Musical notes and the Riemann hypothesis

Let $\lambda(t) = 16e^{-\pi e^t} - 128e^{-2\pi e^t} + 704e^{-3\pi e^t}...$ be the modular lambda function, and $q = e^{-\pi e^t}$, so $\lambda/q = 16 - 128q + 704q^2....$ It is real valued if t is real, and let's choose a slightly different origin in the t axis, let's use $u = log(\frac{log(16)}{\pi})$. For each real number c in the interval $(0, 1/2)$, let $\sigma(s,t) = f(s, u+t) + f(s, u-t)$, the symmetrization, and $\alpha(s,t) = f(s, u+t) - f(s, u-t)$ the anti-symmetrization, for $f(s,t) = e^{(c-1)t} log(\lambda(t)/q(t))$.

Our choice of origin u makes α take small values.

For each positive real number ω, which we think of as an (angular) frequency, the quanity

$$q(c,\omega) = (\int_0^\infty cos(\omega t)\sigma(s,t)dt)^2 + (\int_0^\infty sin(\omega t)\alpha(s,t)dt)^2$$

depends on c.

1. Conjecture.

$$\frac{d}{dc} q(c,\omega) < 0$$

for all $\omega > 10$ and all $c \in (0, 1/2)$.

2. Theorem. The conjecture implies Riemann's hypothesis.

Proof. Let $L(s)$ be that L function which counts the number of ways of expressing a natural number as a sum of four squares of integers

$$L(s) = 8\zeta(s)\zeta(s-1)(1-4)^{1-s}$$

Let χ be the sign character, so

$$L(s,\chi) = \frac{4-2^s}{2+2^s} L(s) = -8\zeta(s)\zeta(s-1)4^{-s}(4-2^s)(2-2^s)$$

and

$$L(s,\chi)\Gamma(s)\pi^{-s} = \int_{-\infty}^{\infty} e^{(s-1)t} d\ log(\lambda/q).$$

Then

$$q(c,\omega) = |L(s,\chi)(\frac{\Gamma(s)}{\pi^s})(\frac{\pi}{1-s})(e^{-(c-1)u})|^2.$$

The last two factors are from integration by parts and change of origin.

The leftmost zeroes of $L(s, \chi)$ are the nontrivial zeroes of Riemann's function ζ, note there is also a pole at 0.

If the magnitude can always be decreased by increasing c, it can never be zero for any ω or any $c \in (0, 1/2)$. QED

3. Remark. Here is an intuitive way to analyze the condition. As c approaches $1/2$ the asymmetric term α tends to become smaller. This is one contribution. Then we are looking mainly at the symmetric term. We can think of sending the cosine wave of unit amplitude into an (infinite) electric circuit, depending on made of inductors, capacitors, resistors and linear amplifiers, whose parameters depend on c. The squared value of the integral is the square of the 'stable gain.' That is, we start at time $t = 0$ inputting voltage $cos(\omega t)$ and wait until the output approaches a sinusoidal wave, then the limiting amplitude squared will be the symmetric part of $q(c, \omega)$.