\{y_i\}_{i=1}^n$ and vertex falsity membership sequence$\{z_i\}_{i=1}^n$ in same order, then it said to have vertex single valued neutrosophic sequence and denoted by $\{< x_i , y_i, z_i >\}_{i=1}^n$.

Example 4.11 In Fig. 5 the vertex truth membership, vertex indeterminacy membership and vertex falsity membership sequence of G is{<0.4, 0.4, 0.5>, < 0.2,0.3,0.5>,< 0.1,0.2,0.6> , < 0.5,0.4,0.8>, < 0.4,0.5,0.4>, < 0.3,0.1,0.7> }.

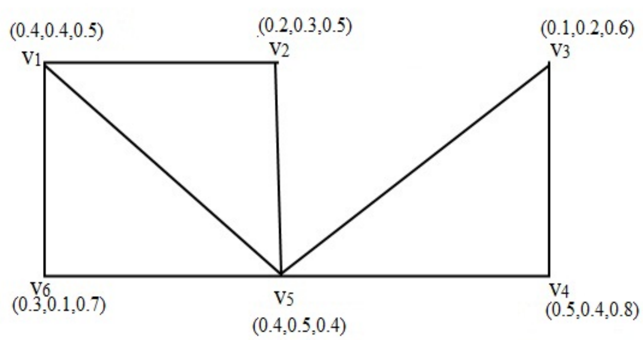

Fig. 5. Vertex single valued neutrosophic sequence.

The properties of vertex truth membership, vertex indeterminacy membership and vertex falsity sequences of complete SVNGs are discussed below:

Theorem 4.12 Let G=(V,E) be a complete SVNG with$|V|$ = n. Then

1) If the vertex truth membership sequence of G is of the form $\{{x_1}^{n-1}$,x_2 }, vertex indeterminacy membership sequence of G is of the form $\{{y_1}^{n-1}$,$y_2\}$ and vertex falsity membership sequence of G is of the form { ${z_1}^{n-1}$,$z_2\}$, then

a.δ_{td_T} (G) = n.T_1 and Δ_{td_T}(G)= $\sum_{i=1}^n T_i$

b. Δ_{td_I} (G) = n.I_1 and δ_{td_I}(G)=$\sum_{i=1}^n I_i$

c.Δ_{td_F}(G)= n.F_1 and δ_{td_F}(G) = $\sum_{i=1}^n F_i$

2) If the vertex truth membership sequence of G is of the form $\{{x_1}^{r_1}, {x_2}^{n-r_1}\}$, vertex indeterminacy membership of G is of the form $\{{y_1}^{r_1}, {y_2}^{n-r_1}\}$ and vertex falsity membership sequence of G is of the form $\{{z_1}^{r_1}, {z_2}^{n-r_1}\}$ with 0< $r_1 \leq$ n-2, then there exists exactly r_1 vertices with minimum T- total degree δ_{td_T} (G), maximum I-total degree $\Delta_{td_I}(G)$ and maximum F-total degree Δ_{td_F} and exactly (n-r_1) vertices with maximum T- total degree Δ_{td_T}(G) , minimum I- total degree δ_{td_I}(G) and minimum F- total degree δ_{td_F}(G).

3) If the vertex truth membership sequence of G is of the form $\{{x_1}^{r_1}, {x_2}^{r_2} , {x_3}^{r_3}, ... , {x_k}^{r_k}\}$, vertex indeterminacy membership sequence of G is of the form $\{{y_1}^{r_1}, {y_2}^{r_2} , {y_3}^{r_3}, . . . , {y_k}^{r_k}\}$ and vertex falsity membership sequence of G is of the form $\{{z_1}^{r_1}, {z_2}^{r_2} , {z_3}^{r_3}, . . . , {z_k}^{r_k}\}$ with $r_k > 1$ and k > 2, then there exists exactly r_1 vertices with minimum T- total degree δ_{td_T}(G), maximum I- total degree $\Delta_{td_I}$and maximum F-total degree Δ_{td_F} . Also, there exists exactly r_k vertices with maximum T- total degree Δ_{td_T}(G), minimum I- total degree δ_{td_I}(G) and minimum F- total degree δ_{td_F}(G).

Proof: The proof of (1) and (2) are obvious. 3 Let $v_i^{(j)}$ be the set of vertices in G, for j = 1, 2, 3, . . . , r_i , 1 ≤ i ≤ k. Then by the **Theorem3.31**

$td_T(v_1^{(j)})$=$\delta_{td_T}(G)$=n.T_1=n. x_1,

$td_I(v_1^{(j)})$=$\Delta_{td_I}(G)$= n.I_1=n. y_1, and

$td_F(v_1^{(j)})$=$\delta_{td_F}(G)$= n.F_1=n. z_1, , for j= 1, 2, 3,. . , r_1.

Since T($v_i^{(j)}$,$v_{i+1}^{(l)}$) = T($v_i^{(j)}$)>x_1 for 2 ≤ i ≤ k , j= 1, 2, 3,. .. . , r_i , l = 1, 2, 3, . . . , r_{i+1} , no vertex with truth membership more than x_1 can have degree δ_{td_T}(G),

I($v_i^{(j)}$,$v_{i+1}^{(l)}$) = I($v_i^{(j)}$) < y_1 for 2 ≤ i ≤ k , j = 1, 2, 3, . . . , r_i , l = 1, 2, 3, . . . , r_{i+1} , no vertex with indeterminacy membership less than y_1 can have degree Δ_{td_I}(G)

And F($v_i^{(j)}$,$v_{i+1}^{(l)}$) = F($v_i^{(j)}$) < z_1 for 2 ≤ i ≤ k , j = 1, 2, 3, . . . , r_i , l = 1, 2, 3, . . . , r_{i+1} , no vertex with falsity membership less than z_1 can have degree Δ_{td_F}(G).

Thus, there exist exactly r_1 vertices with degree δ_{td_T}(G), Δ_{td_I}(G), Δ_{td_F}(G).

To prove $td_T(v_k^{(t)})$ =$\Delta_{td_T}(G)$,

$td_I(v_k^{(t)})$ =δ_{td_I} (G) and

$td_F(v_k^{(t)})$ =δ_{td_F} (G), t=1,2,3 . . , r_k.

Since, T($v_k^{(t)}$) is maximum vertex truth membership,

T($v_k^{(t)}$,$v_k^{(j)}$) = x_k , t ≠j, t, j=1, 2, 3,…,r_k

T($v_k^{(t)}$,$v_i^{(j)}$) = min { T($v_k^{(t)}$) , T($v_i^{(j)}$)} = T($v_i^{(j)}$) for t= 1,2,3. . . , r_k , j = 1, 2, 3, . . . , r_i, i=1,2,3,… , k-1

Thus for t= 1,2,3, . . . , r_k,

$td_T(v_k^{(t)}) = \sum_{i=1}^{k}\sum_{j=1}^{r_i} \mathrm{T}(v_i^{(j)}) + (r_k - 1)x_k$

$= \sum_{i=1}^{n} T_i$

$= \Delta_{td_T}$(G) by **Theorem 3.31**

Now, if v_m is vertex such that $T_m = x_{k-1}$, then

$td_T(v_m) = \sum_{i=1}^{k-2}\sum_{j=1}^{r_i} \mathrm{T}(v_m, v_i^{(j)}) + (r_{k-1} - 1 + r_k)x_{k-1} + T_m$

$= \sum_{i=1}^{k-2}\sum_{j=1}^{r_i} \mathrm{T}(v_i^{(j)}) + \sum_{j=1}^{r_{k-1}} \mathrm{T}(v_{k-1}^{(j)}) + (r_k - 1)x_{k-1} + T_m$

$< \sum_{i=1}^{k-2}\sum_{j=1}^{r_i} \mathrm{T}(v_i^{(j)}) + \sum_{j=1}^{r_{k-1}} \mathrm{T}(v_{k-1}^{(j)}) + (r_k - 1)x_k + T_m$

$= \Delta_{td_T}$(G)

Thus, there exist exactly r_k vertices with degree Δ_{td_T}(G).

To prove $td_I(v_k^{(t)}) = \delta_{td_I}$(G) , for t = 1, 2, 3, . . . , r_k

Since I($v_k^{(t)}$) is minimum vertex indeterminacy membership,

$\mathrm{I}(v_k^{(t)}, v_k^{(j)}) = y_k$, t ≠ j , t, j = 1, 2, 3, . . . , r_k

$\mathrm{I}(v_k^{(t)}, v_i^{(j)}) = \max\{ \mathrm{I}(v_k^{(t)}) , \mathrm{I}(v_i^{(j)})\} = \mathrm{I}(v_i^{(j)})$ for t = 1, 2, 3, . . . , r_k , j = 1, 2, 3, . . . , r_i,

i = 1, 2, 3, . . . , k-1.

Thus for t = 1, 2, 3, . . . , r_k,

$td_I(v_k^{(t)}) = \sum_{i=1}^{k}\sum_{j=1}^{r_i} \mathrm{I}(v_i^{(j)}) + (r_k - 1)y_k$

$= \sum_{i=1}^{n} I_i$

$= \delta_{td_I}$(G) by **Theorem 3.31**

Now, if v_m is vertex such that $I_m = y_{k-1}$, then

$td_I(v_m) = \sum_{i=1}^{k-2}\sum_{j=1}^{r_i} \mathrm{I}(v_m, v_i^{(j)}) + (r_{k-1} - 1 + r_k)y_{k-1} + I_m$

$= \sum_{i=1}^{k-2}\sum_{j=1}^{r_i} \mathrm{I}(v_i^{(j)}) + \sum_{j=1}^{r_{k-1}} \mathrm{I}(v_{k-1}^{(j)}) + (r_k - 1)y_{k-1} + I_m$

$< \sum_{i=1}^{k-2}\sum_{j=1}^{r_i} \mathrm{I}(v_i^{(j)}) + \sum_{j=1}^{r_{k-1}} \mathrm{I}(v_{k-1}^{(j)}) + (r_k - 1)y_k + I_m$

$= \delta_{td_I}$(G)

So, there exist exactly r_k vertices with degree δ_{td_I}(G).

Similarly, it can be proved that $td_F(v_k^{(t)}) = \delta_{td_F}$(G) , for t = 1, 2, 3, . . . , r_k

Since $\mathrm{F}(v_k^{(t)})$ is minimum vertex falsity membership,

$\mathrm{F}(v_k^{(t)}, v_k^{(j)}) = z_k$, t ≠ j , t, j = 1, 2, 3, . . . , r_k

$\mathrm{F}(v_k^{(t)}, v_i^{(j)}) = \max\{ \mathrm{F}(v_k^{(t)}), \mathrm{F}(v_i^{(j)})\} = \mathrm{F}(v_i^{(j)})$ for t = 1, 2, 3, . . . , r_k , j = 1, 2, 3, . . . , r_i, i = 1, 2, 3, . . . , k-1.

Thus for t = 1, 2, 3, . . . , r_k,

$td_F(v_k^{(t)}) = \sum_{i=1}^{k}\sum_{j=1}^{r_i} \mathrm{F}(v_i^{(j)}) + (r_k - 1)z_k$

$= \sum_{i=1}^{n} F_i$

$= \delta_{td_F}$(G) by **Theorem 3.31**

Now, if v_m is vertex such that $F_m = z_{k-1}$, then

$td_F(v_m) = \sum_{i=1}^{k-2}\sum_{j=1}^{r_i} \mathrm{F}(v_m, v_i^{(j)}) + (r_{k-1} - 1 + r_k)F_{k-1} + F_m$

$= \sum_{i=1}^{k-2}\sum_{j=1}^{r_i} \mathrm{F}(v_i^{(j)}) + \sum_{j=1}^{r_{k-1}} \mathrm{F}(v_{k-1}^{(j)}) + (r_k - 1)z_{k-1} + F_m$

$< \sum_{i=1}^{k-2}\sum_{j=1}^{r_i} \mathrm{F}(v_i^{(j)}) + \sum_{j=1}^{r_{k-1}} \mathrm{F}(v_{k-1}^{(j)}) + (r_k - 1)z_k + F_m$

$= \delta_{td_F}$(G)

So, there exist exactly r_k vertices with degree δ_{td_F}(G).

V. Conclusion

In this paper, the idea of strong degree is imposed on the existing concepts of degrees in SVNGs. After that, we defined the vertex truth-membership, vertex indeterminacy-membership and vertex falsity membership sequence in SVNG with proofs and suitable examples. In the next research, the proposed concepts can be extended to labeling neutrosophic graph and also characterize the corresponding properties.

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