A Proof of the Electromagnetic Nature of the Nuclear Binding Force

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ABSTRACT

This paper carries out an electric field theory analysis of the interaction between two polar electric fields that are bounded at a fixed radius. As the neutron is known to have a positive electric field bounded at a radius of about 0.8 x 10^{-15} m the analysis applies to the interaction between two neutrons. This analysis demonstrates that the electrostatic interaction between two such bounded electric fields is push-pull in nature and accounts for the nuclear binding force, further predicting how the force changes as the separation between the neutrons is reduced far below the push-pull balance point.

INTRODUCTION

This paper is in three parts. In the first part the methodology for examining the interaction between two charged particles is developed. In the second part the interaction between two electrons is examined in detail, and the interaction is shown to have a structure. In the third part truncating the electric fields at a certain radius - as happens with the neutron - is shown to cause this same structure to create the push-pull force/distance relationship demonstrated by the nuclear binding force.

CALCULATING THE FORCES BETWEEN TWO ELECTRONS FROM FIRST PRINCIPLES

When there are multiple sources of electric field such as two electrons, the composite electric field vector created by them at a point in space must be calculated by the vector addition of the field from each source at that point, so if E is the electric field vector:-

$$E_{resultant} = E_1 + E_2$$

Consider a pair of electric charges q_1 and q_2 whose electric fields extend to infinity, as is the case with electrons. Separate them by a distance 'r' and choose the co-ordinates so that one charge is at [-r/2,0,0] and the other at [r/2,0,0] as shown in Fig. 1. The inset shows the electric field vectors at point P[x,y,z].

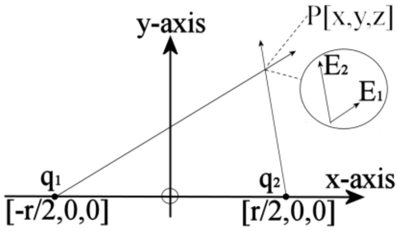

Figure 1. The potential energy density between two charges.

The x-axis is drawn left to right on the sheet. The y-axis is vertical, and the z-axis points out of the page. The total energy in the interaction is 'W' and the energy density dW/(dx dy dz) at point P[x,y,z] is given by:-

$$\frac{dW}{dx\;dy\;dz} = \frac{\varepsilon(|E_{resultant}|^2)}{2}$$

$$= \frac{\varepsilon(|E_1 + E_2|^2)}{2}$$

$$= \frac{\varepsilon(E_1 \bullet E_1 + \; 2E_1 \bullet E_2 + E_2 \bullet E_2)}{2}$$

$$= \frac{\varepsilon E_1 \bullet E_1}{2} + \; \varepsilon(E_1 \bullet E_2) + \frac{\varepsilon E_2 \bullet E_2}{2}$$

Now the first term is the energy density that the first electric field vector would have at that point if the first charge was alone in the universe. The third term is the energy density the second electric field vector would have at that point if the second charge was alone in the universe. The second term is the change in energy density caused by bringing the two charges together to interact. So if we subtract the energy density of each field $\mathbf{E}_1$ and $\mathbf{E}_2$ as it would be at infinite separation we are left with the second term as the "change" component.

When this term is integrated over all of space we get the potential energy ‘U’ between the two charges in moving from an infinite separation to their current separation. Hence this change term is the “potential energy density” at an arbitrary point P[x,y,z] for a charge separation of **r**, and ‘dU/(dx dy dz)’ is the potential energy density at a point, whilst $E_1 \bullet E_2$ is the vector dot product at that point.

$$\frac{dU}{dx\ dy\ dz} = \varepsilon(E_1 \bullet E_2)$$

To find the total potential energy U at a separation r, first find the potential energy density at a point.

$$E_1 = \frac{q_1}{4\pi\varepsilon l_1^2} = \frac{q_1}{4\pi\varepsilon\left(\left(x+\frac{r}{2}\right)^2 + y^2 + z^2\right)}$$

$$E_2 = \frac{q_2}{4\pi\varepsilon l_2^2} = \frac{q_2}{4\pi\varepsilon\left(\left(x-\frac{r}{2}\right)^2 + y^2 + z^2\right)}$$

Then integrate dU/(dx dy dz) over all space.

$$U = \varepsilon \iiint_{-\infty}^{+\infty} (E_1 \bullet E_2)\ dx\ dy\ dz$$

$$= \frac{q_1 q_2}{16\pi^2\varepsilon} \iiint_{-\infty}^{+\infty} \left(\frac{\left(\left(x+\frac{r}{2}\right)\left(x-\frac{r}{2}\right) + y^2 + z^2\right)}{\left(\left(\left(x+\frac{r}{2}\right)^2 + y^2 + z^2\right)\left(\left(x-\frac{r}{2}\right)^2 + y^2 + z^2\right)\right)^{3/2}} \right) dz\ dy\ dx$$

(1)

We cannot integrate this for the analysis of two intersecting neutron-charge spheres because the limits are difficult to handle, as will become apparent in due course, being the intersection volume of two spheres. Further, there is an infinity in the equation at the centre of each charge. We know this cannot exist in reality as it would lead to the particle’s electric field having an infinite energy so there must be an inner limit to the charge distribution. There is little point in continuing with the integration because of problems with the limits when we come to look at the interaction between two neutrons and instead we need to use Finite Element Analysis which allows us to carry out a summation instead.

The above equation allows us to calculate the potential energy. From this we can derive the force.

USING THE FINITE ELEMENT SUMMATION FOR THE FORCE BETWEEN TWO ELECTRONS

When we use the above approach for the force between two electrons q_1 and q_2 the result agrees with the value provided by the standard equation for the force between two charges, with minor computational errors from the Finite Element approach.

$$F = \frac{q_1 q_2}{4\pi\varepsilon r^2}$$

In the Lorentz Force Equation form where one point charge is conceptually seen as lying in the distributed electric field of another we have $\boldsymbol{F} = q_1\boldsymbol{E}_2$. Here $\mathbf{E}_2$ is the electric field vector generated by charge q_2 as seen at the centre of q_1. Conceptually, q_1 is a point charge and the distributed field from q_2 permeates all space. We can equally choose q_2 to be a point charge in the distributed field from q_1. This does not mean that one charge has miraculously become a point charge while the other remains a distributed electric field!

THE STRUCTURE OF THE ELECTROSTATIC INTERACTION BETWEEN TWO NEUTRONS

Let us look in more detail at the energy interaction between two electrons. Consider the numerator from (1):-

$$\left(\left(x+\frac{r}{2}\right)\left(x-\frac{r}{2}\right)+y^2+z^2\right)$$

This expands to:-

$$\left(x^2+y^2+z^2-\frac{r^2}{4}\right)$$

Inspection shows that the numerator is zero on the circumference of a sphere whose diameter ‘r’ is the line joining the two charges. The sign of the function changes as we move from inside the sphere to the outside; this tells us that there are attractive forces inside this region and repulsive forces outside it. Between two electrons Finite Element analysis tells us that this “Sphere of Attraction” generates attractive forces whose strength is about 22% of the repulsive forces outside the sphere, leading to a net repulsion. Likewise between two opposite charges such as the electron and positron this sphere is a region of repulsion and in a similar way this repulsion is overcome by the stronger attractive forces outside the sphere.

Consider now what happens if the charges’ fields do not extend to infinity but are suddenly truncated at some finite radius. Equation (1) above is still valid, but because the limits of the interaction are truncated the limits of the integration changes and the result is no longer the Lorentz one. The neutron is such a particle, known to have a positive electric field truncated at a radius of about 10^{-15}. Consider Fig. 2, showing the interaction between two neutrons at three different separations. At each separation the black circles show the limits of the electric field of the two neutrons and the dashed circle indicates the limits of the Sphere of Attraction. The Finite Element summation limits cover only that region where the neutron fields overlap and therefore interact, namely that region which is simultaneously within the field boundary of both neutrons. We can reasonably assume that the field strength of the neutron follows the classic $1/r^2$ profile inside its charge radius. This region is shown dotted-in in Fig. 2 and the Finite Element summation limits cover those parts of the neutrons' fields that are inside the boundary radius of both fields.

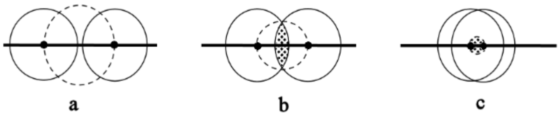

Figure 2. The interaction between two neutrons at different separations.

In Fig. 2a the neutron fields do not overlap, so at all separations greater than twice the truncated charge radius there is no interaction. In Fig. 2b there is a partial overlap of the neutrons’ fields, and so there is an interaction; however, this interaction is entirely contained within the Sphere of Attraction and so at this separation the forces are entirely attractive. In Fig. 4c the Sphere of Attraction is small compared with the total overlap, and bearing in mind that most of the forces are generated inside a radius of about five times the separation it can be seen that the region of repulsion dominates and is tending to the classic $1/r^2$ force profile, where ‘r’ is separation between the centres of the neutrons. Finite Element Analysis gives the force profile shown in Fig. 5. This figure compares the force/separation curve of two neutrons of radius ‘t’. For reference, the $1/r^2$ curve that would apply if the neutron’s electric field extended to an infinite radius is shown as a dotted line.

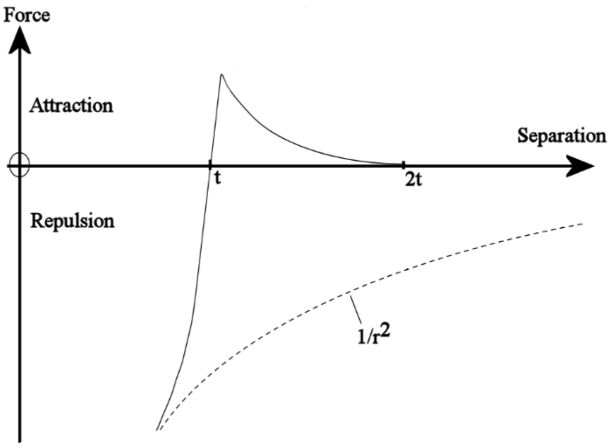

Figure 3. The force between two neutrons as a function of their separation.

There is no interaction at a separation of more than twice the neutron's charge radius 't' as the fields do not overlap. Below this, as the neutrons come together, the attractive force rapidly climbs to a sharp peak then dramatically reverses into a very steep part of the curve, passing through zero force at a separation of 1.00 neutron radii and continuing falling to converge with the $1/r^2$ curve. Two neutrons would therefore come to rest at a centre-to-centre separation of one neutron radius.

Measurements of how the neutron-neutron force changes with separation generally show a softer curve than that shown in Fig. 3, having a soft peak to right of the theoretical peak. There are two possibilities for this difference. The first possibility is that nuclear kinetics limits the accuracy of measurement, softening the curve as if viewed through a low-pass filter and essentially smoothing out the sharp peak and instead showing just the broad outline of the curve. The second possibility is that the neutron's electric field does not abruptly drop to zero at the limiting radius but fades to zero over a small shift in that radius (our calculation in Fig. 3 assumed a hard truncation of the electric field).

This theory gives a very good match to the attraction/repulsion forces between neutrons. This demonstrates that the electrostatic forces between neutrons are in fact the nuclear binding forces. It also confirms the inapplicability of the Lorentz Force Equation in this situation.