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Models for Multiple Attribute Decision-Making with Dual Generalized Single-Valued Neutrosophic Bonferroni Mean Operators

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Abstract: In this article, we expand the dual generalized weighted BM (DGWBM) and dual generalized weighted geometric Bonferroni mean (DGWGBM) operator with single valued neutrosophic numbers (SVNNs) to propose the dual generalized single-valued neutrosophic number WBM (DGSVNNWBM) operator and dual generalized single-valued neutrosophic numbers WGBM (DGSVNNWGBM) operator. Then, the multiple attribute decision making (MADM) methods are proposed with these operators. In the end, we utilize an applicable example for strategic suppliers selection to prove the proposed methods.

Keywords: multiple attribute decision making (MADM); single-valued neutrosophic numbers (SVNNs); dual generalized weighted BM (DGWBM) operator; dual generalized weighted Bonferroni geometric mean (DGWGBM) operator; strategic supplier selection

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1. Introduction

Smarandache [1,2] introduced a neutrosophic set (NS) from a philosophical point of view to express indeterminate and inconsistent information. In an NS A, its truth–membership function $T_A(x)$, indeterminacy–membership $I_A(x)$ and falsity–membership function $F_A(x)$ are represented independently, which lie in real standard or nonstandard subsets of $]^-0,1^+[$, that is, $T_A(x) : X \to]^-0,1^+[, I_A(x) : X \to]^-0,1^+[$ and $F_A(x) : X \to]^-0,1^+[$. The main advantage of NSs is to depict inconsistent and indeterminate information. An NS has more potential power than other fuzzy mathematical modeling tools, such as fuzzy set [3], intuitionistic fuzzy set (IFS) [4] and interval valued neutrosophic fuzzy set (IVIFS) [5]. However, it is not easy to use NSs in solving practical problems. Therefore, Smarandache [2] and Wang et al. [6,7] defined the single-valued neutrosophic set (SVNS) and an interval neutrosophic set (INS). Hence, SVNSs and INSs can express much more information than fuzzy sets, IFSs and IVIFSs. Ye [8] presented the correlation and correlation coefficient of single-valued neutrosophic sets (SVNSs) based on the extension of the correlation of intuitionistic fuzzy sets and demonstrates that the cosine similarity measure is a special case of the correlation coefficient in SVNS. Broumi and Smarandache [9] investigated the correlation coefficient with interval neutrosophic numbers(INNs). Biswas et al. [10] proposed a new approach for multi-attribute group decision-making problems by extending the technique for order preference by similarity to ideal solution to single-valued neutrosophic environment. Liu et al. [11] proposed the generalized neutrosophic number Hamacher weighted averaging (GNNHWA) operator, generalized neutrosophic number Hamacher ordered weighted averaging (GNNHOWA) operator, and generalized neutrosophic number Hamacher hybrid averaging (GNNHHA) operator, and explored some properties of these operators and analyzed some special cases of them. Sahin and Liu [12] proposed the maximizing deviation models for solving the multiple attribute decision-making problems with the single-valued neutrosophic information or interval neutrosophic information. Ye [13] developed the Hamming and Euclidean distances between interval neutrosophic sets (INSs) and proposed the similarity measures between INSs based on the relationship between similarity measures and distances. Zhang et al. [14] developed the interval neutrosophic number weighted averaging (INNWA) operator and interval neutrosophic number weighted geometric (INNWG) operator. Ye [15] proposed a simplified neutrosophic set (SNS) as a more general concept including SVNS and INS. Many researchers have given them attention to SNSs. For example, Peng et al. [16] defined some basic operational laws for SNNs and developed simplified neutrosophic information aggregation operators. Additionally, Peng et al. [17] developed a new outranking approach for multi-criteria decision-making (MCDM) problems in the context of a simplified neutrosophic environment, where the truth-membership degree, indeterminacy-membership degree and falsity-membership degree for each element are singleton subsets in [0, 1], and then Zhang et al. [18] gave an extended version of Peng's approach to interval neutrosophic environment. Liu and Liu [19] developed generalized weighted power operators with SVNNs. Deli and Subas [20] discussed a method to rank SVNNs. Peng et al. [21] introduced the multi-valued neutrosophic sets (MVNSs), which allowed the truth-membership, indeterminacy–membership and falsity-membership degree have a set of crisp values between zero and one, respectively, and then defined the operations of multi-valued neutrosophic numbers (MVNNs) based on Einstein operations, the multi-valued neutrosophic power weighted average (MVNPWA) operator and the multi-valued neutrosophic power weighted geometric (MVNPWG) operator. Zhang et al. [22] presented a new correlation coefficient measure that satisfies the requirement of this measure equaling one if and only if two interval neutrosophic sets (INSs) are the same and presented an objective weight of INSs. Chen and Ye [23] proposed the Dombi operations of single-valued neutrosophic numbers (SVNNs) based on the operations of the Dombi T-norm and T-conorm and then proposed the single-valued neutrosophic Dombi weighted arithmetic average (SVNDWAA) operator and the single-valued neutrosophic Dombi weighted geometric average (SVNDWGA) operator to deal with the aggregation of SVNNs and investigates their properties. Liu and Wang [24] proposed a single-valued neutrosophic normalized weighted Bonferroni mean (SVNNWBM) operator on the basis of Bonferroni mean, the weighted Bonferroni mean (WBM), and the normalized WBM and developed the models solve the multiple attribute decision-making problems with SVNNs based on the SVNNWBM operator. Wu et al. [25] defined the prioritized weighted average operator and prioritized weighted geometric operator for simplified neutrosophic numbers (SNNs) and then proposed two novel effective cross-entropy measures for SNSs and proposed the ranking methods for SNSs to solve MADM problems based on the proposed prioritized aggregation operators and cross-entropy measures. Li et al. [26] proposed the improved generalized weighted Heronian mean (IGWHM) operator and improved generalized weighted geometric Heronian mean (IGWGHM) operator based on crisp numbers, and prove that they can satisfy some desirable properties, such as reducibility, idempotency, monotonicity and boundedness and proposed the single valued neutrosophic number improved generalized weighted Heronian mean (NNIGWHM) operator and single valued the neutrosophic number improved generalized weighted geometric Heronian mean (NNIGWGHM) operator for multiple attribute group decision-making (MAGDM) problems in which attribute values take the form of SVNNs.

Obviously, these established SVNN aggregation operators cannot be used to fuse the arguments that are correlated. Meanwhile, the Bonferroni mean (BM) [27–34] is a very practical tool to tackle the arguments that are correlated. How to effectively extend the mature BM mean to the SVNN environment is a significant research task.

The structure of this manuscript is given. Section 2 reviews SVNSs and basic definitions. Section 3 introduces the extended DGWBM and DGWGBM, which can be used to fuse the SVNNs, and describes some properties of these operators. Section 4 illustrates the functions of the proposed operators with an example for strategic supplier selection in supply chain management area. Section 5 presents the conclusions.

2. Basic Concepts

Smarandache [1,2] proposed Neutrosophic sets (NSs). Wang et al. [6,7] further proposed the SVNSs.

Definition 1 [6,7]. *Let X be a space of points (objects) with a generic element in fix set X, named by x. An SVNS A in X is depicted as the following:*

$$A = \{(x, T_A(x), I_A(x), F_A(x)) | x \in X\} \oplus, \tag{1}$$

where $T_A(x)(0 \le T_A(x) \le 1)$ *is truth–membership function,* $I_A(x)(0 \le I_A(x) \le 1)$ *is indeterminacy–membership and* $F_A(x)(0 \le F_A(x) \le 1)$ *is falsity–membership function, and* $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Zhang et al. [14] gave the order between two SVNNs.

Definition 2 [14]. *Let* $A_1 = (T_{A_1}, I_{A_1}, F_{A_1})$ *and* $A_2 = (T_{A_2}, I_{A_2}, F_{A_2})$ *be two SVNNs,* $s(A_1) = \frac{(2+T_{A_1} - I_{A_1} - F_{A_1})}{3}$ *and* $s(A_2) = \frac{(2+T_{A_2} - I_{A_2} - F_{A_2})}{3}$ *be the scores of* A_1 *and* A_2*, respectively, and let* $H(A_1) = T_{A_1} - F_{A_1}$ *and* $H(A_2) = T_{A_2} - F_{A_2}$ *be the accuracy degrees of* A_1 *and* A_2*, respectively, then if* $S(A_1) < S(A_2)$*,* $A_1 < A_2$*; if* $S(A_1) = S(A_2)$*, then (1) if* $H(A_1) = H(A_2)$*,* $A_1 = A_2$*; (2) if* $H(A_1) < H(A_2)$*,* $A_1 < A_2$*.*

Definition 3 [6]. *Let* $A = (T_{A_1}, I_{A_1}, F_{A_1})$ *and* $A_2 = (T_{A_2}, I_{A_2}, F_{A_2})$ *be two SVNNs and* λ *be a positive real number, some operations of SVNNs are defined:*

1. $A_1 \oplus A_2 = (T_{A_1} + T_{A_2} - T_{A_1} T_{A_2}, I_{A_1} I_{A_2}, F_{A_1} F_{A_2})$;
2. $A_1 \otimes A_2 = (T_{A_1} T_{A_2}, I_{A_1} + I_{A_2} - I_{A_1} I_{A_2}, F_{A_1} + F_{A_2} - F_{A_1} F_{A_2})$;
3. $\lambda A_1 = (1 - (1 - T_{A_1})^{\lambda}, (I_{A_1})^{\lambda}, (F_{A_1})^{\lambda}), \lambda > 0$;
4. $(A_1)^{\lambda} = ((T_{A_1})^{\lambda}, 1 - (1 - I_{A_1})^{\lambda}, 1 - (1 - F_{A_1})^{\lambda}), \lambda > 0$.

Zhang et al. [34] develop the dual generalized WBM (DGWBM) operator and dual generalized WGBM (DGWGBM) operator.

Definition 4 [34]. *Let* $b_i (i = 1, 2, \ldots, n)$ *be a set of nonnegative crisp numbers with the weight* $w = (w_1, w_2, \ldots, w_n)^T$, $w_i \in [0, 1]$ $(i = 1, 2, \ldots, n)$ *and* $\sum_{i=1}^n w_i = 1$*, if*

$$\mathrm{DGWBM}_w^R(b_1, b_2, \ldots, b_n) = \left(\sum_{i_1, i_2, \ldots, i_n = 1}^{n} \left(\prod_{j=1}^{n} w_{i_j} b_{i_j}^{r_j} \right) \right)^{1/\sum_{j=1}^n r_j}, \tag{2}$$

where $R = (r_1, r_2, \ldots, r_n)^T$ *is the parameter vector with* $r_i \ge 0 (i = 1, 2, \ldots, n)$*.*

Several special cases can be obtained given the change of the parameter vector.
If $R = (\lambda, 0, 0, \ldots, 0)$, then we obtain

$$\mathrm{DGWBM}^{(\lambda,0,0,\ldots,0)}(b_1, b_2, \ldots, b_n) = \left(\sum_{i=1}^{n} w_i b_i^{\lambda}\right)^{1/\lambda}, \tag{3}$$

which is the generalized weighted averaging operator.

If $R = (s, t, 0, 0, \ldots, 0)$, then we obtain

$$\mathrm{DGWBM}^{(s,t,0,0,\ldots,0)}(b_1, b_2, \ldots, b_n) = \left(\sum_{i,j=1}^{n} w_i w_j b_i^s b_j^t\right)^{1/(s+t)}, \tag{4}$$

which is the weighted BM.

If $R = (s, t, r, 0, 0, \ldots, 0)$, then we obtain

$$\mathrm{DGWBM}^{(s,t,r,0,0,\ldots,0)}(b_1, b_2, \ldots, b_n) = \left(\sum_{i,j,k=1}^{n} w_i w_j w_k b_i^s b_j^t b_k^r\right)^{1/(s+t+k)}. \tag{5}$$

Definition 5 [34]. *Let $b_i (i = 1, 2, \ldots, n)$ be a set of nonnegative crisp numbers with weight being $w = (w_1, w_2, \ldots, w_n)^T$, $w_i \in [0, 1]$ $(i = 1, 2, \ldots, n)$ and $\sum_{i=1}^{n} w_i = 1$, if*

$$\mathrm{DGWBGM}^R(b_1, b_2, \ldots, b_n) = \frac{1}{\sum_{j=1}^{n} r_j}\left(\prod_{1,i_2,\ldots,i_n=1}^{n}\left(\sum_{j=1}^{n}\left(r_j b_{i_j}\right)\right)\right)^{\prod_{j=1}^{n} w_{i_j}}, \tag{6}$$

where $R = (r_1, r_2, \ldots, r_n)^T$ is the parameter vector with $r_i \geq 0 (i = 1, 2, \ldots, n)$.

Similar to the DGWBM, we can consider some special cases given the change of the parameter vector.

(1) If $R = (\lambda, 0, 0, \ldots, 0)$, then we obtain

$$\mathrm{DGWBGM}^{(\lambda,0,0,\ldots,0)}(b_1, b_2, \ldots, b_n) = \frac{1}{\lambda}\left(\prod_{i=1}^{n}(\lambda b_i)^{w_i}\right). \tag{7}$$

(2) If $R = (s, t, 0, 0, \ldots, 0)$, then we obtain

$$\mathrm{DGWBGM}^{(s,t,0,0,\ldots,0)}(b_1, b_2, \ldots, b_n) = \frac{1}{s+t}\prod_{i,j=1}^{n}(s b_i + t b_j)^{w_i w_j}. \tag{8}$$

(3) If $R = (s, t, r, 0, 0, \ldots, 0)$, then we obtain

$$\mathrm{DGWBGM}^{(s,t,r,0,0,\ldots,0)}(b_1, b_2, \ldots, b_n) = \frac{1}{s+t+r}\prod_{i.j.k=1}^{n}(s b_i + t b_j + r b_k)^{w_i w_j w_k}. \tag{9}$$

3. DGSVNNWBM Operator and DGSVNNWGBM Operator

This section extends DGWBM and DGWGBM to fuse the SVNNs, and proposes the dual generalized SVNN weighted BM (DGSVNNWBM) operator and dual generalized SVNN weighted GBM (DGSVNNWGBM) operator.

Definition 6. *Let $a_i = (T_i, I_i, F_i)$ $(i = 1, 2, \ldots, n)$ be a set of SVNNs with weight $w_i = (w_1, w_2, \ldots, w_n)^T$, $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$.*

Thereafter, the dual generalized SVNN weighted BM (DGSVNNWBM) operator is defined as

$$\mathrm{DGSVNNWBM}_w^R(a_1, a_2, \cdots, a_n) = \left(\bigoplus_{i_1,i_2,\ldots,i_n=1}^{n}\left(\bigotimes_{j=1}^{n} w_{i_j} a_{i_j}^{r_j}\right)\right)^{1/\sum_{i=1}^{n} r_j}, \tag{10}$$

where $R = (r_1, r_2, \ldots, r_n)^T$ is the parameter vector with $r_i \geq 0$ $(i = 1, 2, \ldots, n)$. We can get Theorem 1.

Theorem 1. *Let $a_i = (T_i, I_i, F_i)$ $(i = 1, 2, \ldots, n)$ be a set of SVNNs. Hence, the aggregated result of DGSVNNWBM is a SVNN and*

$$\mathrm{DGSVNNWBM}_w^R(a_1, a_2, \cdots, a_n) = \begin{pmatrix} \left(1 - \prod\limits_{i_1, i_2, \ldots i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - T_{i_j}^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}, \\ 1 - \left(1 - \prod\limits_{i_1, i_2, \ldots, i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \left(1 - I_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}, \\ 1 - \left(1 - \prod\limits_{i_1, i_2, \ldots, i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \left(1 - F_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j} \end{pmatrix} \tag{11}$$

Proof.

$$a_{i_j}^{r_j} = \left(T_{i_j}^{r_j}, 1 - \left(1 - I_{i_j}\right)^{r_j}, 1 - \left(1 - F_{i_j}\right)^{r_j}\right). \tag{12}$$

Thus,

$$w_{i_j} a_{i_j}^{r_j} = \left(1 - \left(1 - T_{i_j}^{r_j}\right)^{w_{i_j}}, \left(1 - \left(1 - I_{i_j}\right)^{r_j}\right)^{w_{i_j}}, \left(1 - \left(1 - F_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right). \tag{13}$$

Thereafter,

$$\underset{j=1}{\overset{n}{\otimes}} w_{i_j} a_{i_j}^{r_j} = \begin{pmatrix} \prod\limits_{j=1}^{n} \left(1 - \left(1 - T_{i_j}^{r_j}\right)^{w_{i_j}}\right), \\ 1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \left(1 - I_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right), \\ 1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \left(1 - F_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right) \end{pmatrix}. \tag{14}$$

Furthermore,

$$\underset{i_1, i_2, \ldots, i_n = 1}{\overset{n}{\oplus}} \left(\underset{j=1}{\overset{n}{\otimes}} w_{i_j} a_{i_j}^{r_j}\right) = \begin{pmatrix} 1 - \prod\limits_{i_1, i_2, \ldots, i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - T_{i_j}^{r_j}\right)^{w_{i_j}}\right)\right), \\ \prod\limits_{i_1, i_2, \ldots, i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \left(1 - I_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right), \\ \prod\limits_{i_1, i_2, \ldots, i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \left(1 - F_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right). \end{pmatrix}. \tag{15}$$

Therefore,

$$\left(\underset{i_1, i_2, \ldots, i_n = 1}{\overset{n}{\oplus}} \left(\underset{j=1}{\overset{n}{\otimes}} w_{i_j} a_{i_j}^{r_j}\right)\right)^{1/\sum_{i=1}^{n} r_j} = \begin{pmatrix} \left(1 - \prod\limits_{i_1, i_2, \ldots i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - T_{i_j}^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}, \\ 1 - \left(1 - \prod\limits_{i_1, i_2, \ldots, i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \left(1 - I_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}, \\ 1 - \left(1 - \prod\limits_{i_1, i_2, \ldots, i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \left(1 - F_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}. \end{pmatrix}. \tag{16}$$

Hence, Label (11) is maintained.

Thereafter,

$$
\begin{aligned}
&0 \leq \left(1-\prod_{i_1,i_2,\ldots i_n=1}^{n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-T_{i_j}^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j} \leq 1,\\
&0 \leq 1-\left(1-\prod_{i_1,i_2,\ldots,i_n=1}^{n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-\left(1-I_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j} \leq 1,\\
&0 \leq 1-\left(1-\prod_{i_1,i_2,\ldots,i_n=1}^{n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-\left(1-F_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j} \leq 1.
\end{aligned}
\tag{17}
$$

In addition,

$$
\begin{aligned}
&0 \leq \left(1-\prod_{i_1,i_2,\ldots i_n=1}^{n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-T_{i_j}^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}\\
&+1-\left(1-\prod_{i_1,i_2,\ldots,i_n=1}^{n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-\left(1-I_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}\\
&+1-\left(1-\prod_{i_1,i_2,\ldots,i_n=1}^{n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-\left(1-F_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j} \leq 3,
\end{aligned}
\tag{18}
$$

thereby completing the proof. □

Moreover, DGSVNNWBM has the following properties.

Property 1. (Monotonicity). *Let* $a_i = (T_{a_i}, I_{a_i}, F_{a_i})(i = 1, 2, \ldots, n)$ *and* $b_i = (T_{b_i}, I_{b_i}, F_{b_i})$ $(i = 1, 2, \ldots, n)$ *be two sets of SVNNs. If* $T_{a_i} \leq T_{b_i}$ *and* $I_{a_i} \geq I_{b_i}$ *and* $F_{a_i} \geq F_{b_i}$ *holds for all* i, *then*

$$
\mathrm{DGSVNNWBM}_w^R(a_1, a_2, \cdots, a_n) \leq \mathrm{DGSVNNWBM}_w^R(b_1, b_2, \cdots, b_n). \tag{19}
$$

Proof. Let $\mathrm{DGSVNNWBM}_w^R(a_1, a_2, \cdots, a_n) = (T_a, I_a, F_a)$, $\mathrm{DGSVNNWBM}_w^R(b_1, b_2, \cdots, b_n) = (T_b, I_b, F_b)$.

Given that $T_{a_i} \leq T_{b_i}$, we can obtain

$$
(1 - T_{a_{i_j}}^{r_j})^{w_{i_j}} \geq (1 - T_{a_{i_j}}^{r_j})^{w_{i_j}}. \tag{20}
$$

Therefore,

$$
1 - (1 - T_{a_{i_j}}^{r_j})^{w_{i_j}} \leq 1 - (1 - T_{b_{i_j}}^{r_j})^{w_{i_j}}. \tag{21}
$$

Thus,

$$
1-\prod_{j=1}^{n}\left(1-\left(1-T_{a_{i_j}}^{r_j}\right)^{w_{i_j}}\right) \geq 1-\prod_{j=1}^{n}\left(1-\left(1-T_{b_{i_j}}^{r_j}\right)^{w_{i_j}}\right). \tag{22}
$$

Thereafter,

$$
1-\prod_{i_1,i_2,\ldots,i_n=1}^{n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-T_{a_{i_j}}^{r_j}\right)^{w_{i_j}}\right)\right) \leq 1-\prod_{i_1,i_2,\ldots,i_n=1}^{n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-T_{b_{i_j}}^{r_j}\right)^{w_{i_j}}\right)\right). \tag{23}
$$

Then,

$$
\begin{aligned}
&\left(1-\prod_{i_1,i_2,\ldots i_n=1}^{n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-T_{a_{i_j}}^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}\\
&\leq \left(1-\prod_{i_1,i_2,\ldots i_n=1}^{n}\left(1-\prod_{j=1}^{n}\left(1-\left(1-T_{b_{i_j}}^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j},
\end{aligned}
\tag{24}
$$

which means $T_a \le T_b$. Similarly, we can obtain $I_a \ge I_b$ *and* $F_a \ge F_b$.

If $T_a < T_b$ *and* $I_a \ge I_b$ *and* $F_a \ge F_b$, then
$\mathrm{DGSVNNWBM}_w^R(a_1, a_2, \cdots, a_n) < \mathrm{DGSVNNWBM}_w^R(b_1, b_2, \cdots, b_n)$;
If $T_a = T_b$ *and* $I_a > I_b$ *and* $F_a > F_b$, then
$\mathrm{DGSVNNWBM}_w^R(a_1, a_2, \cdots, a_n) < \mathrm{DGSVNNWBM}_w^R(b_1, b_2, \cdots, b_n)$;
If $T_a = T_b$ *and* $I_a = I_b$ *and* $F_a = F_b$, then
$\mathrm{DGSVNNWBM}_w^R(a_1, a_2, \cdots, a_n) = \mathrm{DGSVNNWBM}_w^R(b_1, b_2, \cdots, b_n)$.
Therefore, the proof of property 1 is completed. □

Property 2. (Boundedness). *Let* $a_i = (T_{a_i}, I_{a_i}, F_{a_i})(i = 1, 2, \ldots, n)$ *be a set of SVNNS. If* $a^+ = (\max_i(T_i), \min_i(I_i), \min_i(F_i))$ *and* $a^- = (\min_i(T_i), \max_i(I_i), \max_i(F_i))$, *then*

$$\begin{array}{l} \mathrm{DGSVNNWBM}_w^R(a_1^-, a_2^-, \cdots, a_n^-) \\ \le \mathrm{DGSVNNWBM}_w^R(a_1, a_2, \cdots, a_n) \quad . \\ \le \mathrm{DGSVNNWBM}_w^R(a_1^+, a_2^+, \cdots, a_n^+) \end{array} \tag{25}$$

Proof. From Theorem 1, we can obtain

$$\begin{array}{l} \mathrm{DGSVNNWBM}_w^R(a^-, a^-, \ldots, a^-) \\ = \left(\begin{array}{l} \left(1 - \prod\limits_{i_1, i_2, \ldots i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \min T_{i_j}^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}, \\ 1 - \left(1 - \prod\limits_{i_1, i_2, \ldots, i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \left(1 - \min I_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}, \\ 1 - \left(1 - \prod\limits_{i_1, i_2, \ldots, i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \left(1 - \min F_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}. \end{array} \right) \end{array} \tag{26}$$

$$\begin{array}{l} \mathrm{DGSVNNWBM}_w^R(a, a, \ldots, a) \\ = \left(\begin{array}{l} \left(1 - \prod\limits_{i_1, i_2, \ldots i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - T_{i_j}^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}, \\ 1 - \left(1 - \prod\limits_{i_1, i_2, \ldots, i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \left(1 - I_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}, \\ 1 - \left(1 - \prod\limits_{i_1, i_2, \ldots, i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \left(1 - F_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}. \end{array} \right) \end{array} \tag{27}$$

$$\begin{array}{l} \mathrm{DGSVNNWBM}_w^R(a^+, a^+, \ldots, a^+) \\ = \left(\begin{array}{l} \left(1 - \prod\limits_{i_1, i_2, \ldots i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \max T_{i_j}^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}, \\ 1 - \left(1 - \prod\limits_{i_1, i_2, \ldots, i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \left(1 - \max I_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}, \\ 1 - \left(1 - \prod\limits_{i_1, i_2, \ldots, i_n = 1}^{n} \left(1 - \prod\limits_{j=1}^{n} \left(1 - \left(1 - \left(1 - \max F_{i_j}\right)^{r_j}\right)^{w_{i_j}}\right)\right)\right)^{1/\sum_{i=1}^{n} r_j}. \end{array} \right) \end{array} \tag{28}$$

From Property 1, we can obtain

$$\begin{array}{l} \mathrm{DGSVNNWBM}_w^R(a^-, a^-, \ldots, a^-) \\ \le \mathrm{DGSVNNWBM}_w^R(a_1, a_2, \ldots, a_n) \\ \le \mathrm{DGSVNNWBM}_w^R(a^+, a^+, \ldots, a^+). \end{array} \tag{29}$$

Evidently, the DGSVNNWBM operator lacks the property of idempotency.

Furthermore, we extend DGWBGM to SVNNS and propose the dual generalized SVNN weighted GBM (DGSVNNWGBM) operator. □

Definition 7. *Let $a_i = (T_i, I_i, F_i)$ $(i = 1, 2, \ldots, n)$ be a set of SVNNs with their weight vector being $w_i = (w_1, w_2, \ldots, w_n)^T$, thereby satisfying $w_i \in [0,1]$ and $\sum_{i=1}^n w_i = 1$. If*

$$\mathrm{DGSVNNWGBM}_w^R(a_1, a_2, \cdots a_n) = \frac{1}{\sum_{j=1}^n r_j}\left(\underset{i_1, i_2, \ldots, i_n = 1}{\overset{n}{\otimes}}\left(\underset{j=1}{\overset{n}{\oplus}}\left(r_j a_{i_j}\right)\right)^{\prod_{j=1}^n w_{i_j}}\right), \tag{30}$$

where $R = (r_1, r_2, \ldots, r_n)^T$ is the parameter vector with $r_i \geq 0 (i = 1, 2, \ldots, n)$. Then, $\mathrm{DGSVNNWGBM}_w^R$ *is called DGSVNNWGBM.*

We can derive Theorem 2.

Theorem 2. *Let $a_i = (T_i, I_i, F_i)(i = 1, 2, \ldots, n)$ be a set of SVNNs. The aggregated value by DGSVNNWGBM is also a SVNN and*

$$\begin{aligned} &\mathrm{DGSVNNWGBM}_w^R(a_1, a_2, \cdots a_n) \\ &= \left(\begin{array}{l} 1 - \left(1 - \prod_{i_1, i_2, \ldots i_n = 1}^{n}\left(1 - \prod_{j=1}^n \left(1 - T_{i_j}\right)^{r_j}\right)^{\prod_{j=1}^n w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^n r_j}}, \\ \left(1 - \prod_{i_1, i_2, \ldots i_n = 1}^{n}\left(1 - \prod_{j=1}^n I_{i_j}^{r_j}\right)^{\prod_{j=1}^n w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^n r_j}}, \\ \left(1 - \prod_{i_1, i_2, \ldots i_n = 1}^{n}\left(1 - \prod_{j=1}^n F_{i_j}^{r_j}\right)^{\prod_{j=1}^n w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^n r_j}}. \end{array}\right) \end{aligned} \tag{31}$$

Proof.

$$r_j a_{i_j} = \left(1 - \left(1 - T_{i_j}\right)^{r_j}, I_{i_j}^{r_j}, F_{i_j}^{r_j}\right), \tag{32}$$

$$\underset{j=1}{\overset{n}{\oplus}}\left(r_j a_{i_j}\right) = \left(1 - \prod_{j=1}^n \left(1 - T_{i_j}\right)^{r_j}, \prod_{j=1}^n I_{i_j}^{r_j}, \prod_{j=1}^n F_{i_j}^{r_j}\right). \tag{33}$$

Thereafter,

$$\left(\underset{j=1}{\overset{n}{\oplus}}\left(r_j a_{i_j}\right)\right)^{\prod_{j=1}^n w_{i_j}} = \left(\begin{array}{c} \left(1 - \prod_{j=1}^n (1 - T_{i_j})^{r_j}\right)^{\prod_{j=1}^n w_{i_j}}, \\ 1 - \left(1 - \prod_{j=1}^n I_{i_j}^{r_j}\right)^{\prod_{j=1}^n w_{i_j}}, \\ 1 - \left(1 - \prod_{j=1}^n F_{i_j}^{r_j}\right)^{\prod_{j=1}^n w_{i_j}}. \end{array}\right) \tag{34}$$

Therefore,

$$\underset{i_1,i_2,\ldots,i_n=1}{\overset{n}{\otimes}}\left(\overset{n}{\underset{j=1}{\oplus}}\left(r_j a_{i_j}\right)\right)^{\prod_{j=1}^{n} w_{i_j}} = \left(\begin{array}{l} \prod\limits_{i_1,i_2,\ldots,i_n}^{n}\left(1-\prod\limits_{j=1}^{n}\left(1-T_{i_j}\right)^{r_j}\right)^{\prod_{j=1}^{n} w_{i_j}}, \\ 1-\prod\limits_{i_1,i_2,\ldots,i_n}^{n}\left(\left(1-\prod\limits_{j=1}^{n} I_{i_j}^{r_j}\right)^{\prod_{j=1}^{n} w_{i_j}}\right), \\ 1-\prod\limits_{i_1,i_2,\ldots,i_n}^{n}\left(\left(1-\prod\limits_{j=1}^{n} F_{i_j}^{r_j}\right)^{\prod_{j=1}^{n} w_{i_j}}\right). \end{array}\right) \tag{35}$$

Thus,

$$\begin{aligned} &\frac{1}{\sum_{j=1}^{n} r_j}\left(\underset{i_1,i_2,\ldots,i_n=1}{\overset{n}{\otimes}}\left(\overset{n}{\underset{j=1}{\oplus}}\left(r_j a_{i_j}\right)\right)^{\prod_{j=1}^{n} w_{i_j}}\right) \\ &= \left(\begin{array}{l} 1-\left(1-\prod\limits_{i_1,i_2,\ldots i_n=1}^{n}\left(1-\prod\limits_{j=1}^{n}\left(1-T_{i_j}\right)^{r_j}\right)^{\prod_{j=1}^{n} w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{n} r_j}}, \\ \left(1-\prod\limits_{i_1,i_2,\ldots i_n=1}^{n}\left(\left(1-\prod\limits_{j=1}^{n} I_{i_j}^{r_j}\right)^{\prod_{j=1}^{n} w_{i_j}}\right)\right)^{\frac{1}{\sum_{j=1}^{n} r_j}}, \\ \left(1-\prod\limits_{i_1,i_2,\ldots i_n=1}^{n}\left(\left(1-\prod\limits_{j=1}^{n} F_{i_j}^{r_j}\right)^{\prod_{j=1}^{n} w_{i_j}}\right)\right)^{\frac{1}{\sum_{j=1}^{n} r_j}}. \end{array}\right) \end{aligned} \tag{36}$$

Hence, Label (31) is maintained.
Thereafter,

$$\begin{aligned} &0 \le 1-\left(1-\prod_{i_1,i_2,\ldots i_n=1}^{n}\left(1-\prod_{j=1}^{n}\left(1-T_{i_j}\right)^{r_j}\right)^{\prod_{j=1}^{n} w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{n} r_j}} \le 1, \\ &0 \le \left(1-\prod_{i_1,i_2,\ldots i_n=1}^{n}\left(\left(1-\prod_{j=1}^{n} I_{i_j}^{r_j}\right)^{\prod_{j=1}^{n} w_{i_j}}\right)\right)^{\frac{1}{\sum_{j=1}^{n} r_j}} \le 1, \\ &0 \le \left(1-\prod_{i_1,i_2,\ldots i_n=1}^{n}\left(\left(1-\prod_{j=1}^{n} F_{i_j}^{r_j}\right)^{\prod_{j=1}^{n} w_{i_j}}\right)\right)^{\frac{1}{\sum_{j=1}^{n} r_j}} \le 1. \end{aligned} \tag{37}$$

Therefore,

$$\begin{aligned} &0 \le 1-\left(1-\prod_{i_1,i_2,\ldots i_n=1}^{n}\left(1-\prod_{j=1}^{n}\left(1-T_{i_j}\right)^{r_j}\right)^{\prod_{j=1}^{n} w_{i_j}}\right)^{\frac{1}{\sum_{j=1}^{n} r_j}} \\ &+\left(1-\prod_{i_1,i_2,\ldots i_n=1}^{n}\left(\left(1-\prod_{j=1}^{n} I_{i_j}^{r_j}\right)^{\prod_{j=1}^{n} w_{i_j}}\right)\right)^{\frac{1}{\sum_{j=1}^{n} r_j}} \\ &+\left(1-\prod_{i_1,i_2,\ldots i_n=1}^{n}\left(\left(1-\prod_{j=1}^{n} F_{i_j}^{r_j}\right)^{\prod_{j=1}^{n} w_{i_j}}\right)\right)^{\frac{1}{\sum_{j=1}^{n} r_j}} \le 3, \end{aligned} \tag{38}$$

thereby completing the proof. □

Similar to DGSVNNWBM, DGSVNNWGBM has the same properties. The proofs are omitted to save space.

Property 3. *Let $a_i = (T_i, I_i, F_i)$ $(i = 1, 2, \ldots, n)$ be a set of SVNNs.*

(1) *(Monotonicity). Let $a_i = (T_{a_i}, I_{a_i}, F_{a_i})$ $(i = 1, 2, \ldots, n)$ and $b_i = (T_{b_i}, I_{b_i}, F_{b_i})$ $(i = 1, 2, \ldots, n)$ be two sets of SVNNs. If $T_{a_i} \le T_{b_i}$ and $I_{a_i} \ge I_{b_i}$ and $F_{a_i} \ge F_{b_i}$ holds for all i, then*

$$\mathrm{DGSVNNWGBM}_w^R(a_1, a_2, \cdots, a_n) \le \mathrm{DGSVNNWGBM}_w^R(b_1, b_2, \cdots, b_n). \tag{39}$$

(2) *(Boundedness). Let $a_i = (T_i, I_i, F_i)(i = 1, 2, \ldots, n)$ be a set of SVNNS. If*

$$a^+ = (\max_i(T_i), \min_i(I_i), \min_i(F_i)),$$

$$a^- = (\min_i(T_i), \max_i(I_i), \max_i(F_i)).$$

Then,

$$a^- \le \mathrm{DGSVNNWGBM}_w^R(a_1, a_2, \cdots, a_n) \le a^+. \tag{40}$$

4. Numerical Example and Comparative Analysis

4.1. Applicable Example

In this section, we shall present a numerical example to select strategic suppliers under supply chain risk with SVNNs in order to illustrate the method proposed in this paper. There is a panel with five possible strategic suppliers $O_i(i = 1, 2, 3, 4, 5)$ to select. The experts select four attributes to evaluate the five possible strategic suppliers: ① C_1 is the technology level; ② C_2 is the service level; ③ C_3 is the risk managing ability; and ④ C_4 is the enterprise environment risk. The five possible strategic suppliers $O_i(i = 1, 2, 3, 4, 5)$ are to be evaluated using the SVNNs by the decision-maker under the four above attributes (whose weighting vector $\omega = (0.10, 0.40, 0.35, 0.15)^T$), as listed in the following matrix:

$$\widetilde{R} = \begin{bmatrix} (0.6, 0.9, 0.2) & (0.7, 0.4, 0.4) & (0.4, 0.7, 0.2) & (0.6, 0.8, 0.3) \\ (0.8, 0.5, 0.2) & (0.8, 0.6, 0.3) & (0.8, 0.5, 0.5) & (0.9, 0.6, 0.2) \\ (0.7, 0.8, 0.3) & (0.6, 0.8, 0.4) & (0.6, 0.4, 0.2) & (0.7, 0.4, 0.3) \\ (0.9, 0.2, 0.4) & (0.7, 0.4, 0.5) & (0.5, 0.5, 0.3) & (0.6, 0.7, 0.3) \\ (0.7, 0.6, 0.5) & (0.5, 0.9, 0.2) & (0.8, 0.7, 0.3) & (0.6, 0.9, 0.4) \end{bmatrix}.$$

Then, we utilize the proposed operators to select the best strategic suppliers under supply chain risk.

Step 1. According to w and SVNNs $O_{ij}(i = 1, 2, 3, 4, 5, j = 1, 2, 3, 4)$, we can aggregate all SVNNs O_{ij} by using the DGSVNNWBM (DGSVNNWGBM) operator to derive the SVNNs $O_i(i = 1, 2, 3, 4, 5)$ of the alternative O_i. The aggregating results are in Table 1.

Table 1. The aggregating results of strategic suppliers by the DGSVNNWBM and DGSVNNWGBM ($R = (1, 1, 1, 1)$.).

	DGSVNNWBM	**DGSVNNWGBM**
O_1	(0.5720, 0.6135, 0.2929)	(0.5692, 0.6209, 0.2951)
O_2	(0.8157, 0.5549, 0.3423)	(0.8150, 0.5552, 0.3454)
O_3	(0.6253, 0.5980, 0.3033)	(0.6249, 0.6066, 0.3051)
O_4	(0.6377, 0.4583, 0.3888)	(0.6345, 0.4610, 0.3903)
O_5	(0.6431, 0.7999, 0.2927)	(0.6395, 0.8064, 0.2951)

Step 2. According to Table 1, the scores of the strategic suppliers are shown in Table 2.

Table 2. The scores of the strategic suppliers.

	DGSVNNWBM	DGSVNNWGBM
O_1	0.5552	0.5511
O_2	0.6395	0.6381
O_3	0.5747	0.5711
O_4	0.5969	0.5944
O_5	0.5168	0.5127

Step 3. According to the Table 2 and the scores, the order of the strategic suppliers is listed in Table 3, and the best strategic suppliers is O_2.

Table 3. Order of the strategic suppliers.

	Order
DGSVNNWBM	$O_2 > O_4 > O_3 > O_1 > O_5$
DGSVNNWGBM	$O_2 > O_4 > O_1 > O_3 > O_5$

4.2. Influence Analysis

To show the effects on the ranking results by altering the parameters of DGSVNNWBM (DGSVNNWGBM) operators, the corresponding results are shown in Tables 4 and 5.

Table 4. Order for different parameters of DGSVNNWBM.

R	$S(O_1)$	$S(O_2)$	$S(O_3)$	$S(O_4)$	$S(O_5)$	Order
(1,1,1,1)	0.5552	0.6395	0.5747	0.5969	0.5168	$O_2 > O_4 > O_3 > O_1 > O_5$
(2,2,2,2)	0.7701	0.8604	0.7861	0.8158	0.7326	$O_2 > O_4 > O_3 > O_1 > O_5$
(3,3,3,3)	0.8281	0.9127	0.8395	0.8660	0.8096	$A_2 > A_4 > A_3 > A_1 > A_5$
(4,4,4,4)	0.8508	0.9290	0.8583	0.8831	0.8455	$O_2 > O_4 > O_3 > O_1 > O_5$
(5,5,5,5)	0.8621	0.9351	0.8669	0.8915	0.8656	$O_2 > O_4 > O_3 > O_5 > O_1$
(6,6,6,6)	0.8689	0.9378	0.8715	0.8971	0.8785	$O_2 > O_4 > O_5 > O_3 > O_1$
(7,7,7,7)	0.8734	0.9392	0.8743	0.9015	0.8873	$O_2 > O_4 > O_5 > O_3 > O_1$
(8,8,8,8,)	0.8767	0.9400	0.8763	0.9055	0.8938	$O_2 > O_4 > O_5 > O_1 > O_3$
(9,9,9,9)	0.8792	0.9406	0.8778	0.9091	0.8987	$O_2 > O_4 > O_5 > O_1 > O_3$
(10,10,10,10)	0.8811	0.9411	0.8789	0.9124	0.9026	$O_2 > O_4 > O_5 > O_1 > O_3$

Table 5. Order for different parameters of DGSVNNWGBM.

R	$S(O_1)$	$S(O_2)$	$S(O_3)$	$S(O_4)$	$S(O_5)$	Order
(1,1,1,1)	0.5511	0.6381	0.5711	0.5944	0.5127	$O_2 > O_4 > O_3 > O_1 > O_5$
(2,2,2,2)	0.4108	0.4996	0.4227	0.4517	0.3726	$O_2 > O_4 > O_3 > O_1 > O_5$
(3,3,3,3)	0.3619	0.4291	0.3670	0.3985	0.3208	$O_2 > O_4 > O_3 > O_1 > O_5$
(4,4,4,4)	0.3385	0.3920	0.3405	0.3728	0.2962	$O_2 > O_4 > O_3 > O_1 > O_5$
(5,5,5,5)	0.3245	0.3702	0.3252	0.3572	0.2817	$O_2 > O_4 > O_3 > O_1 > O_5$
(6,6,6,6)	0.3149	0.3562	0.3153	0.3462	0.2716	$O_2 > O_4 > O_3 > O_1 > O_5$
(7,7,7,7)	0.3076	0.3467	0.3084	0.3377	0.2641	$O_2 > O_4 > O_3 > O_1 > O_5$
(8,8,8,8,)	0.3018	0.3398	0.3032	0.3308	0.2582	$O_2 > O_4 > O_3 > O_1 > O_5$
(9,9,9,9)	0.2978	0.3347	0.3002	0.3251	0.2531	$O_2 > O_4 > O_3 > O_1 > O_5$
(10,10,10,10)	0.4150	0.3307	0.4182	0.3203	0.2505	$O_3 > O_1 > O_2 > O_4 > O_5$

4.3. Comparative Analysis

Then, we compare our proposed operators with single valued neutrosophic weighted averaging (SVNWA) operator and single valued neutrosophic weighted geometric (SVNWG) operator [35]. The comparative results are depicted in Table 6.

Table 6. Order of the strategic suppliers.

	Order
SVNWA	$O_2 > O_4 > O_3 > O_1 > O_5$
SVNWG	$O_2 > O_4 > O_3 > O_1 > O_5$

From above, we can we get the same results to show the effectiveness and practicality of the proposed operators. However, the existing aggregation operators, such as SVNWA operator and SVNWG operators, don't take into account the relationship between aggregated arguments, and thus cannot eliminate the influence of unfair arguments on decision results. Our proposed DGSVNNWBM and DGSVNNWGBM operators consider the information about the relationship among multiple arguments being aggregated.

5. Conclusions

In this paper, we focused on SVNN information aggregation operators, as well as their application in MADM. To aggregate the SVNNs, the DGSVNNWBM and DGSVNNWGBM operators have been developed. We have conducted further research into these two operator's several desirable properties. In addition, we demonstrated the effectiveness of the DGSVNNWBM and DGSVNNWGBM operators in practical MADM problems. At the end of this study, we use an applicable example for supplier selection in the supply chain management process to show applicability of these two operators; meanwhile, the analysis of the comparison as the parameters take different values have also been studied. In our future studies, we shall expand the proposed models to other uncertain environments [36–57] and fuzzy MADM problems [58–80].

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