Electron spin 1/2 is "hidden" electromagnetic field angular momentum

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Abstract

This is to present and discuss an alternative method for precise analytical determination of electron spin angular momentum $\hbar/2$. The method is based on the Lorentz-force acting on a point-like charge moved through the entire magnetic dipole-field of the electron. The result $\hbar/2$ coincides with a previous result based on Lagrangian electrodynamics and confirms the "hidden" electromagnetic origin of spin angular momentum. Both methods reveal a key role of the "classical" electron radius.

1 Introduction

This analysis is to determine the electrons "hidden" electromagnetic angular momentum $\hbar/2$ (equivalent to its spin angular momentum), embodied in the electromagnetic vacuum-field outside of a sphere of classical electron radius r_e, by means of Lorentz-electrodynamics. The term "hidden" refers to the invisibility of electromagnetic field angular momentum, corresponding to the canonical angular momentum generated by a charge located in a magnetic field. [[1]], [[2]]

According to the Abraham-Lorentz conjecture the electron mass m_e is relativistically determined electrostatic Coulomb-field mass located outside of a sphere of classical electron radius r_e. Hence it is obvious that the electrons spin angular momentum also is embodied in its electromagnetic field-mass, as suggested in [[3]]. To prove this conjecture it has been attempted to determine electron spin angular momentum $\hbar/2$ by analysis of the Poynting-vector field surrounding an electron outside of a sphere of classical electron radius r_e, by volume integration of its assignable electromagnetic field angular momentum. However in this case the envisaged spin angular momentum $\hbar/2$ did not match up with the lower integration limit imposed by the classical electron radius r_e - as expected - even if vacuum-polarization was considered. [[3]] A specific advantage of the Lorentz-force approach presented here is that it is solely based on the quantums of charge e and magnetic moment μ_b (Bohr's magneton) thus avoiding consideration of the Coulomb- and Poynting-vector fields presumably affected by vacuum-polarization.

Notwithstanding the perfect result obtained with Lagrange-electrodynamics [[4]] there were raised some doubts with respect to the validity of the Lagrangian method in a non-uniform dipole field. Therefore an alternative Lorentz-force method will be used here to confirm the results achieved with Lagrangian electrodynamics.

2 The Lorentz-force method

This method is based on determination of the Lorentz-force, -torque and -angular momentum produced by a point-like charge during and after guided radial motion through the whole magnetic vacuum-field of an uncharged point-like dipole μ located at the origin (0,0,0), proceeding from "infinity" towards classical electron radius r_e where the motion is halted. A specific advantage of Lorentz-electrodynamics is that it doesn't require uniformity of the magnetic field. Instead, the Lorentz-force can be determined by the local induction field $\vec{B}(r)$ in the equatorial plane of μ (where the charge moves along), irrespective of its structure, in this case a dipole-field.
This analysis aims at a general determination of total (Lorentz-) angular momentum $\vec{L}_l$ transmitted by a point-like charge q to its guiding slide or similar device, after having traversed the whole dipole-field $\vec{B}(r)$.

Let p_e designate the equatorial (x-y) plane of a magnetic dipole $\vec{\mu}$ centered at its origin $(0,0,0)$, such that the dipole axis of $\vec{\mu}$ coincides with the z-axis.
Then the induction-field $\vec{B}(r,\Theta)$ of a point-like dipole μ is generally given by

$$\vec{B} \approx \frac{\mu_0\,\vec{\mu}}{4\pi\,r^3}\Big(2\,cos\,\Theta\,+sin\,\Theta\Big) \tag{1}$$

where $\mu_0 = 1/\epsilon_0 c^2$ is the vacuum permeability, $\vec{\mu}$ the dipole-moment, Θ the polar angle and r the radial distance of a point from the origin.

For any point in the equatorial plane p_e the following applies:
$\Theta = \pi/2 \rightarrow\; cos\,\Theta = 0,\; sin\,\Theta = 1,\; \vec{B}(\vec{r}) \parallel z$ and $\vec{B}(\vec{r}) \perp p_e$.

Thus in p_e the first term in brackets in (1) vanishes yielding

$$|\vec{B}(\vec{r})| = \frac{\mu_0\,\vec{\mu}}{4\,\pi\,r^3} \tag{2}$$

Now imagine a point-like probe charged with q initially being at rest on the equatorial plane p_e, at very large distance $r \approx \infty$ from the origin where $\vec{\mu}$ is located.
Then let this probe be accelerated and conducted in radial motion with velocity

$\vec{v}$ towards the origin $0,0,0$ until it reaches r_e where it is halted.
(It can be shown that such radial motion is no absolute requirement but substantially simplifies the calculus. What counts the the final position of q)

As soon as the charged probe is in radial motion with $\vec{v}$ through the induction-field $\vec{B}(r)$ it immediately is subjected to Lorentz-force

$$\vec{F_l} = q\,\vec{v} \times \vec{B}(r) \tag{3}$$

oriented perpendicular to its instantaneous velocity $\vec{v}$ and the local induction $\vec{B}(r)$.
While the moving probe is subjected to F_l it delivers an instantaneous Lorentz-torque

$$\vec{T_l} = \vec{r} \times F_l = q\,\vec{r} \times (\vec{v} \times \vec{B}(r)) \tag{4}$$

into its radial guiding slide.
Now imagine $\vec{T_l}$ would act during a time-differential dt. Then an angular momentum differential $d\vec{L_l}$ was imposed on the guiding slide

$$d\vec{L_l} = \vec{T_l}\,dt = \vec{r} \times \vec{F_l}\,dt = q\,\vec{r} \times (\,\vec{v} \times \vec{B}(r))\,dt = q\,\vec{r} \times\,(\vec{dr} \times \vec{B}(r)) \tag{5}$$

Note that in (5) the time-differential dt has been substituted by the radius-differential dr using $\vec{v} = d\vec{r}/dt$ or $dt = dr/v$. Thus the angular momentum increment $d\vec{L_l}$ in (5) becomes independent of $\vec{v}$, only depending on $d\vec{r}$ and $\vec{B}(r)$.

Substitution of $\vec{B}(r)$ in (5) with (2) yields:

$$dL_l = \frac{q\,\mu_0\,\mu}{4\,\pi}\,\frac{dr}{r^2} \tag{6}$$

Then total angular momentum L_l along the whole pathway from $r = \infty$ to $r = r_0$ generally is

$$L_l = \int_{r_0}^{\infty} dL_l = \frac{q\,\mu_0\,\mu}{4\,\pi} \int_{r_0}^{\infty} \frac{dr}{r^2} = \frac{q\,\mu_0\,\mu}{4\,\pi r_0} \tag{7}$$

Further substitution in (7) with the electron characteristics:

$q = e$, $\mu = \mu_B = e\hbar/4\pi m_e$, $r_0 = r_e = e^2/4\pi\epsilon_0 m_e c^2$ and $\mu_0 = 1/\epsilon_0 c^2$

yields the final result coinciding with the established spin 1/2 angular momentum

$$\vec{L_l} = \frac{e\,\mu_0\,\vec{\mu}_B}{4\,\pi\,r_e} = \vec{\hbar}/2 \tag{8}$$

or equivalently

$$\vec{L_l} = e\,\frac{\vec{\Phi_0}}{2\,\pi} = \vec{\hbar}/2 \tag{9}$$

where r_e is the classical electron radius and $\vec{\Phi_0} = \vec{h}/2e$ the magnetic flux quantum (fluxon) passing through the equatorial plane p_e, outside of a delimiting circle of radius r_e in p_e.

In this particular case total magnetic flux $\vec{\Phi}$ traversing the equatorial plane p_e outside of a circle of radius r_e can be determined by integration of the magnetic flux differential $d\Phi = 2\pi r B(r)$ from r_e outwards to ∞.:

$$\vec{\Phi} = \int_{r_e}^{\infty} d\vec{\phi} = 2\pi \int_{r_e}^{\infty} r\vec{B(r)}dr = \frac{\mu_0\, \mu_B}{2} \int_{r_e}^{\infty} r^{-2}dr = \vec{\Phi_0} \qquad (10)$$

precisely amounting one magnetic flux-quantum as shown in [[4]]. Note that (10) also proves that Bohr's magneton μ_B comprises one magnetic flux quantum Φ_0.

3 Reactive angular momentum is "hidden" in the electromagnetic field

Conservation of angular momentum and symmetry aspects require that force, torque and angular momentum must always emerge pairwise with mutually opposed directions. Thus "hidden" angular momentum $\vec{L}_h$ is reactive angular momentum corresponding to the Lorentz-angular momentum $\vec{L}_l$ caused by a moving charge e in simultaneous interaction with the guiding slide and a magnetostatic field $\vec{B}(r)$ (1) according to:

$$\vec{L_h} = -\vec{L_l} \qquad (11)$$

In other words, any Lorentz-angular momentum $\vec{L_l}$ introduced into the radial slide must be counterbalanced by a reactive angular momentum of opposed sense $-\vec{L_l}$, to be absorbed and stored by a system not mechanically interacting with the slide. Hence the only other physical entity interacting with the charge and being able to absorb and store the reactive angular momentum $-\vec{L_l}$ is the electromagnetic field which is being built up by the charge e moving towards the origin where the dipole μ_B is located. If e would come into contact with μ_B at r_e it was charged up with e and simulate an electron. [[1]]

4 Summary and Comment

A new conditional equation (8) for electron spin $\hbar/2$ has been derived by Lorentz-electrodynamics.
The analytical method is based on a thought experiment comprising an uncharged point-like magnetic dipole $\vec{\mu}$ (of equal magnitude as Bohr's magneton) at rest and a point-like charge e in a probe moving through the dipole-field, from a remote starting point towards the dipole $\vec{\mu}$ where it is halted at a distance of classical electron radius from the origin. All along on its way, the Lorentz-force acting on the moving charge e is used to determine the pair of active and

reactive ("hidden") electromagnetic angular momentum brought into a radial guiding slide and into the dipole-field, both of which amounting $\pm\hbar/2$.
In other words, if e is halted on arrival at r_e the induced electromagnetic angular momentum coincides with spin angular momentum $\hbar/2$ (8) being identical with that determined in [4] by Lagrangian electrodynamics.
Generally, the approach of a point-like charge to a dipole is not restricted to the equatorial plane thus can take any arbitrary pathway from a remote starting point through the dipole-field to the location of the dipole.
Remarkably, the moving charge e must not come into contact with the dipole $\vec{\mu}$ to charge it up, in order to generate "hidden" angular momentum. Instead, this begins to be generated as soon as charge e enters into the dipole field of $\vec{\mu}$ and is subjected to Lorentz-force.
Eq. (9) indicates how "hidden" electromagnetic angular momentum (canonical angular momentum) also is determined by (total) magnetic flux Φ_0 of a dipole field.
Instead of a mechanical guiding apparatus like a slide it would also be possible to inject a ballistic electron aiming at the origin where the dipole μ is located. The initial angular momentum of the injected electron would be equivalent to the "hidden" angular momentum after impact on the dipole μ where the electrons angular momentum vanishes. (with the origin as reference point)
In support of above result it deserves a final note that spin angular momentum $\hbar$ of a photon also is carried by its electromagnetic field as "hidden" angular momentum. [[5]]

References

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