DEMONSTRATING THE RELATIONSHIP BETWEEN QUANTUM MECHANICS AND RELATIVITY

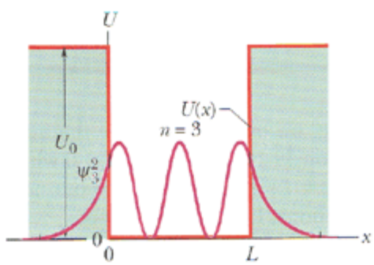
John Cipolla, February 8, 2018

Abstract

This **thought experiment** is an attempt to unify Einstein's theory of general relativity, which governs gravitational fields around massive objects and quantum mechanics, or the wave nature of matter and energy. String theory postulates that electrons and quarks in atoms are one-dimensional oscillating line segments. The first elementary particle string models were called **bosonic strings** because only **bosons** or force carriers like photons; gluons and the Higgs particles were modeled. Later, **superstring theory** was developed that predicted a connection or "super-symmetry" between bosons and **fermions** where fermions are elementary particles like electrons, protons and quarks that compose all ordinary matter. This analysis assumes all massive objects can be modeled as **heterotic** or closed superstrings. Further, demonstrating the relationship between string theory and general relativity is simplified because general relativity approaches the Newtonian limit for small velocity. These results indicate string theory and general relativity are related.

Background

In this analysis we attempt to relate the gravitational forces predicted by string theory, Einstein's theory of general relativity and classical Newtonian gravity. Establishing the rudimentary relationship between general relativity and quantum mechanics requires a simple "model equation" that represents the basic physics but at a very simplistic level.



Using the concept of a "model equation" for this analysis is required because the full string theory equations are far too complex at the present time. This basic technique is used in computational fluid dynamics (CFD) where solution methods are tested using one-dimensional "model equations" and later extended to the full three dimensional nature of fluid flow. For this analysis the one-dimensional string[1] illustrated in Figure-2 represents the **wave equation** in Figure-3 and its solution. This is a notional string theory solution and not the "real" solution. But, the basic physics of any problem can sometimes be revealed using the "model equation" technique. The solution for the wave equation[1] in Figure-3 reveals the traveling **wave velocity** of a disturbance on a string equals the square root of the **tension** on a string divided by the **linear mass density** of the string.

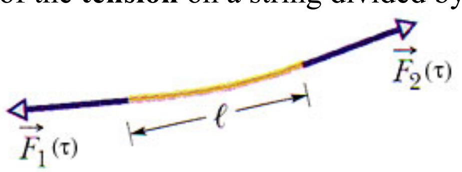


Figure-2, String segment

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \textit{where} \quad v = \sqrt{\tau/\mu}$$

Figure-3, String wave equation and solution

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad \textit{where} \quad \psi(x) = Ae^{ikx} + Be^{-ikx}$$

Figure-4, Schrödinger's wave equation for a free particle[1] in a potential well and probability function, ψ

After acknowledging the **wave equation** in Figure-3 and its solution for wave velocity, ν represents the physics of a vibrating string it becomes clear the **Schrödinger's wave equation** for a free particle in Figure-4 has a clear analogy with the physics of a vibrating string. Therefore, its seems reasonable to approach the unification of general relativity and quantum mechanics by using physics of the string wave equation as a starting point to develop a model equation for a Theory of Everything (TOE).

GENERAL RELATIVITY IN THE NEWTONIAN LIMIT FOR $|\nu| \ll c$

Solutions using general relativity approach solutions using Newtonian mechanics when mass-energy generated gravitational fields produce velocities, ν much less than the speed of light. $|\varphi| \ll 1$ *and* $|\nu| \ll c$. For the low velocity approximation, general relativity must make the same predictions as Newtonian mechanics. In addition, it can be shown using tensor mathematics that the Einstein field equation described by Equation-1 for relativistic motion in strong gravity fields reduces to the Newtonian solution for $|\nu| \ll c$ in Equation-2 and Equation-3.

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}. \tag{1}$$

$$\nabla^2 \Phi = 4\pi G \rho. \tag{2}$$

The solution of Equation-2 for a single gravitating body in Newtonian mechanics clearly shows the following potential field equation describes Newtonian gravity when $|\nu| \ll c$.

$$\Phi = -\frac{GM}{r}. \tag{3}$$

Finally, the acceleration of gravity in the Newtonian limit is easily derived and is used in this analysis to determine the gravitational force of attraction and acceleration of gravity for comparing string theory to Newtonian mechanics when $|\nu| \ll c$.

$$g = \frac{\mathrm{d}\Phi}{\mathrm{dr}} = \frac{GM}{r^2}. \tag{4}$$

$$F = mg. \tag{5}$$

GRAVITATIONAL FORCE OF ATTRACTION USING CLASSICAL THEORY

Derivation of the non-relativistic gravitational field using the equations of general relativity in the **Newtonian limit** follows. Please note the form of $\boldsymbol{F} = \boldsymbol{ma}$[1, 2] reduce to Newtonian mechanics for $v \ll c$ in the equations of general relativity[4, 5] as previously discussed. The Newtonian limited equations of general relativity are defined when three requirements are achieved. First, particles are moving slowly relative to the speed of light. Second, gravitational fields are weak meaning space-time can be considered flat. Third, gravitational fields are static and unchanging. These derivations will illustrate the equivalence between gravitational fields around massive objects for classical mechanics and the gravitational fields generated by **heterotic** strings when $v \ll c$. The Newtonian limited equations of general relativity for the classical gravitational force of attraction between planet Earth and the Sun is known to be.

$$F_{Newtonian} = \frac{G\, m_e M_{sun}}{{R_{sun}}^2}. \tag{6}$$

Where, the classical Newtonian acceleration of gravity at radius, $R_{sun}$is.

$$g_{Newtonian} = \frac{G\, m_e}{{R_{sun}}^2}. \tag{7}$$

GRAVITATIONAL FORCE OF ATTRACTION FOR A STRING

This analysis assumes a **heterotic** or closed superstring[3] circles the Sun at a distance equal to the orbital radius of the Earth around the Sun. The total mass of the circular superstring is equal to the mass of the Earth having uniform mass-energy density along the string. Motion of the Earth particle is notionally like that of matter-wave trapped in a potential well as depicted in Figure-1. In this case the probability of finding the Earth in any particular location around the Sun is described by Schrödinger's wave equation of a free particle[1] in a potential well where the probability function and its solution is described in Figure-4. In this case the potential well where the Earth is "trapped" is the gravitational potential well generated by the mass of the Sun which curves space-time as described by the Einstein field equation in Equation-1. Also, the energy required for the Earth to escape the potential well where it is "trapped" is equal to the escape velocity from the Earth's location orbiting around the Sun. The velocity, V_{escape} is critical to describe the exact solution required for this string solution to match classical theories of gravitation. The following derivation describes in detail how the string solution produces forces of gravitational attraction from tension in the superstring. First, the length of the matter-wave representing the Earth is derived from the geometrized mass of the Earth. The remaining steps develop the proposed string theory force of gravitational attraction and string theory gravitational acceleration toward the Sun. The equations for force of gravitational attraction and acceleration between each planet in the solar system and the Sun indicate excellent agreement between string theory and classical gravitational theory.

Length of string using **geometrized mass** length units for the Earth.

$$L_s = \frac{G\,M_e}{c^2}\,. \tag{8}$$

Linear mass density of string at distance Rsun from the Sun.

$$\mu = \frac{M_e}{L_s}. \tag{9}$$

String **wave velocity** set equal to the **escape velocity** from the solar system at Rsun.

$$v = V_{escape}. \tag{10}$$

Where, the escape velocity at distance Rsun is the following.

$$V_{escape} = \sqrt{\frac{2\,G\,M_{sun}}{R_{sun}}}\,. \tag{11}$$

Then, the well-known wave velocity[1] (Equation-1) of a heterotic string follows.

$$v = \sqrt{\frac{\tau}{\mu}}\,. \tag{12}$$

Solving, tension in a heterotic string due to gravitational force at radius, Rsun becomes.

$$\tau = \mu\,v^2. \tag{13}$$

Compute string force of attraction to the Sun by first determining string end-point slope.

$$tan(\alpha) = \frac{x}{\sqrt{R_{sun}{}^2 - x^2}}\,.\ \text{ Where, } x = {}^{L_s}\!/_2. \tag{14}$$

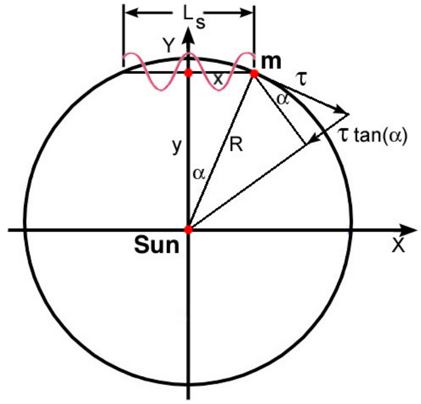


Figure-5, Gravitational attraction using string tension, τ and end-point slope, tan (τ) at distance R from Sun

Finally, string gravitational force of attraction and acceleration to the Sun is found to be.

$$F_{string} = \tau\ tan(\alpha) \quad \text{And} \quad g_{string} = \frac{F_{string}}{M_{sun}}. \tag{15}$$

After inserting the required input variables into the equation for $F_{classical}$ and F_{string} a MathCAD analysis predicts an exact match between classical results for gravitational force and acceleration and string theory gravitational force and acceleration.

$$\frac{F_{Newtonian}}{F_{string}} = \frac{F}{Fs} = 1 \quad \text{And} \quad \frac{g_{Newtonian}}{g_{string}} = \frac{G}{Gs} = 1. \tag{16}$$

Validation of the Proposed Relationship Using The Solar System

Figure-6 plots the ratio of gravitational force and acceleration computed using general relativity at the Newtonian limit to the gravitational force and acceleration computed by string theory for each planet in the solar system. Please notice the amazing result that the force of gravity and gravitational acceleration for Newtonian gravity and string theory are practically equal making the ratio of the results displayed in Figure-6 to be identically equal. These amazing results indicate the effects of gravity may be modeled as a heterotic or closed string as a first order model equation solution for a unifying theory for general relativity, quantum mechanics and string theory.

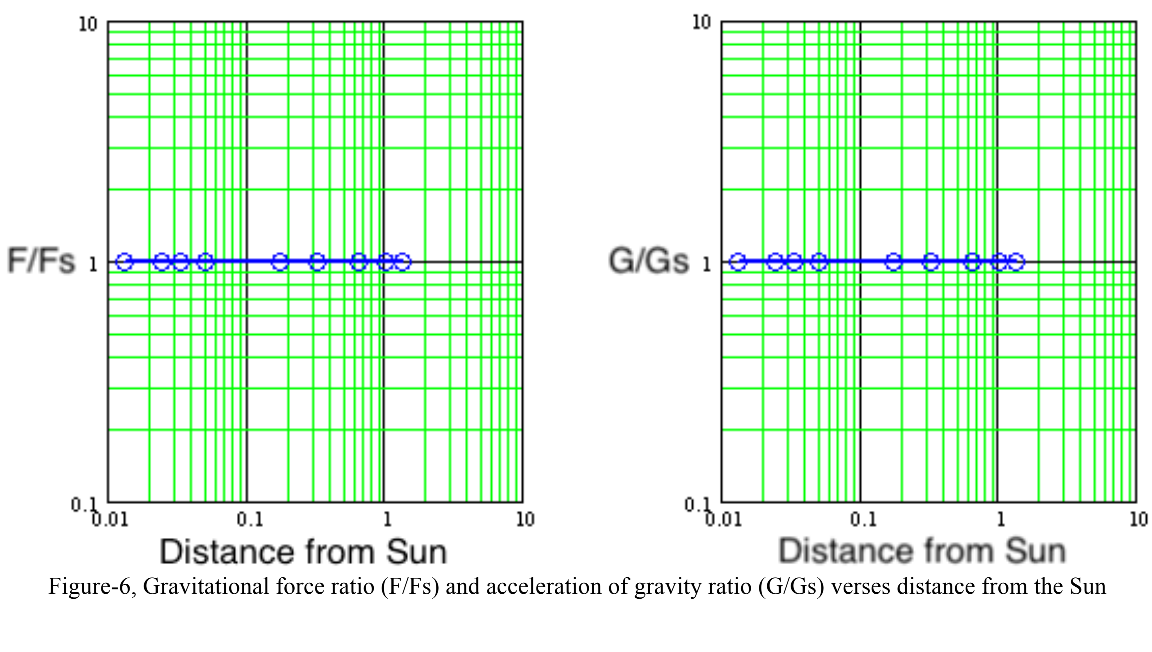


Figure-6, Gravitational force ratio (F/Fs) and acceleration of gravity ratio (G/Gs) verses distance from the Sun

SUMMARY OF RESULTS

This simple thought experiment indicates the tangential force acting in tension; τ on a heterotic or closed string having radius equal to Rsun from the Sun and planetary mass, m has an inward gravitational force, F_{string} equal to the gravitational force, $F_{Newtonian}$ acting on the planet. These interesting results were achieved by using **geometrized mass** units for planetary string length and then by setting planetary **wave velocity** equal to **escape velocity** at each planetary location, Rsun. These results indicate string theory has an embedded field theory relationship to general relativity and quantum mechanics when $|v| \ll c$. Finally, these results demonstrate that a relationship exits between quantum mechanics and general relativity and that string theory may provide a basis for describing the relationship between the gravity, electromagnetism, strong force (nuclear force) and weak force for a theory of everything (TOE).

DERIVATION OF STRING FORCE OF GRAVITY

In Figure-7 tangential force, τ acting on each end of string segment, L_s has a transverse force F_{string} that keeps the string segment in equilibrium and acts as a force of compression toward mass, M. If a hypothetical string having length, L_s is considered an elastic band, the force; F_{string} is acting as a compressive force when the string segment is held in place at radius, Rsun. Where, F_{string} is the force of gravitational attraction to mass, M that is equivalent to the force of gravity, $F_{Newtonian}$. The result that classical and string theory forces of attraction are equal strongly suggests there is a relationship between general relativity, Newtonian theory and string theory. The derivation of F_{string}, the force of attraction or gravitational force predicted by string theory follows.

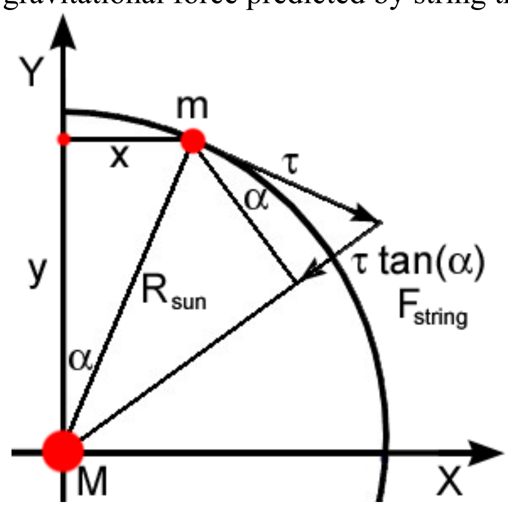


Figure-7, Derivation of F_{string}, the gravitational force of attraction due to string tension, τ

The equation for a circle that represents a **heterotic** or closed string of mass, m circling around the Sun of mass, M follows.

$$x^2 + y^2 = R^2. \tag{17}$$

Solving for variable, y the equation for the Sun-orbiting circle becomes.

$$y = (R^2 - x^2)^{1/2}. \tag{18}$$

Slope of string end-points is determined by first substituting, u for the variable, y.

$$u = y. \tag{19}$$

String end-point slope is determined using equation-8 on page 328 of reference 6.

$$\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}. \tag{20}$$

After finding the derivative the slope of each string end-point is.

$$\frac{dy}{dx} = -\frac{x}{\sqrt{R^2 - x^2}}. \tag{21}$$

Forces acting on the string toward the Sun are determined using string end-point slope.

$$\tau \tan\alpha = \tau \sin\alpha. \tag{22}$$

String end-point slope for a small angle approximation ($\alpha \to 0$) is.

$$\tan\alpha = \frac{dy}{dx}. \tag{23}$$

Numerically, string end-point slope is determined using Equation-21 and Equation-23.

$$tan(\alpha) = \frac{x}{\sqrt{R_{sun}{}^2 - x^2}}\text{ . Where, } x = {}^{L_s}\!/_2. \tag{24}$$

Then, after a little algebra, the string theory force of gravitational attraction, Equation-25 between the Sun and each planet in the solar system is easily determined. Equation-25 is used to determine string gravitational forces of attraction for each planet orbiting the Sun and compared to forces of gravitational attraction determined by general relativity and Newtonian theory when $|v| \ll c$. Finally, gravitational forces of attraction determined by general relativity are compared to string theory results as the ratio, $F_{Newtonian}/F_{string}$. Please see Figure-6, which illustrates an exact match between string theory and classical methods where the plotted ratio of gravitational forces of attraction and gravitational acceleration toward the Sun is exactly 1. These results indicate that classical methods of gravity are related to quantum mechanics and string theory.

$$F_{string} = \tau\ tan(\alpha). \tag{25}$$

REFERENCES

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NOTE: This is a work in progress and will be revised as new information is discovered