Energy Required to Keep The Universe Expanding: An insight on dark energy

By: Keaten Wood
2018
March

1 Abstract

It is popularly known that dark matter is responsible for the accelerating expansion of our universe, as well as many other phenomenon. This could very well be the case. Through many mathematical steps, using mechanical laws and calculus, I have determined the amount of energy needed to expand the universe a certain distance quickly converges, meaning that all the universe needs is a sufficient amount of energy in order to expand infinitely, either given initially by the big bang or gradually by dark matter.

2 Potential energy gained by rising object

Let's start with something simple. We know that the amount of potential energy gained by raising an object is the area under the curve of the gravitational force experienced over its distance from the attracting object.

$$\int_{r_s}^{r_e} (g * m_p * m_o)/r^2 \; dr \;\; = g * m_p * m_o * (1/r_s - 1/r_e) \tag{1}$$

for gravitational constant g, planet mass m_p, object mass m_o, initial distance from planet r_s and final distance from planet r_e

3 Potential energy gained by an expanding planet

In this case, we want to find out how much potential energy each point on the planet gains by moving away from it's center by a certain amount, and add all of those values together. In other words, we will integrate the amount of potential energy gained over each radius r from 0 to r_s, and angles α from 0 to 2π and σ from 0 to π.

Firstly, each point on the planet is going to expand by a different amount, depending on its distance from the center. For example, if a planet's radius increases by E, the particles closer to the surface will expand by E and particles closer to the center of the planet will expand by less. The expansion that a particle r away from the center will undergo can be given in terms of the planet's surface radius r_s and the amount that the planet's radius will increase E

$$E/r_s * r \tag{2}$$

Now, we must reconsider the equation given in section 2 so that it applies to the integral we are planning. In the original equation, m_p represented the mass of the planet whose mass is exerting a gravitational force on the object. Since an object experiences no net gravitational force in a shell, we can deduce that only the mass underneath the object will exert a gravitational pull. Using the ratios of the volumes of the planet and the volume of radius r, we know that the amount of mass acting on a point of a distance r away is

$$r^3/r_s^3 * m_p \tag{3}$$

For the entire planet's mass m_p

In the original equation, m_o represented the mass of the object being lifted, but when we are integrating each piece it will therefore become infinitely small. If we pull m_o outside of the integral, however, it becomes the sum of each of those tiny parts, which is m_p, since we are integrating over every point on the planet. Now we have the integral

$$2 * \pi^2 * g * m_p^2 * \int_0^{r_s} r^3/r_s^3 * (1/r - 1/(r + E/r_s * r)) \, dr \tag{4}$$

which can be simplified to

$$2 * \pi^2 * g * m_p^2 * E/(r_s^3 * (r_s + E)) * \int_0^{r_s} r^2 \, dr \tag{5}$$

solving the integral gives us

$$(2/3) * \pi^2 * g * m_p^2 * E/(r_s + E) \tag{6}$$

we can also acknowledge that for large values of E compared to r_s, the right-most fraction converges toward 1, which would give us

$$(2/3) * \pi^2 * g * m_p^2 \tag{7}$$

This is most certainly believable, the farther you are from the source of gravitational pull, the less energy it takes to move you farther away, so it's not absurd that this amount of energy would converge; As a matter of fact, it's mathematically accurate.

4 Application to big bang and universe expansion

Now, let's use the equation given in section 3 to see how much potential energy it took to expand the universe from some given initial size r_s, since for this application the universe can be treated as an inflating planet. It is obvious that our universe is no exception to the fact that the amount of potential energy needed to expand it a certain distance also converges to

$$(2/3) * \pi^2 * g * m_p^2 \tag{8}$$

for m_p being the mass of the universe.

If we consider the mass of the universe to be $m_p = 10^{53}$, then we realize that the amount of potential energy required to expand the universe by some large radius is about $4.4 * 10^{96}$ Joules.