The Nuclear Force and Limitations to the Lorentz Electrostatic Field Equation

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Abstract

In Electromagnetic Field Theory it is the interaction of distributed electrostatic fields that leads to the forces between charged particles. The field of an electron spreads across all space, so the interaction between two electrons covers all of space. If we calculate the point energy densities of the interaction between these two electrons and sum them over all space, then differentiate it with respect to separation to get the force, it is no surprise that we end up with the electrostatic component of the Lorentz Force Equation. An interesting part of the interaction is that not only do the two electrons have field lines that are broadly aligned and thus create increased energy densities and repulsive forces, but there is a region between the two electrons where the field lines oppose each other, reducing the energy density and giving rise to attractive forces. With the electron – whose field extends to infinity – the repulsive forces dominate at all separations. But now consider the neutron whose electric field is bounded at a miniscule radius. There are obviously no forces between two neutrons outside twice the radius as there is no overlap in the fields. As they close there is a limited range over which the repulsive forces are not in play and the attractive region dominates. This leads to an attractive force between neutrons for separations of two and one times the radius, inside of which repulsive forces finally take over.

An Electromagnetic Model of the Strong Nuclear Force

The appendix gives the basic field interactions and point energy density equations. These are no longer included in some textbooks so can take time to locate.

Introducing the Lorentz Force Equation

How does the Lorentz Force Equation work? There are two parts to it, the first being the forces on an electron placed in an electrostatic field. This is relevant to how electrostatic forces hold the atomic nucleus together. The Lorentz Force Equation is old and dates from the origins of Electromagnetism. The equation is...

$$\mathbf{F} = \mathrm{q}(\mathbf{E} + \mathbf{v}x\mathbf{B})$$

'**F**' is the vector force. It is created by the interaction of a charge 'q' with a local electric field vector '**E**', added to the separate interaction of that same charge 'q' with a vector magnetic field '**B**' through which it is moving on velocity vector '**v**'. The cross product '**v**x**B**' induces an electric field which is normal to both the direction of motion and to the magnetic field, and with which the charge 'q' interacts.

How is this equation derived? In this chapter I consider only the first part of the equation, '**F** = q.**E**'. Normally the electric field '**E**' is generated from an assembly of other charges. This was historically derived as follows. The electric field '**E**' at an arbitrary point around an electron of unit charge q_e is given by...

$$\mathbf{E} = \frac{\mathrm{q_e}}{4\pi\varepsilon\mathbf{r}^2}$$

r is the radial vector from the centre of the charge. 'ε' is permittivity.

Traditionally in the Lorentz equation we then place a second charge – a "test" point charge - q_t "in the field" of the first charge and the force it experiences is the field strength of the first charge at the centre of the test charge, times the strength of the test charge...

$$\mathbf{F} = -\mathrm{q_t}\mathbf{E}$$

$$= -\frac{\mathrm{q_t q_e}}{4\pi\varepsilon\mathbf{r}^2}$$

However, a moment's thought brings up a significant conceptual flaw. We can choose *either* of our charges to be our 'test' point charge, each point charge interacting with the field from the other charge. Even if we add multiple charges in the region, we still treat the solitary "test" charge as a point charge interacting with the vector sum of the fields from all the other charges. The equation clearly treats the test charge as a point object responding to the summated vector field strength at a point, while it treats every other charge as a distributed charge that helps make this summated field. How can the test charge *know* that it is the test charge and react as a point object, whilst the other charges respond as distributed fields? It can't. In fact in field theory, the forces on the test charge are a result of the interaction of its distributed field with the distributed fields of all the other charges. There is no such physical entity as a "point charge" in Electromagnetic Field Theory.

Calculating the potential energy between two charges

So how does the equation manage to work when it treats the test charge as a point object but all other charges as the source of a distributed field? To understand this it is necessary to look at how the distributed field of the test charge interacts with the distributed fields of the other charges. Essentially, we need to analyse the field interactions from first principles, using the Principle of Conservation of Energy, to determine if the equations actually work. To do this we first determine the potential energy density of the interaction between the electric fields at a point in space, and then integrate over all space to get the total potential energy; from this we can determine the forces involved.

Consider a pair of distributed electric charges q_1 and q_2 whose electric fields extend to infinity, as happens with electrons. Separate them by a distance 'r' and choose the co-ordinates so that one charge is at [-r/2,0,0] and the other at [r/2,0,0] as shown in Figure 1.

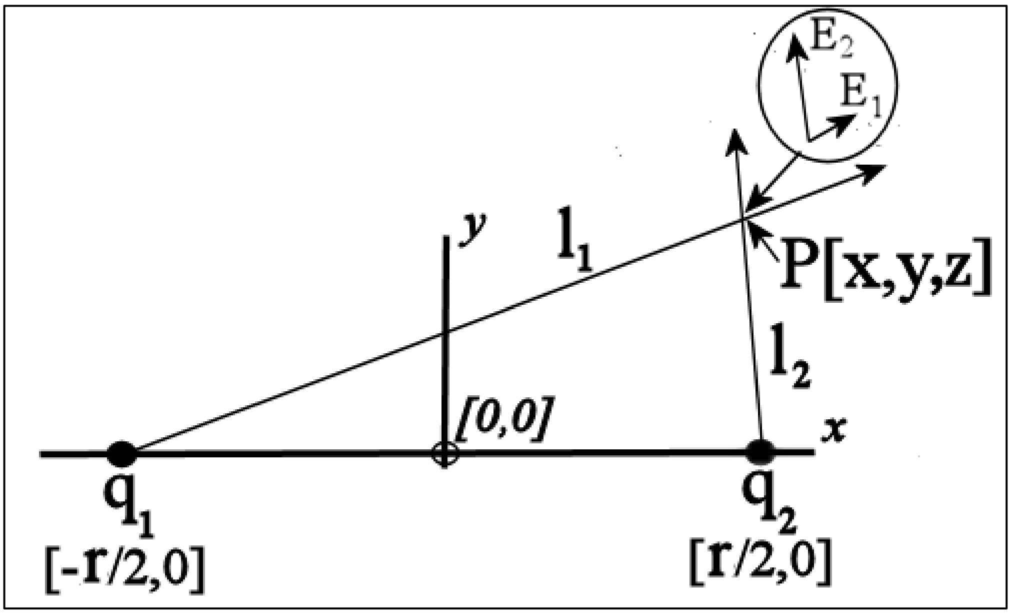

Figure 1

The x-axis is left to right on the sheet. The y-axis is bottom to top, and the z-axis points out of the page. The potential energy density at an arbitrary point P[x,y,z] for a separation **r** is (using the calculations given in the Appendix)...

$$\frac{dU}{ds} = \varepsilon(\mathbf{E}_1 \bullet \mathbf{E}_2)$$

Here 'U' is the total potential energy, 'dU/ds' is the potential energy density at a point, and $\mathbf{E}_1 \bullet \mathbf{E}_2$ is the vector dot product. To find the total potential energy U at a separation **r**, integrate dU/ds over all space.

$$\mathbf{E}_1 = \frac{q_1}{4\pi\varepsilon \mathbf{l}_1^2} = \frac{q_1}{4\pi\varepsilon\left(\left(\mathbf{x} + \frac{r}{2}\right)^2 + \mathbf{y}^2 + \mathbf{z}^2\right)}$$

$$\mathbf{E}_2 = \frac{q_2}{4\pi\varepsilon \mathbf{l}_2^2} = \frac{q_2}{4\pi\varepsilon\left(\left(\mathbf{x} - \frac{r}{2}\right)^2 + \mathbf{y}^2 + \mathbf{z}^2\right)}$$

$$U = \varepsilon \iiint_{-\infty}^{+\infty} (\mathbf{E}_1 \bullet \mathbf{E}_2)\, dx\, dy\, dz$$

$$= \frac{q_1 q_2}{16\pi^2\varepsilon} \iiint_{-\infty}^{+\infty} \left(\frac{\left(\left(x + \frac{\mathbf{r}}{2}\right)\left(x - \frac{\mathbf{r}}{2}\right) + y^2 + z^2\right)}{\left(\left(\left(x + \frac{\mathbf{r}}{2}\right)^2 + y^2 + z^2\right)\left(\left(x - \frac{\mathbf{r}}{2}\right)^2 + y^2 + z^2\right)\right)^{3/2}} \right) dz\, dy\, dx$$

Equation 1

This integration is difficult. There are discontinuities at the centre of each charge and we know that the equation does not apply there or the rest mass of the electron would be infinite (have a search for "The Classical Radius of the Electron" for the argument) so the centre of both electrons must be outside the limits; however the electron field goes on to infinity outside the Classical Radius so there are no other limits. It is much simpler to demonstrate that Numerical Integration gives us a solution for potential energy numerically equal to...

$$U = \frac{q_1 q_2}{4\pi\varepsilon \mathbf{r}}$$

... for separations significantly greater than the Classical Radius. Now since Force is equal to the rate of change of energy with distance (i.e. the separation '**L**'), to find the force between two charges at a separation 'L' we have:-

$$\mathbf{F} = \frac{dU}{d\mathbf{r}} = -\frac{q_1 q_2}{4\pi\varepsilon \mathbf{r}^2}$$

The minus sign indicates that the force opposes any increase in energy. Returning to the Lorentz Force Equation, the field from charge q_2 at a distance **r** is...

$$\mathbf{E}_2 = \frac{q_2}{4\pi\varepsilon \mathbf{r}^2}$$

Finally if we assign $\mathbf{E}_2$ to the background field from charge q_2, and q_1 to the 'test point charge' placed at a separation **r** from the centre of q_2, we derive the Lorentz equation...

$$\mathbf{F} = -q_1\mathbf{E}_2$$

As can be seen, it does not mean that one charge has miraculously become a point charge while the other remains a distributed electric field! It is just happenchance that allows this arbitrary assignment.

Note that the minus sign that appears in this equation is because the force opposes higher-energy configurations so causes like charges to be forced apart. This minus sign is often by convention dropped in the Lorentz Force Equation and a positive charge is then taken to be impelled in line with the field vector, which gives the same result – like charges are forced apart.

This brings us back to the first part of the Lorentz Force Equation and proves that the first part of the equation agrees with - and is derived from - the Principle of Conservation of Energy in this particular geometry. However, it must be born in mind that it works *only* because the potential energy in this arrangement is $q_1q_2/4\pi\varepsilon\mathbf{L}$. This will not be the case if either or both of the electric fields involved do not extend to infinity because the integration limits are then reduced. Before analyzing the consequences of this limitation in more depth, let us look in more detail at the energy interaction between two electrons.

The Structure of the Electrostatic Interaction between two Electrons

Consider the numerator from the triple integration in Equation 1:-

$$\left(\left(x+\frac{\mathbf{r}}{2}\right)\left(x-\frac{\mathbf{r}}{2}\right)+y^2+z^2\right)$$

This expands to:-

$$\left(x^2+y^2+z^2-\frac{r^2}{4}\right)$$

Inspection shows that the numerator is zero on the circumference of a sphere whose diameter is the line joining the two charges. The sign of the function changes as we move from the inside of the sphere to the outside. This implies that between two like charges there is a region inside this sphere where there are attractive forces, even though these forces must be overcome by the stronger repulsion outside the sphere to create a net repulsion. Likewise between two opposite charges there is a region of repulsion inside this sphere even though this repulsion is overcome by the stronger attractive forces outside the sphere.

To get an idea of the structure of attraction and repulsion in the interaction between two like charges, consider Figure 2, which shows how electric field lines from each charge are orthogonal to each other on the sphere whose diameter joins the centres of the two charges.

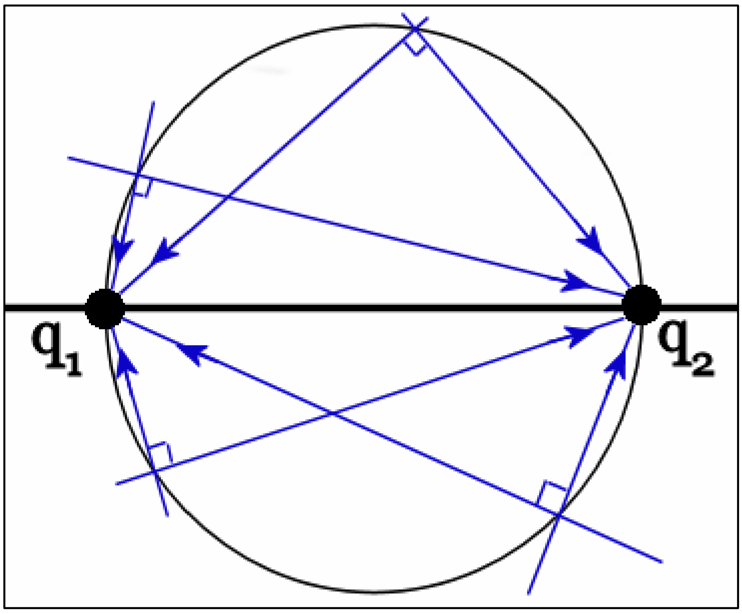

Figure 2

Being orthogonal to each other, there is no interaction on the surface of this sphere between the charges. Inside the circle, however, the field lines intersect at 90 to 270 degrees, so the fields oppose each other and hence tend to cancel each other, even if only partially. This reduces the energy density inside the sphere, leading to attractive forces in this region. Outside the sphere, however, the field vector intersection is between -90 to +90 degrees and the fields augment each other so that the energy density increases and repulsive forces are generated. The net effect is one of repulsion between electrons, as the sum total of the attractive forces inside the sphere is a little over one fifth of the repulsive forces outside the sphere. This applies at all separations way out to infinity, as the electron's fields extend to infinity.

Figure 3 shows the classical field pattern which results from this field cancellation within the "sphere of attraction"...

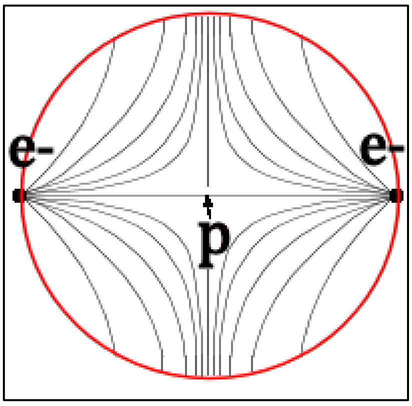

Figure 3

This pattern is sometimes assumed to be a repulsion pattern between the two electrons but nothing could be farther from the truth. At point 'p' in Figure 3 the fields from the electrons are equal and opposite, so the energy density is zero. Field regions vertically up and down from this point have their horizontal component cancelled leaving only their vertical component; the central vertical flux lines runs from the periphery, dropping in strength all the way to point ‘p’ where they meet and are zero. At point ‘p’ two new horizontal flux lines start and run each way, one to the centre of each electron, gaining in strength all the way. Other flux lines adjacent to these are continuous, running from the periphery to the centre of one or other electron, but the nearer they are to point ‘p’ the greater the cancellation and the weaker the field strength.

It can be seen that the distributed forces between two charges make a rather beautiful composite field pattern in space. More complex arrangements of charges create an art form of their own. No matter how complex the charge distribution, provided all the charges have fields that extend to infinity, the gross forces are perfectly described by the Lorentz Force Equation and its “point charge in a distributed field” approach works well.

But what if the charge’s field does *not* extend to infinity? What then?

Failure of the Lorentz Force Equation in dealing with the Electrostatic Forces between two Neutrons

Now consider what happens if the charges do not extend to infinity but are suddenly truncated at some finite radius. Equation 1 used above in deriving the Lorentz Force equation is still valid, but because the limits of the interaction are truncated at a finite radius the result of the integration changes and that result is no longer the Lorentz one. The neutron is such a particle, known to have a positive electric field truncated at a radius of about 10^{-15} meters (there has been much discussion about the exact radius, and this is only an approximate value). Consider Figure 4, showing the interaction between two such neutrons at three different separations. At each separation the black circles show the limits of the electric field of the two neutrons and the red circle indicates the region of the potential “sphere of attraction”. Assume that the field strength of the neutron follows the classic $1/r^2$ profile inside its charge radius.

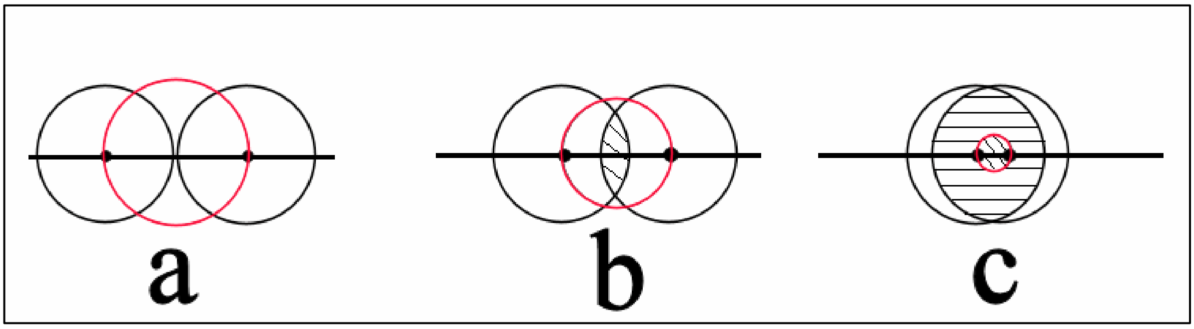

Figure 4

In Figure 4a the neutron fields do not overlap, so at all separations greater than twice the truncated charge radius there is no interaction. In Figure 4b there is a partial overlap of the neutrons’ fields, and so there is an interaction; however, this interaction is entirely contained within the “sphere of attraction” region and so at this separation the forces are entirely attractive. In Figure 4c the “sphere of attraction” region is small compared with the total overlap, and bearing in mind that most of the forces are generated inside a radius of about five times the separation it can be seen that the region of repulsion dominates and is tending to the classic $1/r^2$ force profile, where ‘r’ is separation between the centres of the neutrons. The awkward limits make direct mathematical integration impractical but Numerical Computation gives the force profile shown in Figure 5 (here ‘t’ represents the radius of the neutron’s truncated field limit). This compares the force/separation curve of two neutrons of radius ‘r’ with the $1/r^2$ curve that would apply if the neutron’s electric field extended to infinite radius as it does with the electron.

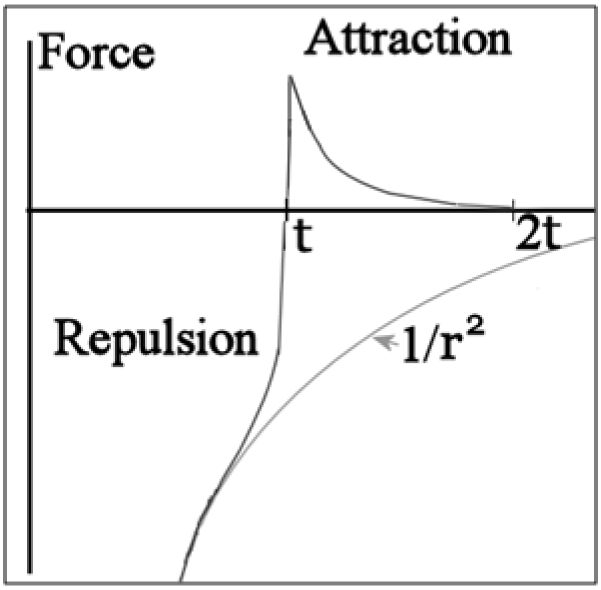

Figure 5

There is no interaction at a separation of more than twice the neutron’s charge radius ‘t’ as the fields do not overlap. Below this, as the neutrons come together, the attractive force rapidly climbs to a peak at a separation of 1.02 neutron radii, then collapses very suddenly, dropping through zero into repulsive forces at a separation of 1.0 neutron radii and continuing falling to converge with the $1/r^2$ curve. Two neutrons would therefore be at rest at a centre-to-centre separation of one neutron radius.

Measurements of how the neutron-neutron force changes with separation generally show a softer curve than that shown in Figure 5, having a soft peak to right of the theoretical peak. There are two possibilities for this difference. The first possibility is that nuclear kinetics limits the accuracy of measurement, softening the curve as if viewed through a low-pass filter and essentially smoothing out the sharp peak and instead showing just the broad outline of the curve. The second possibility is that the softer curve is precise which would mean that the outer edge of the neutron’s electric field does not abruptly drop to zero at the limiting radius but fades to zero over a small shift in that radius (our calculation in Figure 5 assumed a hard sudden truncation of the electric field). However, as we will see later, that is unlikely because the neutron magnetic resonance would be proportionately spread in frequency.

The Force/Separation Profile of the Force holding the nucleus together

Existing studies of the force between two neutrons agree with the right hand side of the plot in Figure 5 but details of the force as it drops below zero do not seem to exist, especially in the region of convergence to a $1/r^2$ curve at separations significantly less than the charge radius 't'. This information would be a real test of this approach. However with the information we already have there seems to be a good match to the attraction/repulsion pattern between neutrons.

It also confirms the limits of the Lorentz Force Equation in this situation. For example in Figure 4b Lorentz predicts that there is no force because the ***centre*** of one neutron is outside the distributed field of the other so is in a region of zero external field strength. Yet there is clearly an interaction from the energy density integration. Hence the Lorentz Force Equation disagrees with the Principle of Conservation of Energy, reporting no forces where the energy density integration clearly predicts forces in Figure 4b. It cannot be emphasised too strongly that the Lorentz Force Equation works only for those particles whose electric fields extend to infinity.

The field strengths of neutrons, and therefore the forces involved in neutron-neutron interactions, are orders stronger than the forces between electrons.

A Model for the Proton

So how is the proton held in the nucleus of the atom? With the electron we know that the electric field continues inwards to a fixed radius at which it is truncated, because if it continued to zero radius the rest mass/energy of the electron would be infinite - look up references to "The classical Radius of the Electron" for the details. So we could perhaps model the proton electromagnetically as a sort of super-electron where the inner radius of the field continues to a smaller radius than that of the electron. However, it is impossible to make such an object hold position inside the atomic nucleus – it would be expelled immediately – and such a model will not work.

However, a model that does work well is where the proton has a neutron-like core with a high neutron-like field strength, surrounded by an electron-strength field that extends from the inner core's outer periphery out to infinity (of course the proton has a positive charge while the electron's is negative). Consider Figure 6, which shows just this layout.

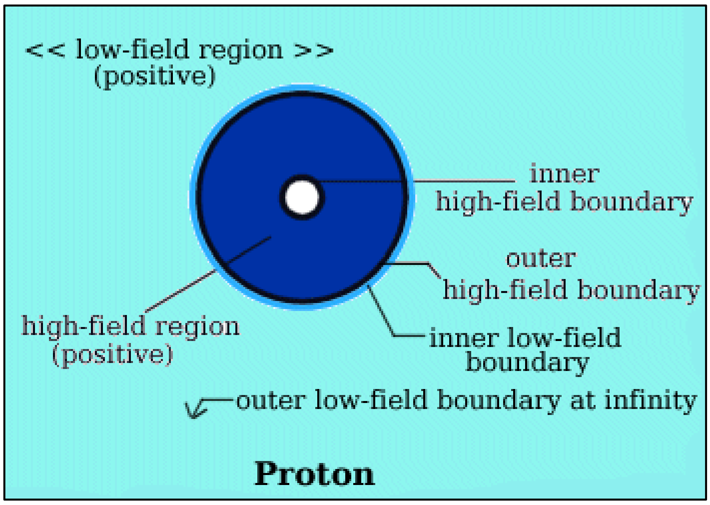

Figure 6

The interaction between a proton and a neutron would be similar to that between two neutrons at close range, but where the neutron and the proton core do not overlap and hence do not interact, but are still within about four times the core radii, Numerical Computation (using energy density integration techniques) predicts that there will be a weak interaction between the neutron's field and the proton's outer field that falls off extremely rapidly with increasing separation. This weak interaction has been recorded, confirming the viability of this electromagnetic model of the proton. The reason is demonstrated graphically in Figure 7. Figure 7a shows a proton whose centre is shown at point 'p' and a neutron whose centre is shown at point 'n'. The proton's outer field extends to infinity, but the neutron's field extends only to a finite radius; the inner core of the proton is not delineated but is assumed to lie entirely outside the neutron field and thus takes no part in the interaction. The interaction between the proton's outer field and the neutron occurs over the whole field of the neutron but nowhere else. Some field lines from the proton that intersect with the neutron's field are shown. The separation in Figure 7a is about two neutron radii. Figure 7b is similar except that the separation is much greater. The arc drawn across the neutron field represents the boundary of the sphere of attraction.

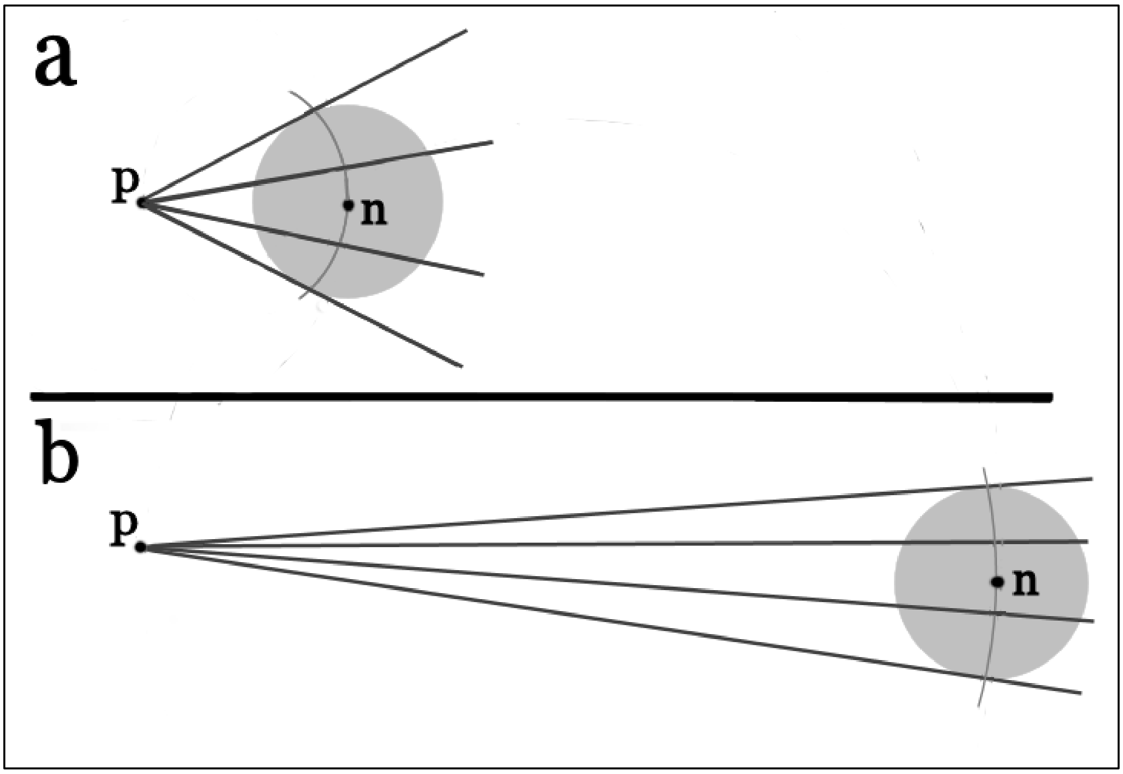

Figure 7

In Figure 7a the field gradient in the region of the proton's outer field that intersects the neutron field is very high, falling off rapidly from left to right in magnitude as the inverse of the square of the separation of the centres of the proton and the neutron. The region inside the sphere of attraction thus dominates, leading to a net attractive force.

In Figure 7b the field strength of the proton's outer field is much lower and the gradient is also much lower, giving almost the same proton field strength on the right of the neutron as on the left. The former makes the force between the particles fall off as the inverse of the square of the separation; the latter causes a second fall-off again as the inverse square. The latter effect is reduced by the fact that the volume of the neutron field is more equally shared between repulsion and attraction in Figure 7b, but the combined effect is still a force that falls of at very roughly the inverse cube of the separation.

So how does the proton structure of a high inner field surrounded by a low outer field hold together? Figure 8 shows the proton inner field (in red) displaced so that its inner field overlaps its outer field (in black).

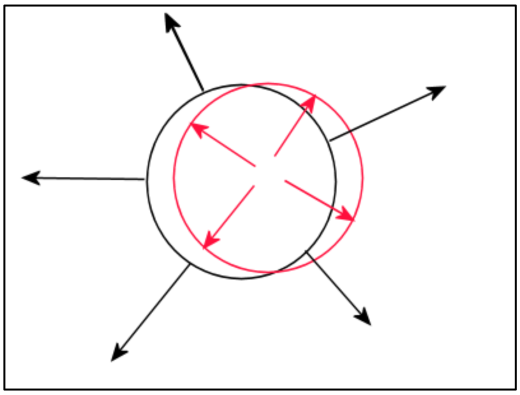

Figure 8

It can be seen that on the left side of Figure 8 the core has moved away from the outer field but there is no change in any energy density there as there never was any overlap between them anyway. On the right, however, the core field now overlaps with the outer field. Where it does so, the fields reinforce each other as they are both positive and this increases the total energy density in the overlap region, giving rise to repulsive forces that tend to restore the inner field to its original position with respect to the outer field. This means that the electromagnetic model indicates that this arrangement is stable. If, however, the core and outer fields had been of different sign any overlap would reduce the energy density, creating attractive forces that would create instability.

Hence a compound model of the proton works well and agrees with the qualitative force/distance interaction between protons and neutrons in an atomic nucleus. Determining the exact values for inner field strength, and the core diameter, needs us to look elsewhere. However, the qualitative behaviour is a good match to measurement. This match requires the use of the energy density integral in analysis rather than the Lorentz Force Equation which gives results known to be at odds with measurement. This demonstrates that the Lorentz Force Equation violates the Principle of Conservation of Energy in this instance.

Appendix: The Basic Electromagnetic Field Equations

Introduction to the Electromagnetic Field

In this appendix we examine the fundamental energy and induction equations from which everything else in this book is derived. Little else is needed. They are presented here because over the latter half of the 20th century these equations have been increasingly dropped from textbooks. All equations are based on SI units.

Within Electromagnetic Field Theory, the interaction between charged particles is in fact the interaction of their distributed Electromagnetic Fields. Here we introduce those basic field equations. No matter how complex the problem we can analyse the behaviour of any assembly of charged particles by the interaction of their fields and the corresponding changes in total field energy as they move. This is often best done by Numerical Computation, also termed Finite Element Analysis, where the energy density at a point is integrated over all space to give a complete result.

The Electric Field

The electric field at a point in space is described by the electric field vector **E**, giving the magnitude and direction of the electric field. There is energy in any electric field, and to describe the amount of energy associated with that point in space we have to use the energy density, which describes the energy density in terms of energy per unit volume (such as Joules/comic metre). The total energy associated with a single point in space is of course zero as a point in space has zero length in all spatial dimensions; it is by integrating the energy density over all space that we get actual energy values.

Deriving electromagnetic equations from first principles is straightforward, and is based on the equation...

$$\frac{dW}{ds} = \frac{\varepsilon|\mathbf{E}|^2}{2}$$

$$= \frac{\varepsilon\,\mathbf{E} \bullet \mathbf{E}}{2}$$

...where dW/ds is the absolute electric energy density per volume ‘ds’ at a point in space where the vector electric field is **E**.

The *potential energy* is a measure of the work we can extract from a system, and is generally much more useful than the absolute energy. From the above equation we can work out the *potential* (as opposed to the absolute) energy density. When two electric fields E_1 and E_2 interact the absolute energy density at a point in space is derived from the vector sum of the two fields...

$$\frac{dW}{ds} = \frac{\varepsilon(|\mathbf{E_1} + \mathbf{E_2}|^2)}{2}$$

$$= \frac{\varepsilon(\mathbf{E}_1 \bullet \mathbf{E}_1 + 2\mathbf{E}_1 \bullet \mathbf{E}_2 + \mathbf{E}_2 \bullet \mathbf{E}_2)}{2}$$

$$= \frac{\varepsilon\mathbf{E}_1 \bullet \mathbf{E}_1}{2} + \boldsymbol{\varepsilon}(\mathbf{E}_1 \bullet \mathbf{E}_2) + \frac{\varepsilon\mathbf{E}_2 \bullet \mathbf{E}_2}{2}$$

Now since the first term and the third term are simply the absolute energy densities of the individual and separate fields $\mathbf{E}_1$ and $\mathbf{E}_2$, it follows that the second term is purely the interaction energy – that is, the *potential* energy associated by bringing the two fields together to interact. Hence, at a point in space where two fields interaction, the "potential energy density" dU/ds is the vector dot product of the fields times the permittivity...

$$\frac{\mathrm{dU}}{\mathrm{ds}} = \varepsilon(\mathbf{E}_1 \bullet \mathbf{E}_2)$$

The integral of this function over all space is the total potential energy. This is the potential energy seen when (say) bringing two electrons together from infinite separation and appears often in electrostatic field calculations. Differentiating either the total energy or the total potential energy with separation '**l**' gives the force, **F**=dU/d**l**. It is generally easier to do the latter.

The Magnetic Field and Potential

Magnetic fields operate in the same way. The energy density dW/ds for a magnetic field of field strength **B** is:-

$$\frac{\mathrm{dW}}{\mathrm{ds}} = \frac{\mathbf{B}^2}{2\mu}$$

The magnetic field, with its energy density, links regions of different magnetic potential. Regions with a constant magnetic potential have zero energy density.

It is trivial to derive the magnetic potential energy of two interacting magnetic fields in a similar way to the derivation of the electric field potential energy of two interacting electric fields. It is...

$$\frac{\mathrm{dU}}{\mathrm{ds}} = \frac{(\mathbf{B}_1 \bullet \mathbf{B}_2)}{\mu}$$

Moving fields

A magnetic field moving in our own stationary frame of reference induces an electric field in that stationary frame. The strength of the induced electric field is...

$$\mathbf{E} = \mathbf{v}x\mathbf{B}$$

Here **B** is the vector magnetic field strength and **v** is the vector velocity so that the induced electric field **E** is the cross product **v***x***B**.

Equally, an electric field moving in our own stationary frame of reference induces a magnetic field in that stationary frame. The strength of the induced magnetic field **B** is...

$$\mathbf{B} = \mu\varepsilon(\mathbf{v}x\mathbf{E})$$

$$= (\mathbf{v}x\mathbf{E})/\mathbf{c^2}$$

Force

Force is an effect that appears when energy is extracted from or entered into an energy storage mechanism and essentially couples different energy systems. Force is anonymous in that a pure force by itself carries no information as to what generated it; you have to look further into how it arose although you may get clues by its spatial distribution. Force is an energy conservation mechanism as without it no energy could ever be exchanged. Equally, without at least possible energy changes there can be no forces. It is a vector quantity defined by the equation:-

$$\mathbf{F} = \frac{\mathrm{dU}}{\mathrm{d}\mathbf{s}}$$

That is, force **F** is equal to rate of change of energy U with vector distance **s**. This is a universal equation and applies whatever the energy mechanism might be. You can use either absolute or potential energy (in the former case the symbol ‘W’ is commonly used), but the result is identical.