On the Non-Trivial Zeros of the Riemann Zeta Function

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Abstract

In this document, we present several important insights concerning the Riemann-Zeta function and the locations of its zeros. More importantly, we prove that we should be awarded the \$1 000 000 prize for proving or disproving the Riemann hypothesis.

1 Introduction

The Riemann Zeta function

$$\zeta(z) = \sum_{n=0}^{\infty} \frac{1}{n^z} \tag{1}$$

is a function with a \$1 000 000 prize attached to it. In this paper, we prove that we should be awarded this prize by proving or disproving it.

2 Prize

Theorem 2.1. *We should be awarded the \$1 000 000 prize for proving or disproving the Riemann hypothesis.*

Proof. We prove by exhaustion of cases.

Case 1: Assume that the Riemann zeta function has non-trivial zeros only of the form $\zeta(z) = 0, \Re(z) = \frac{1}{2}$. Then we have demonstrated that the Riemann hypothesis holds and are entitled to the prize.

Case 2: Assume that there exists a non-trivial root of the Riemann zeta function of the form $\zeta(z_0) = 0, \Re(z_0) \neq \frac{1}{2}$. Then we have demonstrated that z_0 is a counterexample to the Riemann hypothesis and are entitled to the prize.

By the law of the excluded middle, cases 1 and 2 are exhaustive.

Therefore we are entitled to the prize. □

3 Conclusion

As we have succinctly proven, the Riemann Hypothesis has been resolved via this extension of casework. Casework, while not new, has certainly not seen wide-spread use in mathematics. However, this approach can be widely used in areas such as analysis, algebra, and computer science. A simple proof resolving P vs. NP, for example, quickly follows by using this technique.