The Simplest and Accurate Theory of Proton and Neutron Based on Only Six Parameters that are Experimental Values

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Abstract: Here, we present the simplest version of the atom-like structure of baryons. We use six parameters only that are experimental values. They are the three fundamental physical constants, the mass of electron and the two masses of pions. There do not appear free parameters. We calculated masses of nucleons and their magnetic moments. Obtained results are in very good consistency with experimental data. For example, calculated magnetic moment of proton expressed in the nuclear magneton is +2.7928471 (the experimental value is +2.792847351(9) [2]) so the 7 first significant digits are the same. The same is for the mass of proton - we obtained 938.272065 MeV (the experimental result is 938.272081(6) MeV [2]). Here we apply the experimental central values for five from the six parameters because accuracy of the experimental mass of neutral pion is very low. To obtain the perfect results, we used the theoretical mass of neutral pion (134.97678 MeV) which overlaps with the interval defined by experiments: 134.9770(5) MeV [2]. Due to future more precise data for mass of neutral pion, we will able to verify presented here theory of nucleons. Emphasize that our results are much, much better than values obtained within the Standard Model despite the fact that our model contains at least 5 times less parameters.

1. Introduction

Within the Scale-Symmetric Theory (SST), among many other things concerning the particle physics and cosmology, we described in details internal structure of nucleons using the initial conditions which define the inflation field [1]. The original set of the 7 initial parameters which define the inflation field and spacetime leads to the secondary set containing the four fundamental physical constants (i.e. the gravitational constant $G$, unitary spin $\hbar$, speed of light in “vacuum” $c$, and the elementary charge $e$) and three masses (i.e. the mass of electron $m_{\text{electron}}$, of neutral pion $m_{\text{pion(0)}}$, and charged pion $m_{\text{pion}}$). All obtained values from the original set are very, very close to experimental ones.

Here we neglect the transition from the original set to the secondary one. What’s more, we significantly simplified the mathematical description. Generally, presented here and in paper [1] calculations are similar. But due to the new initial formula which contains the all used here 6 experimental values/parameters we significantly simplified the theory of nucleons. The six parameters lead to very precise masses and magnetic moments of nucleons. Here we
described also the small corrections which follow from the electromagnetic and weak interactions.

Here, the parameters are the three fundamental physical constants, mass of electron and the two masses of pions (we neglect the gravitational interactions). All of them are the experimental values [2]. Moreover, five of them are the experimental central values [2] whereas mass of the neutral pion lies within the interval defined by experiments.

Below are listed the parameters applied in this paper.

*Unitary spin: $\hbar = 1.054571726 \cdot 10^{-34}$ Js
*Speed of light in spacetimes: $c = 2.99792458 \cdot 10^8$ m/s
*Electric charge of electron: $e = 1.602176565 \cdot 10^{-19}$ C
*Mass of electron: $m_{\text{electron}} = 0.5109989461$ MeV
*Mass of charged pion: $m_{\text{pion}(\pm)} = 139.57061$ MeV
*Mass of neutral pion: $m_{\text{pion}(0)} = 134.97678$ MeV (experiments give 134.9770(5) MeV [2] so the interval is $<134.9765, 134.9775>$).

Within the atom-like structure of baryons described within SST, we calculated already as well the other properties of nucleons such as the running coupling constant for the nuclear strong interactions [1], the bottle and beam lifetimes of neutron [3], the electron radius and muon-radius of proton [4], we described the V – A theory of nucleons [5], and we calculated the masses of quarks – they are the loops or condensates [1], [6]. Here we present the very, very precise calculations of masses of nucleons and of their magnetic moments. The results obtained are amazing with their accuracy and the presented model is shocking that it is semi-classical and non-perturbative. It suggests that we should verify our views on the internal structure of nucleons and other baryons.

2. The model

The successive phase transitions of the inflation field, described within the SST, lead to the atom-like structure of nucleons [1]. There is the spin-1/2 core of nucleons with a mass of $H^+$. It consists of the spin-1/2 electric-charge/torus $X^+$ and the spin-0 central condensate $Y$ both composed of the Einstein-spacetime (Es) components – the Es components are the spin-1 neutrino-antineutrino pairs. Such picture follows from the dynamics of the inflation field [1].

We know that following equation defines a torus:

$$(x^2 + y^2 + z^2 - a^2 - b^2)^2 = 4 b^2 (a^2 - z^2) .$$

![Image](https://via.placeholder.com/150)
Physical tori are most stable when \( b = 2a \) (see Fig. titled “Stable tori”) [1]. In nucleons there is \( a + b = A \) [1]. Such torus is most stable because along each diameter of the equator there appears regular spin wave with length \( 4A/3 \) – the \( 4/3 \) factor solves the problem of mass of electron in classical theory.

The spin-1 large loops with mass \( m_{LL} \) and with a radius of \( b = 2A/3 \) (where \( A \) is the equatorial radius of the electric-charge/torus) are produced inside the electric-charge/torus – the neutral pions are built of two such loops with antiparallel spins. In the \( d = 1 \) state (it is the \( S \) state i.e. the azimuthal quantum number is \( l = 0 \) – see Paragraph 3) there is a relativistic pion – radius of the orbit is \( (A + B) \).

Why the electric-charge is a torus? It follows from the fact that the components of the residual-inflation-field (of the Higgs field) have infinitesimal spin [1] – it leads to conclusion that spinning objects composed of the Higgs-field components must have internal helicity – torus is the simplest surface which can have internal helicity (a sphere cannot have internal helicity).

The large loops are responsible for the internal, nuclear strong interactions. They cause that nucleons are the modified black holes in respect of the nuclear strong interactions with the Schwarzschild radius equal to \( 2A \) which is bigger than \( A + B \). It leads to conclusion that in nucleons, the system of the core plus relativistic pion cannot decay due to the nuclear strong interactions. It is not true that inside fields having internal helicity (the nuclear strong fields and the large loops have internal helicity [1]) in defined quantum state can be infinite number of pions. It follows from the fact that in the nuclear strong field, in the ground state for pions (it is the \( A + B \) state) the binary systems of the large loops (i.e. the neutral pions) behave as the electron-electron pair in the ground state of an atom i.e. in the \( A + B \) state can be only one pion.

3. Masses of the proton and neutron

It was the most difficult problem to solve to find the relation which ties the mass of the large loops, \( m_{LL} \), with the three masses-parameters. I finally noticed that the relative change in the electromagnetic energy for neutral pion (i.e. the ratio of the electromagnetic binding energy of the two large loops with a radius of \( 2A/3 \) to mass of the neutral pion) is equal to the ratio of the electromagnetic energy of the electron \( \alpha_{em}m_{electron} \) (where \( \alpha_{em} = e^2/c/(10^7\hbar) = 1/137.03599905 \) is the fine structure constant) to its energy in the charged pion

\[
(2 m_{LL} - m_{pion(o)}) / m_{pion(o)} = \alpha_{em}m_{electron} / (m_{pion(+)} - 2 m_{LL}).
\]  

Notice that in formula (2) there are the all 6 parameters plus mass of the large loop – it suggests that we cannot formulate theory of particles applying less than 6 parameters. In theories of particles, we neglect the gravitational interactions so the Theory of Everything must start from 7 parameters as it is in SST (we must add the gravitational constant \( G \)). Emphasize that in SST, the all basic physical constants are derived from the more fundamental initial conditions.

From formula (2) we obtain two values for \( m_{LL} \) i.e. \( m_{LL} = 67.544545084 \text{ MeV} \) and \( m_{LL}^* = 69.729149916 \text{ MeV} \). The transition from the original set to the secondary one shows that only the first solution is valid.

All hadrons and charged leptons are built of the Einstein-spacetime (Es) components – they are the neutrino-antineutrino pairs. The Es components can be entangled or confined, their spin can rotate or not, and their speed is equal to the speed of light in “vacuum”.
The large loops consist of the entangled Es components. We can use the formula for virtual loops \( ET = \hbar \) (a virtual loop with energy \( E \) multiplied by the period of spinning \( T \) has unitary spin) to calculate the radius \( A \)

\[
ET = [(m_{LL} F) c^2] \left[ 2 \pi \left( \frac{2 A}{3} \right) / c \right] = \hbar . \tag{3}
\]

From this formula we obtain \( A = 0.69744113895 \) fm, where \( F \) is

\[
F = e \cdot 10^6 / c^2 = 1.7826618449 \cdot 10^{-30} \text{ kg/MeV}. \tag{4}
\]

The large loops are responsible for the nuclear strong interactions inside nucleons whereas their binary systems (i.e. the neutral or charged pions) are responsible for the strong interactions between nucleons. When a nucleon is accelerated then period of spinning \( T \) increases (resultant velocity, i.e. the sum of the linear and spinning velocity, must be equal to \( c \)) so from formula (3) results that energy of the loop \( E \) decreases so the coupling constant of the nuclear strong interaction decreases as well i.e. it is the running coupling [1]. The large loops have internal helicity – in baryons they have the left-handed internal helicity whereas in anti-baryons they are right-handed.

From properties of the most stable tori follows that the mean radius of the spin-1/2 electric-charge/torus is 2\( A \)/3 and the mean spin speed is 2\( c \)/3. This leads to the mass of the electric charge \( X^+ \) (it is a classical object)

\[
(X^+ F) \left( \frac{2 c}{3} \right) \left( \frac{2 A}{3} \right) = \hbar / 2 . \tag{5}
\]

From this formula we obtain \( X^+ = 318.29616994 \) MeV. From formulae (3) and (5) we obtain \( X^+ = 3 \pi m_{LL} / 2 \) and \( A = 9 \hbar / (8 c X^+ F) \).

The unitary spin in formula (3) and the half-integral spin in formula (5) follow from the dynamics of the inflation field [1].

The \( \alpha \)-order correction for the radiation energy, \( m_{em} c^2 \), created in the interactions of the virtual or real electron-positron pairs (it is a virtual or real photon emitted by an electrically charged particle) is

\[
m_{em} c^2 = k e^2 / \lambda_C , \tag{6}
\]

where \( k = c^2 / 10^7 \) and \( \lambda_C \) is the Compton wavelength of a charged fermion.

The Compton wavelength of electrically charged fermion with a bare mass \( m_{bare} \) is

\[
\lambda_C = 2 \pi \hbar / (c m_{bare}) , \tag{7}
\]

Then from (6) and (7) we obtain

\[
m_{em} = C m_{bare} , \tag{8}
\]

where \( C = e^2 c / (2 \pi 10^7 \hbar) = 0.0011614097331 \) and \( m_{bare} = m / (1 + C) \).
Mass of the bound neutral pion, $m^{*}_{\text{pion(o)}}$, is equal to mass of two large loops minus binding energy which is the radiation energy of charged muon which can appear occasionally in the neutral pion decay

$$m^{*}_{\text{pion(o)}} = 2 \, m_{ LL} - m_{\text{muon}} \frac{C}{(1 + C)} = 134.96632492 \, \text{MeV}. \quad (9)$$

The same result we obtain from the condition that the muon and electron which can appear in the neutral-pion decay are in distance equal to the $2\pi$ multiplied by the reduced Compton length of the bare muon.

On surface of the core of baryons, so on its equator as well, there appear virtual bosons that to equalize their number density in spacetime are emitted. Assume that the radius of the equator of the core of baryons is $A$, and that the range of a virtual boson is $B$. At distance $A + B$ there is symmetrical decay of the virtual boson to two identical parts. One part is moving towards the equator whereas the second one is moving in the opposite direction. It means that in the place of decay there is produced a hole in the field surrounding the core. When the first part reaches the equator then the second one stops and decays to two identical parts – it takes place in distance $A + 2B$. Next decay takes place in distance $A + 4B$. A statistical distribution of the holes in the field in the plane of the equator (of the circular tunnels in the field) is defined by following formula

$$R_d = A + dB, \quad (10)$$

where $R_d$ denotes the radii of the circular tunnels, the $A$ denotes the external radius of the torus/core, $d = 0, 1, 2, 4$ (there is the upper limit for $d$, i.e. $d = 4$, which follows from the size of the nuclear strong interactions – see the further explanations in this Paragraph), the $B$ denotes the distance between the second tunnel ($d = 1$) and the first tunnel ($d = 0$). The first tunnel is in contact with the equator of the torus. Formula (10) is the Titius-Bode law for the nuclear strong interactions.

The nuclear strong field of baryons consists of the open virtual large loops that are the gluons. Moreover, the core of baryons as a whole is the modified black hole in respect of the nuclear strong interactions i.e. gluons on the equator have spin speed equal to the $c$. We will show that only the $d = 0$ and $d = 1$ states are placed under the Schwarzschild surface for the nuclear strong interactions.

In nucleons, in the $d = 1$ state, that is placed under the Schwarzschild surface, is relativistic pion that interacts with the core due to the nuclear strong interactions i.e. via the large loops – it is the reason that this pion cannot appear in decays of nucleons.

The relativistic pion in the $d = 1$ state exchanges a lepton pair (positron plus electron-neutrino in proton or electron plus electron-antineutrino in neutron) with the core. It causes that there are two different mass states in each nucleon and that the mean square charge for nucleons is fractional.

In the Standard Model is assumed that many pions can simultaneously occupy the same state. The SST shows that it is untrue in fields having internal helicity i.e. it does not concern the nuclear strong fields. A neutral pion consists of two large loops (in the charged pions, among other particles, there are two large loops also – they can be virtual) both having left-handed internal helicity. Due to the internal helicity of the nuclear strong fields, the pions behave as the electron-electron pairs in the ground state in atoms i.e. the unitary spins of the two large loops must be antiparallel. This means that the selection rules for the pions and loops created in baryons appears – they lead to the hyperons which contain one, two or three
relativistic pions in the $d = 2$ state but due to their interactions with gluons, their mass states are different [1].

Why all the $d$ states of the relativistic pions in baryons are the $S$ states i.e. why all the azimuthal/secondary quantum numbers of the relativistic pions are $l = 0$? It results from the fact that a pion in defined $d$ state behaves as follows. Centre of mass of a pion disappears in one point of defined circular orbit/tunnel and appears in another one, and so on, but senses of the spin velocities of the pion change randomly – it causes that resultant angular momentum on the circular orbit is equal to zero.

There are two different types of motion of loops or binary systems of them in the $d$ states.

*Centre of mass of the relativistic pion is on $d$ orbit and there are not loops overlapping with the orbits. Such configuration leads to the nucleons and hyperons – the hyperons decay “slowly” due to the orbits/tunnels. Masses of such pions will be denoted by $W_d$.

*Besides the $W_d$ pions, there can be one or more loops overlapping with the orbits/tunnels. The states of the additional loops are the short-lived resonance states. Masses of such additional loops will be denoted by $S_d$.

Hyperons arise very quickly because of the nuclear strong interactions.

The distance $B$ we can calculate on the condition that the relativistic charged pion in the $d = 1$ state, which is responsible for the properties of nucleons, should have unitary angular momentum because this state is the ground state for $W_d$ pions:

$$m_{W(\pm),d=1} (A + B) \nu_{d=1} = \hbar,$$

where $\nu_{d=1}$ denotes the spin speed of the $W_d$ pion on the $d = 1$ orbit.

We can calculate the relativistic mass of the $W_d$ pions using Einstein’s formula

$$m_{W(\pm),d} = m_{\text{pion}(\pm)}^* / (1 - v_d^2 / c^2)^{1/2}, \quad \text{(12)}$$

where $m_{\text{pion}(\pm)}^* = m_{\text{pion}(\pm)}$.

We know that the square of the speed is inversely proportional to the radius $R_d$ (for $d = 1$ is $v_{d=1}^2 = c^2 A / (A + B)$) so from (12) we have:

$$m_{W(\pm),d} = m_{\text{pion}(\pm)}^* (1 + A / (d B))^{1/2}. \quad \text{(13)}$$

Since we know the $A$ then from formulae (11)–(13) we can obtain the $B = 0.50183535473$ fm. We see that the $d = 1$ state is lying under the Schwarzschild surface for the nuclear strong interactions. Circumference of the large loop is $R_C = 2.9214344$ fm so the last orbit for the strong interactions has radius $A + 4B = 2.7048$ fm i.e. the $d = 4$ is the last state ($A + 4B < R_C$). We can see that the second solution $A + B' = 0.6974411 + 0.9692903 = 1.6667314$ fm lies outside the Schwarzschild surface for the nuclear strong interactions. The orbit with a radius of $A + B'$ is close to the orbit with radius $A + 2B = 1.7011$ fm and relativistic mass on the second one is lower so it is the dominating state (the mass distance between $A + B'$ and $A + B$ is about $1.3$ MeV).

Creation of a resonance is possible when loops overlap with tunnels. Such bosons I call $S_d$ bosons because they are associated with the nuclear strong interactions. Their masses are
denoted by $m_{S(d)+o,d}$. The spin speeds of $S_d$ bosons (they are equal to the $c$) differ from the speeds calculated on the basis of the Titius-Bode law for strong interactions.

The mass of the core of resting baryons is denoted by $m_{H(+)-o}$. The maximum mass of a virtual $S_d$ boson cannot be greater than the mass of the core so we assume that the mass of the $S_d$ boson, created in the $d = 0$ tunnel, is equal to the mass of the core. As we know, the ranges of virtual particles are inversely proportional to their mass. As a result, from (11) we obtain:

$$m_{H(+)-o}A = m_{S(d)+o,d}(A + dB).$$

(14)

There is some probability that virtual $S_d$ boson arising in the $d = 0$ tunnel decays to two parts. One part covers the distance $A$ whereas the remainder covers the distance $4B$. The large loops arise as binary systems (i.e. as the neutral pions). The part covering the distance $A$ consists of four virtual neutral pions (i.e. of the eight large loops). Then the sum of the mass of the four neutral pions (~539.87 MeV) and the mass of the remainder (~187.57 MeV) is equal to the mass of the core of baryons and is equal to the mass of $S_d$ boson in the $d = 0$ state (~727.44 MeV – see formula (16).

Denote the mass of the remainder (it is the $S_d$ boson) by $m_{S(d)+d=4}$, then:

$$m_{S(d)+d=4} = m_{H(+)-o} - 4m_{pion(o)}.$$

(15)

Using formulae (14) and (15) we have

$$m_{H(+)} = m_{pion(o)}(A/B + 4) = 727.43890621 \text{ MeV}.$$  

(16)

The nucleons and pions are respectively the lightest baryons and mesons interacting strongly, so there should be some analogy between the carrier of the electric charge interacting with the core of baryons (it is the distance of masses between the charged and neutral cores) and the carrier of an electric charge interacting with the charged pion (this is the electron)

$$\left(\frac{m_{H(+)} - m_{H(o)}}{m_{H(+)}}\right) / m_{pion(+)} = m_{electron} / m_{pion(+)}.$$ 

(17)

The results obtained from formulae (13)–(17), with the value $A/B = 1.3897807964$, are collected in Table 1 (the masses are provided in MeV).

<table>
<thead>
<tr>
<th>$d$</th>
<th>$m_{H(+)}$</th>
<th>$m_{H(o)}$</th>
<th>$m_{W(+)}$</th>
<th>$m_{W(o)}$</th>
</tr>
</thead>
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<td>727.43890621</td>
<td>724.77559110</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>423.043</td>
<td>421.494</td>
<td>215.76103168</td>
<td>208.64330612</td>
</tr>
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<td>298.244</td>
<td>297.152</td>
<td>181.704</td>
<td>175.710</td>
</tr>
<tr>
<td>4</td>
<td>187.574</td>
<td>186.889</td>
<td>162.013</td>
<td>156.668</td>
</tr>
</tbody>
</table>

There is a probability $y$ that the proton is composed of $H^+$ and $W_{(o),d=1}$ and a probability $1-y$ that is composed of $H^o$ and $W_{(+),d=1}$. From the Heisenberg uncertainty principle follows that the probabilities $y$ and $1-y$, which are associated with the lifetimes of protons in the
above-mentioned states, are inversely proportional to the relativistic masses of the \(W_d\) pions so from this condition and (13) we have

\[
y = \frac{m_{\text{pion}(+)} - m_{\text{pion}(o)}}{m_{\text{pion}(+)} + m_{\text{pion}(o)}} = 0.50838554762 , \\
1 - y = \frac{m_{\text{pion}(o)}}{m_{\text{pion}(+)} + m_{\text{pion}(o)}} = 0.49161445238 .
\]  

There is a probability \(x\) that the neutron is composed of \(H^+\) and \(W_{(-),d=1}\) and a probability \(1-x\) that is composed of \(H^\ast\), resting neutral pion and \(Z^\ast\). The mass of the last particle is \(m_{Z(o)} = m_{W(o),d=1} - m_{\text{pion}(o)}\) (the pion \(W_{(o),d=1}\) decays because in this state both particles, i.e. the torus and the \(W_{(o),d=1}\) pion, are electrically neutral). Since the \(W_{(o),d=1}\) pion only occurs in the \(d = 1\) state and because the mass of resting neutral pion is greater than the mass of \(Z^\ast\) (so the bound neutral pion lives shorter) then

\[
x = \frac{m_{\text{pion}(o)}}{m_{W(-),d=1}} = 0.62553614926 , \\
1 - x = 0.37446385074 .
\]  

The mass of the baryons is equal to the sum of the mass of the components because the binding energy associated with the nuclear strong interactions cannot abandon the nuclear strong field.

The mass of the proton is

\[
m_{\text{proton}} = (m_{H(+) + W(0),d=1}) y + (m_{H(0) + W(+),d=1}) (1-y) = \\
= 938.27206489 \text{ MeV}.
\]  

The mass of the neutron should be

\[
m_{\text{neutron}}^* = (m_{H(+) + W(-),d=1}) x + (m_{H(0) + m_{\text{pion}(0)} + m_{Z(0)})} (1-x) .
\]  

But there appears a correction concerning the mass distance between the neutron and proton. To calculate it, because neutron decays due to the nuclear weak interactions, we must know value of the coupling constant for the nuclear weak interactions \(\alpha_{w(proton)}\). Such constant is defined by properties of the central condensate \(Y\).

According to SST, gluons are the rotational energies of the Einstein-spacetime (Es) components, which are the neutrino-antineutrino pairs, or rotational energies of their groups (then the Es components of a group are entangled). It leads to conclusion that simplest neutral pion consists of four neutrinos or four groups of them). Spin of pions is zero whereas of muon is 1/2. On the other hand, muon is a result of decay of charged pion which can consist of neutral pion, electron and electron-antineutrino. It suggests that mass of bound muon should be equal to mass distance between charged pion and one fourth of bound neutral pion. Applying formula (9) we obtain

\[
m_{\text{muon}}^* = m_{\text{pion}(+)} - m_{\text{pion}(o)} / 4 = 105.826415 \text{ MeV} .
\]
Mass of the central condensate $Y$ is the sum of masses of the $X^+$ and bound muon. To such a conclusion leads the fact that the origin of the muon must be associated with internal structure of nucleons. We have

$$Y = X^+ + m^*_{\text{muon}} = 424.12258494 \text{ MeV}.$$  

(25)

The weak binding energy of $Y$ and $X^+$ is

$$\Delta E_{\text{binding}} = (X^+ + Y) - H^+ = 14.97984 \text{ MeV}.$$  

(26)

Notice that the ratio of the mass distance between the charged pion and bound neutral pion to the mass of electron is close to the ratio of masses of the charged core of baryons $H^+$ and $Z^+$, where $m_{Z^+} = m_{W^+,d=1} - m^*_{\text{pion(o)}}$ (the ratios are about 9.0). This should have some deeper meaning. Assume that the increase in the mass of electrons and $Z^+$ boson is realized in the $d = 0$ state because this tunnel has some width resulting from the diameter of the virtual condensates $Y$ created near the equator of the torus of the core of baryons. The width of the $d = 0$ tunnel means that the mentioned particles in this tunnel do not move with a speed equal to the $c$. The relativistic masses of the $W_d$ pions can be calculated using Einstein’s formula.

Definition of the coupling constant for the strong-weak interactions $\alpha_{\text{sw}}$ (the core of baryons is the modified black hole with respect to the strong interactions i.e. on the equator of torus the spin speed is equal to the $c$) leads to following formula

$$\alpha_{\text{sw}} = G_{\text{sw}} M m / (c s_d) = m v_d^2 r_d / (c s_d) = v_d / c,$$  

(27)

where $G_i$ denotes the strong-weak constant, $s_d = m v_d r_d$ is the angular momentum of particle in the $d$ state whereas $v_d$ is the spin speed in the $d$ tunnel. For example, for the large loop responsible for the strong interactions is $s_d = \hbar$ and $v_d = c$ – it leads to $\alpha_S = 1$.

From formulae (12) and (27) we obtain

$$\alpha_{\text{sw}(Z^+,d=0)} = v_{d=0} / c = (1 - (m_{Z^+} / m_{H^+})^2)^{1/2} = 0.99381308.$$  

(28)

The $r_{p(\text{proton})}$ denotes the radius of the condensate $Y$ and the range of the weak interactions of the condensate. Because $v^2 = G_{\text{sw}} m_{H^+} / r$ and because the particle $Z_{(+,-),d=0}$ is in distance $r = r_{p(\text{proton})} + A$ from the centre of torus then from formula (28) we obtain

$$A / (r_{p(\text{proton})} + A) = (v_{d=0} / c)^2 = 1 - (m_{Z^+} / m_{H^+})^2.$$  

(29)

From it we obtain $r_{p(\text{proton})} = 0.87110615994 \cdot 10^{-17} \text{ m}$. Since on the equator of the condensate, the spin speed of the binary systems of neutrinos must be equal to the $c$ then we can calculate the constant for the weak interactions

$$G_w = c^2 r_{p(\text{proton})} / Y = 1.0355048130 \cdot 10^{27} \text{ m}^3 \text{s}^{-2} \text{kg}^{-1}.$$  

(30)

The coupling constant for weak interactions of protons, $\alpha_{w(\text{proton})}$, can be calculated using the formula-definition
\[ \alpha_{w(\text{proton})} = G_w \frac{Y^2}{(c \ h)} = 0.018723025693 . \]  

(31)

Here the \( Y \) is both the mass of the source and the carrier of the nuclear weak interactions. Notice that the ratio

\[ \frac{(\alpha_{w(\text{proton})} + \alpha_{em})}{\alpha_{w(\text{proton})}} = 1.389752847 \approx A / B \]  

(32)

is very close to the ratio that follows from formulae (3) and (11): 1.3897807964 (five significant digits are the same). It suggests that the lepton pair inside the core of nucleon interacts only weakly with the core whereas outside it interacts weakly and electromagnetically – the stronger external interactions cause that the range \( B \) is shorter than the range \( A \).

Neutron consists of positively charge core and relativistic neutral pion which exchange the lepton pair composed of electron and electron-antineutrino.

Notice that creation of a condensate due to the weak interactions causes that there is emitted electromagnetic energy of its weak mass. It leads to conclusion that outside the core of neutron we have following correction for the mass distance \( m_{\text{neutron}}^* - m_{\text{proton}} \)

\[ M_1 = (m_{\text{neutron}}^* - m_{\text{proton}}) (\alpha_{w(\text{proton})} + \alpha_{em} - \alpha_{em} \alpha_{w(\text{proton})}) , \]  

(33)

whereas inside it is

\[ M_2 = (m_{\text{neutron}}^* - m_{\text{proton}}) (\alpha_{w(\text{proton})} - \alpha_{em} \alpha_{w(\text{proton})}) , \]  

(34)

The arithmetic-mean correction is

\[ M_{\text{Mean}} = (m_{\text{neutron}}^* - m_{\text{proton}}) (\alpha_{w(\text{proton})} + \alpha_{em} / 2 - \alpha_{em} \alpha_{w(\text{proton})}) , \]  

(35)

The mass of free neutron is

\[ m_{\text{neutron}} = m_{\text{neutron}}^* + M_{\text{Mean}} = 939.56542415 \text{ MeV} . \]  

(36)

The mean square charge for the proton is

\[ \langle Q_{\text{proton}}^2 \rangle = e^2 \left[ y^2 + (1-y)^2 \right] / 2 . \]  

(37)

The mean square charge for the neutron is

\[ \langle Q_{\text{neutron}}^2 \rangle = e^2 \left[ x^2 + (-x)^2 \right] / (2 x + 3 (1-x)) \]  

(38)

where \((2 x + 3 (1-x))\) defines the mean number of particles in the neutron.

The mean square charge for the nucleon is

\[ \langle Q^2 \rangle = \left[ \langle Q_{\text{proton}}^2 \rangle + \langle Q_{\text{neutron}}^2 \rangle \right] / 2 \approx 0.29e^2 . \]  

(39)

The quark model gives \(-0.28e^2\) so both models lead to similar mean square charges.
4. The observed magnetic moments of nucleons forced by magnetic field

Both the proton and neutron produce two loops of electric current, one loop with radius of $2A/3$ and the second one with radius $(A + B)$. In both nucleons, the first loop is produced by the positive charge $H^+$ (in proton, probability of such state is $y$ while in neutron is $x$) whereas the second loop is produced by the relativistic charged pion $W_{(+),d=1}$ – in the proton, it is positively charged and probability is $(1 - y)$ while in neutron, it is charged negatively and probability is $x$. It leads to following formulae for magnetic moments of nucleons expressed in the nuclear magneton

$$
\mu_{\text{proton}} = \mu_N = m_{\text{proton}} y / m_{H^+} + m_{\text{proton}} (1 - y) / m_{W_{(+),d=1}} = +2.7936, \quad (40)
$$

$$
\mu_{\text{neutron}} = m_{\text{proton}} x / m_{H^+} - m_{\text{proton}} x / m_{W_{(-),d=1}} = -1.9134. \quad (41)
$$

On the other hand, we measure the magnetic moments of nucleons by placing them in the external magnetic field but there is also the internal magnetic field. It increases the centripetal force acting on the charges on the loops. The centripetal force is directly proportional to the product of the central mass $Y$ and the orbiting mass around it, i.e. of the $X^+$ or $W_{(+),d=1}$ and inversely proportional to the squared radius of the loops of electric current. According to SST, spin and charge of the core must be conserved so size of the loop with radius $2A/3$ and the mass $m_{H^+}$ are invariant. The same concerns the loop with radius $(A + B)$ because there is a tunnel in the Einstein spacetime produced by the nuclear strong interactions. It leads to conclusion that external/internal magnetic field forces an increase in mass $W_{(+),d=1}$ only. The radius $(A + B)$ is defined by the internal nuclear strong interactions for which the coupling constant is defined as the ratio of the spin speed of the large loop (for a non-relativistic nucleons, it is equal to $c$ – it is because the large loop consists of the entangled Einstein-spacetime components moving with the resultant speed equal to $c$) to the $c$. It means that the strong coupling constant for the interior of the non-relativistic nucleons is unitary $\alpha_S = 1$ [1].

The nuclear strong mass of $M$ is $M_S = \alpha_S M = M$. Besides the strong interactions, the charged $W_{(+),d=1}$ can interact with the core of nucleons weakly or/and electromagnetically – such interactions, when are forced by external/internal magnetic field, increase mass of the charged $W_{(+),d=1}$. In proton, when the relativistic pion is charged then both the torus and the condensate $Y$ are electrically neutral so $W_{(+),d=1}$ can interact weakly with $X^o$ and next weakly with $Y$, or vice versa. Such interaction increases the strong mass of $m_{W_{(+),d=1}}$ by $\alpha_{w(\text{proton})} \alpha_{w(\text{proton})} m_{W_{(+),d=1}}$ i.e. the resultant mass is

$$
m_{W_{(+),d=1},\text{proton}} = m_{W_{(+),d=1}} \alpha_S + \alpha_{w(\text{proton})}^2 \bigl(1 + \alpha_{w(\text{proton})}^2 \bigr). \quad (42)
$$

In neutron, when the relativistic pion is charged then the torus is charged whereas the condensate $Y$ is electrically neutral so $W_{(+),d=1}$ can interact electromagnetically with $X^+$ and next weakly with $Y$, or vice versa. Such interaction increases the nuclear strong mass of the $m_{W_{(-),d=1}}$ by $\alpha_{\text{em}} \alpha_{w(\text{proton})} m_{W_{(-),d=1}}$ i.e. the resultant mass is

$$
m_{W_{(-),d=1},\text{neutron}} = m_{W_{(-),d=1}} \alpha_S + \alpha_{\text{em}} \alpha_{w(\text{proton})} \bigl(1 + \alpha_{\text{em}} \alpha_{w(\text{proton})} \bigr). \quad (43)
$$

Applying formulae (42) and (43) we can rewrite formulae (40) and (41) as follows
\[
\mu_{\text{proton}} / \mu_N = m_{\text{proton}} y / m_{H(+)} + m_{\text{proton}} (1-y) / m_{W(+)},d=1,\text{proton}^* = \\
= +2.7928471322 \approx +2.7928471 .
\]

Experimental value is +2.792847351(9) [2] so 7 significant digits are the same – there is no other theory which leads to such incredible accuracy!

For neutron we obtain
\[
\mu_{\text{neutron}} / \mu_N = m_{\text{proton}} x / m_{H(+)} - m_{\text{proton}} x / m_{W(-)},d=1,\text{neutron}^* = \\
= -1.9130395424 \approx -1.913040 .
\]

Experimental value is \(-1.9130427(5)\) [2] so 6 significant digits are the same – there is no other theory which leads to such incredible accuracy!

Notice that the increases in mass are in the cost of the virtual field entangled with nucleons so the increases in mass decrease the local mass density of the virtual field – this leads to conclusion that total rest mass of nucleons is invariant.

5. The weak interactions of muon with electron

We calculated the mass \(Y\) in proton and its radius \(r_{p(\text{proton})}\). On the other hand, the mass of the condensate in centre of electron is a half of the bare mass of electron – when we take into account only the \(\alpha\)-order correction for the radiation energy then on the assumption that the condensates have the same density, for the radius of the condensate in electron we obtain
\[
r_{p(\text{electron})} = r_{p(\text{proton})} \left\{ m_{\text{electron}} /[2 (1 + C) Y] \right\}^{1/3} = 0.7354211502 \cdot 10^{-18} \text{ m} ,
\]
where \(m_{p(\text{electron})} \approx m_{\text{electron}} /[2 (1 + C)]\).

The coupling constant for the weak interactions of muon with electron is defined as follows
\[
\alpha_{w(\text{muon-electron})} = m_{p(\text{electron})} r_{p(\text{electron})} c / h = 0.95112056491 \cdot 10^{-18} .
\]

6. The return mass of the free neutral pion (the mass of neutral pion is in this paper the initial parameter)

The bound neutral pion can interact with the core of baryons weakly or electromagnetically so the arithmetic mean is \((\alpha_{w(\text{proton})} + \alpha_{\text{em}})/2\). When the bound neutral pion becomes free then there is the transition to the weak muon-electron interactions so there is following correction
\[
2\alpha_{w(\text{muon-electron})} / (\alpha_{w(\text{proton})} + \alpha_{\text{em}}) \text{ to the strong interactions of the bound neutral pion (} \alpha_S = 1 \text{) – this leads to the mass of free neutral pion (it is the return mass)}
\]
\[
m_{\text{pion(o),return}} = m_{\text{pion(o),bound}} \left[ 1 + 2\alpha_{w(\text{muon-electron})} / (\alpha_{w(\text{proton})} + \alpha_{\text{em}}) \right] = \\
= 134.97619174 \text{ MeV} .
\]

7. The mass of free muon

Muon is created near the \(Y\) so there is the nuclear weak interaction of the mass distance between the bound and free muon – it leads to \((m_{\text{muon,bound}} - m_{\text{muon}})\alpha_{w(\text{proton})}\). On the other
hand, the neutral pion is outside the core of baryons so its binding energy interacts weakly and electromagnetically with the core: $(2 \, m_{LL} - m_{\text{pion(o),bound}})(\alpha_{w(\text{proton})} + \alpha_{\text{em}})$. But the electrically neutral binding energy can interact weakly with the torus/electric-charge and next with the condensate $Y$, or vice versa – such energy is emitted so it leads to:

$$(2 \, m_{LL} - m_{\text{pion(o),bound}})(\alpha_{w(\text{proton})} + \alpha_{\text{em}} - \alpha_{w(\text{proton})}^2).$$

From the equality

$$m_{\text{muon,bound}} - m_{\text{muon}} = (2 \, m_{LL} - m_{\text{pion(o),bound}})(\alpha_{w(\text{proton})} + \alpha_{\text{em}} - \alpha_{w(\text{proton})}^2),$$

we have $m_{\text{muon}} = 105.65810$ MeV.

**Summary**

Here we present the very, very precise calculations of masses of nucleons and of their magnetic moments. The results obtained are amazing with their accuracy and the presented model is shocking that it is semi-classical and non-perturbative. It suggests that we should verify our views on the internal structure of nucleons and other baryons.

Due to future more precise data for mass of neutral pion (the central value should be 134.97678 MeV instead the present value 134.9770(5) MeV) [2], we will able to verify presented here theory of nucleons. Emphasize that our results are much, much better than obtained within the Standard Model despite the fact that our model contains at least 5 times less parameters.

<table>
<thead>
<tr>
<th>Physical quantity</th>
<th>Theoretical value</th>
<th>Experimental value [2]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of proton</td>
<td>938.27206489 MeV</td>
<td>938.272081(6) MeV</td>
</tr>
<tr>
<td>Mass of neutron</td>
<td>939.56542415 MeV</td>
<td>939.565413(6) MeV</td>
</tr>
<tr>
<td>Proton magnetic moment in nuclear magneton</td>
<td>+2.7928471322</td>
<td>+2.792847351(9)</td>
</tr>
<tr>
<td>Neutron magnetic moment in nuclear magneton</td>
<td>−1.9130395424</td>
<td>−1.9130427(5)</td>
</tr>
<tr>
<td>Mass of neutron minus mass of proton</td>
<td>1.2933592651 MeV</td>
<td>1.2933321(5) MeV</td>
</tr>
<tr>
<td>Coupling constant for weak interactions of the proton/baryons</td>
<td>0.018723025693</td>
<td></td>
</tr>
<tr>
<td>Mass of charged pion minus mass of neutral pion</td>
<td>4.59383 MeV</td>
<td>4.5936(5) MeV</td>
</tr>
<tr>
<td>Mass of free muon</td>
<td>105.65810 MeV</td>
<td>105.6583745(24) MeV</td>
</tr>
<tr>
<td>Return mass of free neutral pion</td>
<td>134.97619 MeV</td>
<td></td>
</tr>
<tr>
<td>Mean square charge of nucleon</td>
<td>0.29 $e^2$</td>
<td>0.28 ± 0.03 $e^2$</td>
</tr>
</tbody>
</table>

A history of presented here model is as follows.

At the beginning (this was in 1976), I noticed that the following formula describes how to calculate the mass of a hyperon:

$$m \, [\text{MeV}] = 939 + 176 \, n + 26 \, d,$$

where $n = 0, 1, 2, 3$ and $d = 0, 1, 3, 7$. 
For a nucleon it is $n = 0$ and $d = 0$ which gives 939 MeV. For hyperon Lambda $n = 1$ and $d = 0$ which gives 1115 MeV, for Sigma $n = 1$ and $d = 3$ which gives 1193 MeV, for Ksi $n = 2$ and $d = 1$ which gives 1317 MeV and for Omega $n = 3$ and $d = 7$ which gives 1649 MeV. I later noticed that the mass distances between the resonances and mass distances between the resonances and hyperons are approximately 200 MeV, 300 MeV, 400 MeV, and 700 MeV.

In 1985, I described the atom-like structure of baryons (there appeared the Titius-Bode law for the nuclear strong interactions) but the internal structure of the core of baryons was neglected – there appeared the mass of the core of baryons: 727 MeV.

In 1997, I described the successive phase transitions of the inflation field – such transitions lead to the internal structures of bare particles so of the core of baryons as well.

References
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