ALL SPECIAL RELATIVITY EQUATIONS OBTAINED WITH GALILEAN TRANSFORMATION & EINSTEIN FIELD EQUATIONS OBTAINED ONLY WITH GAUSS CURVATURE AND ZOOM UNIVERSE MODEL CHARACTERISTICS

(ZOOM THEORY)

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Abstract

Demonstration of how to do that the light velocity c be the same independently of the velocity of the observer and obtain the mass-energy equivalence $E = mc^2$ using the Galilean transformations and the 4 dimensions zoom-universe model characteristics.

Demonstration of how to interpret it the time dilation/length contraction typical of the special relativity using the Galilean transformations and the 4 dimensions zoom-universe model characteristics.

Demonstration of how to obtain the typical waves equation with transmission velocity c and how to obtain it through the medium for transmission of light given by the zoom-universe model.

Demonstration of how to obtain the Einstein Field Equations without using the Stress-Energy Tensor, without using the Bianchi Identities and without using the Energy Conservation to obtain it.

Demonstration of how to obtain the Einstein Field Equations only using the Gauss Curvature and the zoom universe model characteristics.

Gravity in zoom universe model.

Special relativity zoom.

General relativity zoom.

Sphere and Hypersphere example to understand it better.

Contents

1. Introduction

We start thinking about the big holes that the current conception of the universe and the theory of relativity have, that are:

1 - Is the universe flat? If so, and if the universe expands, it will have limits or it will be infinite. Is this possible?
2 - If we assume that in the BIG BANG the universe, time, and other dimensions were created, what was there before time? And how were time and other dimensions created?
3 - Regarding the theory of relativity, if light is also an electromagnetic wave, how can a wave move in a vacuum?

1 - In ancient times it was believed that the earth was also flat, the problem was solved with a spherical earth that basically consists in bending a 2 dimensional plane through a third dimension.
We will assume the same with the 3 dimensions of the universe (x, y, z) and we will curve them through a fourth dimension that we will call Zz, with which we will have the whole universe in a hypersphere.

2 - I will use the zoom as that Zz dimension in which the other dimensions are curved to create the hypersphere and to obtain the same accepted equations of special relativity and general relativity using the zoom as a dimension. To not complicate the calculations much I used the time t in the equations and the derivatives (we can think of time as a fifth dimension or we can think of it as the expansion of the

universe through the Zz coordinate in which case time would not really exist), the advantages of considering zoom as a dimension and considering that time does not really exist are enormous:

2.1 - The limits of the hypersphere are perfectly defined, the limit that marks the outer surface is the object with the minimum possible zoom, we think that although the smallest possible object it has will much smaller than an electron it will always be bigger than 0, because if it were 0, it would not exist.

2.2 - The universe, the hypersphere, can be expanded, since the minimum possible object, with the minimum possible zoom level, could always be smaller. Here we would have the outer limit of the universe expanding perfectly defined.

2.3 - If we move in the other direction of the Zz zoom dimension of the radius of the hypersphere we would have the largest possible object that would be the whole universe, with all its energy contained in a single point, it is the same as the BIG BANG but with the advantage that there is no “before” since we have not used time for anything.

2.4 - Having all the universe concentrated in a single point is the same as having it fully expanded, since we are only zooming in, it is like looking at an object through a magnifying glass, the object is the same, only that we are in another level of zoom. and if a point (0 dimensions) in a Euclidean space you expand it in a circle, sphere, hypersphere, etc, the surface and the radius will acquire that number of dimensions and as it I will demonstrated in the form used to obtain $E = mc^2$ this is the mechanism by which dimensions and matter exist

3 - As I will demonstrate with equations, an object, although we increase or decrease as much as possible the zoom, that object will continue to exist, so that object exists at all zoom levels. I take advantage of this feature to show that the light moves like a wave of speed c through this coordinate (Zz). Move along the dimension zoom in the rest of dimensions would be appreciated as a “ball of light” of speed c that each time becomes larger, which would coincide with the observed and with the particle-wave duality of quantum physics (although this paper only focuses on relativistic concepts).

I will use these concepts to deduce all the equations of special relativity and general relativity and thus be able to prove that these concepts are valid.

2. Zoom universe model characteristics

The Euclidean space is composed by 4 dimensions (x,y,z,Zz).

The normal 3 dimensions (x,y,z) $\mapsto$ (α,θ,ϕ) are curved by a fourth dimension creating a hypersphere with radius r.

The coordinates of the surface of the hypersphere embedded can be (x,y,z) intrinsic 3d view or (α,θ,ϕ) with r as radius extrinsic 4d view.

If we represent the total non-infinite energy of that universe with a point in this Euclidean space we have 0 dimensions, if we make a zoom now we have 4 dimensions, 3 surface intrinsic coordinates (x,y,z) and 1 extrinsic coordinate r, with the same non-infinite energy.

An object in a zoom scale, exist in all zoom scales,therefore, we have a medium for transmission of light and electromagnetic waves (r) similar at the transmission of longitudinal waves for a solid object through normal dimensions (x,y,z).

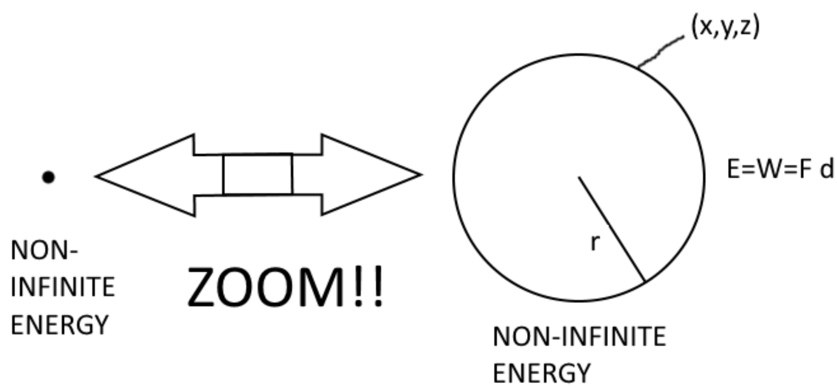

Fig. 1. Zoom universe model draw without 3D perspective and without 4D perspective

3. Galilean transformations

$$x' = x - v_x \mathrm{t} \qquad y' = y - v_y \mathrm{t} \qquad z' = z - v_z \mathrm{t} \quad , \qquad v = v_{\downarrow} = \sqrt{v_x{}^2 + v_y{}^2 + v_z{}^2}$$

$$z_z' = z_z + v_{\downarrow} \mathrm{t} \quad , \qquad z_z' = \sqrt{x^2 + y^2 + z^2} \text{ (if the } z_z' \text{ it's in the same zoom level than O)}$$

$$t' = t \quad , \qquad z_z' = \sqrt{x'^2 + y'^2 + z'^2} \text{ (if the } z_z' \text{ it's in the same zoom level than O')} \tag{1}$$

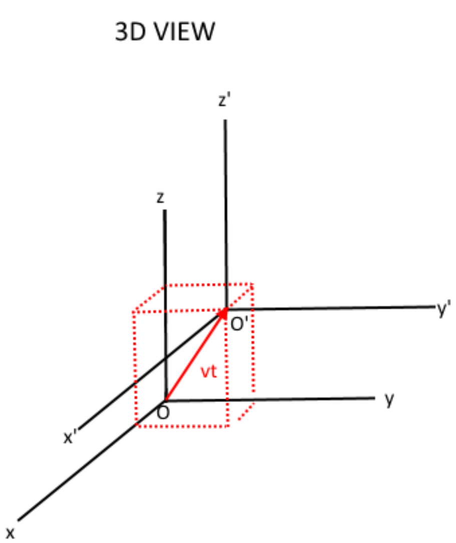

Fig. 2. Galilean transformation in normal 3D view with 3D perspective

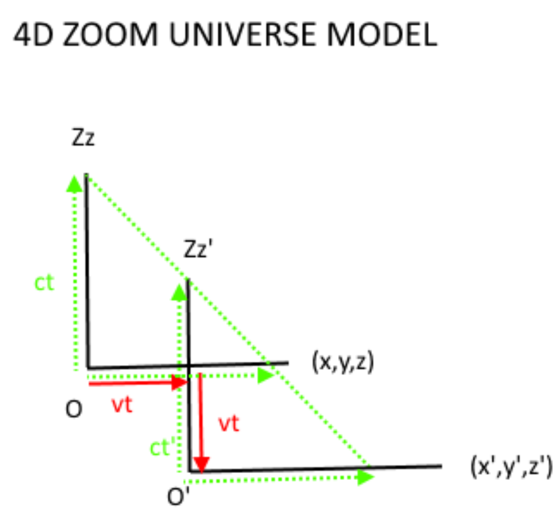

Fig. 3. Galilean transformation in 4D zoom universe model

4. Demonstration of how light velocity c is the same independently of the observer with Galilean transformations

The hypersphere zoom universe model is very large, we can take a “small” interval and interpret it as a plane interval.

From the point of view of the mobile observer O’ the length that the light travels it's ct' ,that it’s the same length that the length of z_z' dimension:

$x'^2 + y'^2 + z'^2 = (ct')^2$ (2)

Apply the Galilean transformations:

$(x - v_x t)^2 + (y - v_y t)^2 + (z - v_z t)^2 = (ct)^2$ (3)

$x^2 - 2 x v_x t + (v_x t)^2 + y^2 - 2 y v_y t + (v_y t)^2 + z^2 - 2 z v_z t + (v_z t)^2 = (ct)^2$ (4)

As it's still at the same zoom level that the mobile observer O':

$(z_z')^2 + (v_\downarrow t)^2 - 2 x v_x t - 2 y v_y t - 2 z v_z t = (ct)^2$ (5)

Is it $(x v_x + y v_y + z v_z)$ equal to $(z_z' v_\downarrow)$?

$(x v_x + y v_y + z v_z) = \sqrt{v_x^2 + v_y^2 + v_z^2} \sqrt{x^2 + y^2 + z^2}$ (6)

$(x v_y - y v_x)^2 (x v_z - z v_x)^2 (y v_z - z v_y)^2 = 0$ (7)

$x v_y = y v_x \longrightarrow \frac{x}{y} = \frac{v_x}{v_y}$

$x v_z = z v_x \longrightarrow \frac{x}{z} = \frac{v_x}{v_z}$

$y v_z = z v_y \longrightarrow \frac{y}{z} = \frac{v_y}{v_z}$ (8)

Yes, in all cases is equal because the light spreads like a sphere, whatever direction of v it is equal to the light spread direction , therefore, $(x v_x + y v_y + z v_z) = (z_z' v_\downarrow)$ (9)

Combining with equation (5) we have:

$(z_z')^2 + (v_\downarrow t)^2 - 2 t (z_z' v_\downarrow) = (ct)^2$ (10)

$(z_z' - v_\downarrow t)^2 = (ct)^2$ (11)

Apply the Galileo transformations

$(z_z)^2 = (ct)^2$ (12)

$x^2 + y^2 + z^2 = (ct)^2$ (13)

We can affirm that the distance traveled and velocity of light is the same independently of the velocity of observer.

5. Length contraction

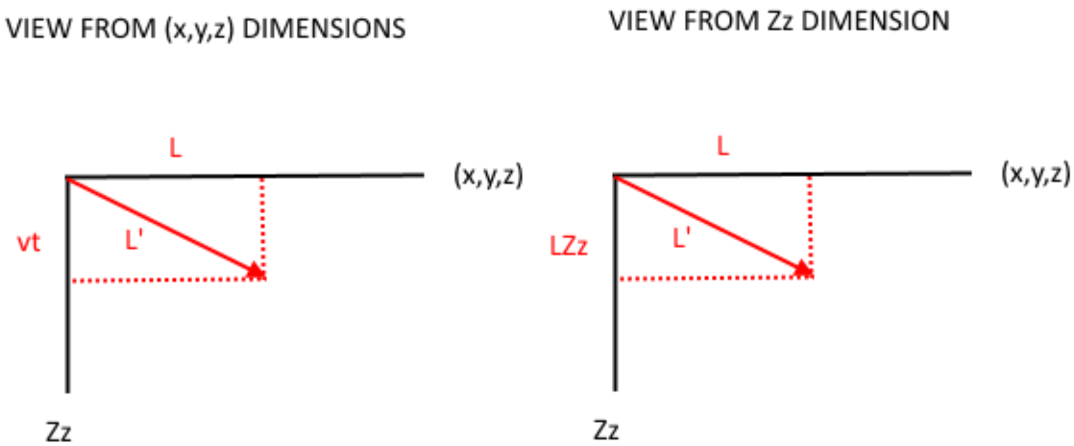

Fig. 4. Length contraction from normal 3D dimensions view and length contraction from fourth dimension Zz view

5.1 View from 3d (x,y,z) dimensions:

First we have an own length called L’ and we use the velocity of light to have a invariance length $L' = \text{ct}$.

From the 3D view we have:

$$L^2 = (L')^2 - (\text{vt})^2 \quad (14)$$

$$\frac{L}{L'} = \sqrt{1 - \frac{v^2}{c^2}} \quad , \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (15)$$

$$L = \frac{L'}{\gamma} \quad (16)$$

It is the same as the special relativity.

5.2 View from Zz dimension:

$$\text{Lz}_z{}^2 = (L')^2 - (L)^2 \quad (17)$$

$$\text{Lz}_z{}^2 = (L')^2 - \left(\frac{L'}{\gamma}\right)^2 \quad (18)$$

$$\frac{\text{Lz}_z}{L'} = \sqrt{1 - \left(1 - \frac{v^2}{c^2}\right)} = \frac{v}{c} \quad (19)$$

$$\text{Lz}_z = \frac{v}{c} L' = \frac{v}{c} L \gamma \quad (20)$$

This result doesn’t exist in special relativity, but it is very important in zoom special relativity.

6. Time dilation

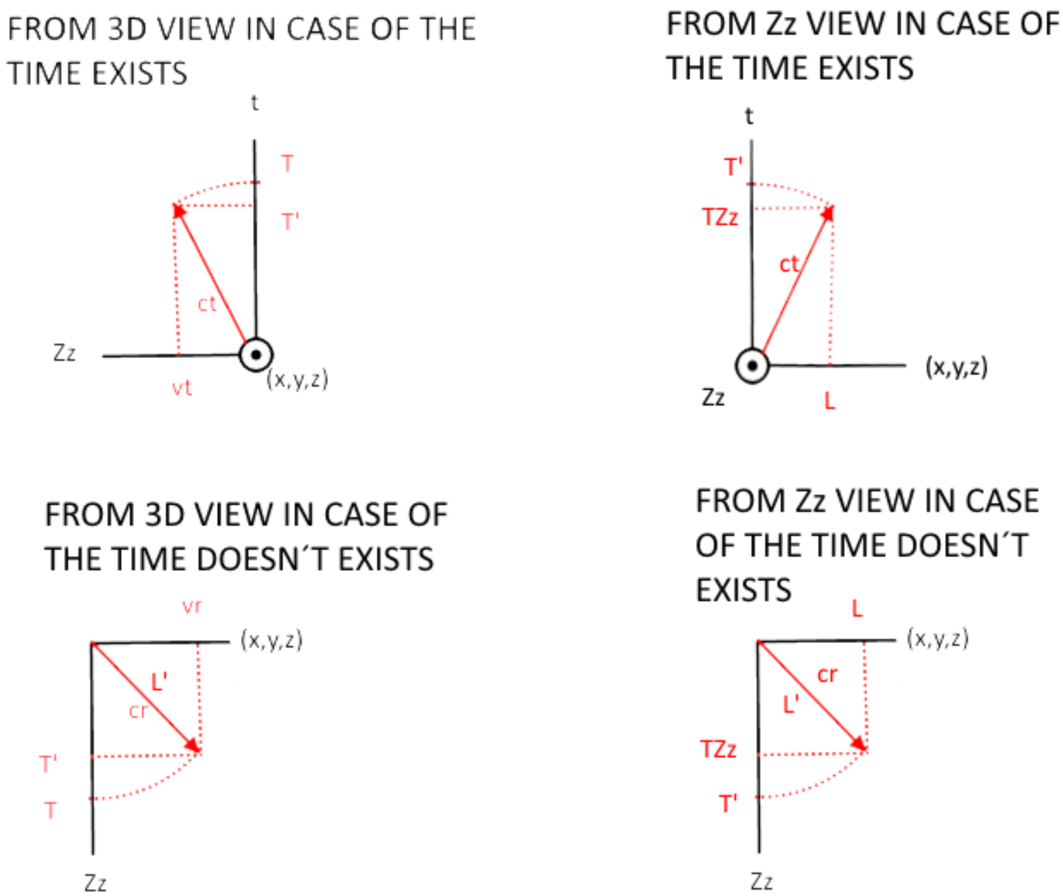

Fig. 5. Time dilation from 3D view and from Zz view in case of the time exists or in case of the time doesn't exists

▪ 6.1 From 3d view in case that the time exists

The time, in case that exist, it's a dimension and it must be perpendicular to the rest of dimensions in a Cartesian coordinate system.

The measurement of the time it's only possible if we have a velocity or a frequency in a known dimensions. If that velocity is through a unknown dimension (Zz) the projection in our known dimension it will be our measure of time.

So for a fixed observer O the time it will be T but for a mobile observer O' the time it will be the projection in a known dimension T' (own time).

$$T'^2 = (T)^2 - (\mathrm{vt})^2 \qquad (21)$$

$$\frac{T'}{T} = \sqrt{1 - \frac{v^2}{c^2}} \quad , \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (22)$$

$$T' = \frac{T}{\gamma} \qquad (23)$$

It is the same as the special relativity.

▪ 6.2 From 3d view in case that the time doesn't exists

In case that the time doesn't exist,our time perception can be the movement given by the Zz dimension caused by a velocity non perceptible by us (the movement of the Earth or the milky way,for example), (centrifuge force causes a movement in zoom dimension Zz caused for a 3d velocity) or maybe by the universe expansion.

In this case Zz it will be the time dimension for us (note that I have replaced the time t for hypersphere radius r in the picture).

The equations are the same as in the previous case and the same as the special relativity.

6.3 From Zz view in case of the time exists

The time for a Zz view (TZz) it will be the projection of the own time T' so:

$\mathrm{Tz}_z^2 = (T')^2 - (L)^2$ (24)

$\mathrm{Tz}_z^2 = (T')^2 - \left(\frac{L'}{\gamma}\right)^2$, $T' = L' = \mathrm{ct}$ (25)

$\frac{\mathrm{Tz}_z}{T'} = \sqrt{1 - \left(1 - \frac{v^2}{c^2}\right)} = \frac{v}{c}$ (26)

$\mathrm{Tz}_z = \frac{v}{c} T' = \frac{v}{c} \frac{T}{\gamma}$ (27)

This result doesn't exist in special relativity, but it is very important in zoom special relativity.

6.4 From Zz view in case of the time doesn't exists

In this case Zz it will be the time dimension for us (note that I have replaced the time t for hypersphere radius r in the picture).

The equations are the same as in the previous case.

7. Mass-energy equivalence

Any movement in (x,y,z) directions it would be equal to a circle movement in our hypersphere zoom universe, we have a centrifuge force and a centrifuge acceleration.

$a = \frac{v_O^2}{R}$ (28)

But this acceleration will be an acceleration from the O observer view, we need the acceleration from the $\mathrm{O}z_z$ observer view (20).

$\mathrm{Lz}_z = \frac{v}{c} L \gamma$ (29)

$v\, t_{z_z} = \frac{v}{c} L \gamma$ (30)

$L = \frac{c\, t_{z_z}}{\gamma}$, $\frac{\mathrm{dL}}{\mathrm{dt}} = v_O = \frac{c}{\gamma}$ (31)

$a = \frac{c^2}{\gamma^2 R}$ (32)

We know for the elasticity Hooke´s law [24] that:

$\mathrm{F} = -\mathrm{K}\, \mathrm{Lz}_z$ (33)

$m \frac{c^2}{\gamma^2 R} = -\mathrm{K}\, v\, t_{z_z}$, $\mathrm{K} = -m \frac{c^2}{\gamma^2 R\, v\, t_{z_z}}$

The intrinsic elastic constant is [24]:

$K_i = -m \frac{c^2}{\gamma^2 v\, t_{z_z}}$

But this constant it is for the $\mathrm{O}z_z$ observer view (27), the same constant from the O observer view:

$\mathrm{Tz}_z = \frac{v}{c} \frac{T}{\gamma}$ (34)

$K_i = -m \frac{c^3}{\gamma v^2 t} = -m \frac{c^3}{\gamma v L}$ (35)

$\mathrm{Lz}_z = \frac{v}{c} L \gamma$ (36)

$K_i = -m \frac{c^2}{\mathrm{Lz}_z}$ (37)

Potential Energy (U) for the hypersphere zoom universe:

$$U = -\int_{-R}^{0} K\,\mathrm{Lz}_z\; dz_z \;=\; -\int_{-R}^{0} \frac{K_i}{R}\,\mathrm{Lz}_z\; dz_z \;=\; \int_{-R}^{0} m\,\frac{c^2}{R}\; dz_z \;=\; \mathrm{mc}^2 \qquad (38)$$

We can conclude that the mass-energy equivalence $E = \mathrm{mc}^2$ it is the potential energy that need the matter to exist in a scale zoom in the zoom universe model.

8. Light transmission medium

We use the typical longitudinal elastic wave transmission method [24].

We take a little piece of length Δ_{Zz} and the elongation of this piece we named $\delta_{\Delta \mathrm{Zz}}$.

$$F(\mathrm{Zz}) = -\,\mathrm{K}\;\delta_{\Delta \mathrm{Zz}} = -\,K_i\;\frac{\delta_{\Delta \mathrm{Zz}}}{\Delta_{\mathrm{Zz}}} = -\,K_i\;\frac{d\,\delta}{d_{\mathrm{Zz}}} \qquad (39)$$

We named $\psi(\mathrm{Zz})$ at displacement of a section of piece, ρ at the mass density and dm at a little mass.

$$\mathrm{dF} = \mathrm{dm}\;a \quad , \qquad \mathrm{dm} = \rho\;\mathrm{dx\;dy\;dz} \qquad (40)$$

$$\mathrm{F(Zz)} - \mathrm{F(Zz + dZz)} = \mathrm{dm}\;\frac{\partial^2 \psi}{(\partial t)^2} \qquad (41)$$

Maclaurin series:

$$\mathrm{dZz}\;\frac{\partial F}{\partial \mathrm{Zz}} = \rho\;\mathrm{dx\;dy\;dz}\;\frac{\partial^2 \psi}{(\partial t)^2} \qquad (42)$$

If (20):

$$\mathrm{Lz}_z = \frac{v}{c}\,L\,\gamma \qquad (43)$$

We can do:

$$\mathrm{dx} = \frac{\mathrm{dZz}\,c}{v\,\gamma} \qquad (44)$$

And:

$$\frac{\partial F}{\partial \mathrm{Zz}} = \rho\;\frac{c}{v\,\gamma}\;\mathrm{dy\;dz}\;\frac{\partial^2 \psi}{(\partial t)^2} \qquad (45)$$

The little elongation of this section of piece we can defined:

$$\mathrm{d}\delta = \psi(\mathrm{Zz + dZz}) - \psi(\mathrm{Zz}) = \frac{\partial \psi}{\partial \mathrm{Zz}}\;\mathrm{dZz} = \mathrm{d}\psi \qquad (46)$$

Equation (39):

$$F(\mathrm{Zz}) = -\,K_i\;\frac{d\,\delta}{d_{\mathrm{Zz}}} = \quad -\,K_i\;\frac{d\psi}{d_{\mathrm{Zz}}} \qquad (47)$$

$$\frac{\partial F}{\partial \mathrm{Zz}} = \quad -\,K_i\;\frac{\partial^2 \psi}{(\partial_{\mathrm{Zz}})^2} \qquad (48)$$

Equation (45):

$$\frac{\partial^2 \psi}{(\partial t)^2} = -\,K_i\;\frac{v\,\gamma}{\rho\,c\,\mathrm{dy\,dz}}\;\frac{\partial^2 \psi}{(\partial_{\mathrm{Zz}})^2} \qquad (49)$$

If (in the O observer view) (36) (37):

$$K_i = -\mathrm{dm}\;\frac{c^3}{\gamma\,v^2\,t} \qquad (50)$$

$$\frac{\partial^2 \psi}{(\partial t)^2} = \frac{\mathrm{dm}}{v\,t}\;\frac{c^2}{\rho\;\mathrm{dy\,dz}}\;\frac{\partial^2 \psi}{(\partial_{\mathrm{Zz}})^2} \quad , \qquad \mathrm{dm} = \rho\;\mathrm{vt\;dy\;dz} \qquad (51)$$

$$\frac{\partial^2 \psi}{(\partial t)^2} = \; c^2\;\frac{\partial^2 \psi}{(\partial_{\mathrm{Zz}})^2} \qquad (52)$$

That is the wave equation with velocity of transmission by the medium (Zz) c, the light velocity.

9. Gauss-Codazzi equations & Riemann Tensor

For a Surface of 2 dimensions embedded in a three-dimensional space $S : R^2 \longrightarrow R^3$ expressed by [29]:

$\vec{s} = (\, x(u_1,u_2),\, y(u_1,u_2),\, z(u_1,u_2)\,)$ (53)

If we do this [29] [16]:

$d\vec{s} = \frac{\partial \vec{s}}{\partial u_1} du_1 + \frac{\partial \vec{s}}{\partial u_2} du_2 = \vec{s}_1 du_1 + \vec{s}_2 du_2 = \vec{e}_1 du^1 + \vec{e}_2 du^2$, $\vec{N} = \frac{\vec{e}_1 \wedge \vec{e}_2}{|\vec{e}_1 \wedge \vec{e}_2|}$ (54)

We have basis vectors and a normal vector in a point of the surface. And we have [29] [16]:

$I = ds^2 = d\vec{s} \bullet d\vec{s} = \vec{e}_1 \bullet \vec{e}_1 (du^1)^2 + \vec{e}_2 \bullet \vec{e}_2 (du^2)^2 + 2\, \vec{e}_1 \bullet \vec{e}_2 (du^1)(du^2) = g_{11}(du^1)^2 + g_{22}(du^2)^2 + 2\, g_{12}(du^1)(du^1)$ (55)

$II = d\vec{s} \bullet d\vec{N} = \vec{e}_1 \bullet \vec{N}_1 (du^1)^2 + \vec{e}_2 \bullet \vec{N}_2 (du^2)^2 + 2\, \vec{e}_1 \bullet \vec{N}_2 (du^1)(du^2) = l_{11}(du^1)^2 + l_{22}(du^2)^2 + 2\, l_{12}(du^1)(du^2)$ (56)

The first and second fundamental form. And we have [29]:

$\vec{N}_1 = - l_1^1 \vec{e}_1 - l_1^2 \vec{e}_2$ (57)

$\vec{N}_2 = - l_2^1 \vec{e}_1 - l_2^2 \vec{e}_2$ (58)

The derivation of the normal vector $\vec{N}$ expressed in a base vector coordinates

And the Gauss curvature K [29]:

$K = \frac{l_{11}\, l_{22} - l_{12}{}^2}{g_{11}\, g_{22} - g_{12}{}^2} = \frac{L}{g}$ (59)

The Gauss-Codazzi equations for 2 dimensions can be written as [29]:

$\frac{\partial \Gamma^i_{ab}}{\partial u_j} - \frac{\partial \Gamma^i_{aj}}{\partial u_b} + \Gamma^m_{ab}\Gamma^i_{mj} - \Gamma^m_{aj}\Gamma^i_{mb} = l_{ab}\, l^i_j - l_{aj}\, l^i_b$, i = 1,2 (60)

The Riemann Tensor definition it is [5]:

$R^i_{ajb} = \frac{\partial \Gamma^i_{ab}}{\partial u_j} - \frac{\partial \Gamma^i_{aj}}{\partial u_b} + \Gamma^m_{ab}\Gamma^i_{mj} - \Gamma^m_{aj}\Gamma^i_{mb}$ (61)

Therefore, for 2 dimensions we can write:

$R^i_{ajb} = l_{ab}\, l^i_j - l_{aj}\, l^i_b$, i = 1,2 (62)

10. Ricci Tensor & Gauss curvature

The Ricci Tensor definition it is (for 2 dimensions) [5]:

$R_{ab} = R^i_{aib} = R^1_{a1b} + R^2_{a2b}$ (63)

Therefore, for 2 dimensions we can write (63) (62):

$R_{ab} = l_{ab}\, l^i_i - l_{ai}\, l^i_b$, i = 1,2 (64)

And we can do [23] [25]:

$R_{ab} = g^{ir}(l_{ab}\, l_{ir} - l_{ai}\, l_{br})$, i = 1,2 (65)

The inverse of the metric tensor, we can obtain with the cofactor of g, in 2 dimensions we have [23]:

$g^{ij} = \frac{G_{ij}}{|g_{ij}|} \longrightarrow g^{11} = \frac{g_{22}}{g_{11}\, g_{22} - g_{12}{}^2}, g^{12} = \frac{-g_{21}}{g_{11}\, g_{22} - g_{12}{}^2}, g^{22} = \frac{g_{11}}{g_{11}\, g_{22} - g_{12}{}^2}$ (67)

$R_{ab} = g^{ir}(l_{ab}\, l_{ir} - l_{ai}\, l_{br}) = g_{ba} \frac{(l_{ab}\, l_{ir} - l_{ai}\, l_{br})}{|g_{ij}|}$ → If a = b , i = 1,2 (68)

$= g_{ba} \frac{(l_{ai}\, l_{br} - l_{ab}\, l_{ir})}{|g_{ij}|}$ → If a ≠ b , i = 1,2 (69)

We can see if (a = b) → (i = r) and if (a ≠ b) → (i ≠ r) therefore, knowing the Gauss curvature definition (59):

$R_{\text{ab}} = g_{\text{ba}}\ [\text{ K }]_{\text{plane } a-b}$ (in 2 dimensions) (70)

We can see that this expression would also be useful for more dimensions **as long as we have a diagonal metric** and add the curvatures in common

$$\text{g} = \begin{pmatrix} g_{11} & 0 & 0 & 0 \\ 0 & g_{22} & 0 & 0 \\ 0 & 0 & g_{33} & 0 \\ 0 & 0 & 0 & g_{44} \end{pmatrix} \qquad (71)$$

$R_{11} = g_{11}$ ($[\text{ K }]_{\text{plane } 1-2} + [\text{ K }]_{\text{plane } 1-3} + [\text{ K }]_{\text{plane } 1-4}$)

$R_{22} = g_{22}$ ($[\text{ K }]_{\text{plane } 2-1} + [\text{ K }]_{\text{plane } 2-3} + [\text{ K }]_{\text{plane } 2-4}$)

$R_{33} = g_{33}$ ($[\text{ K }]_{\text{plane } 3-2} + [\text{ K }]_{\text{plane } 3-1} + [\text{ K }]_{\text{plane } 3-4}$)

$R_{44} = g_{44}$ ($[\text{ K }]_{\text{plane } 4-2} + [\text{ K }]_{\text{plane } 4-3} + [\text{ K }]_{\text{plane } 4-1}$) (72)

11. Ricci Scalar & Gauss curvature

The Ricci Scalar definition it is (for 2 dimensions) [5]:

$R = g^{\text{ab}} R_{\text{ab}} = g^{11} R_{11} + g^{12} R_{12} + g^{21} R_{21} + g^{22} R_{22}$ (73)

Therefore, for 2 dimensions we can write (δ = Kronecker delta) [23] (73) (70):

$R = g^{\text{ab}} R_{\text{ab}} = g^{\text{ab}} g_{\text{ba}}\ [\text{ K }]_{\text{plane } a-b} = \delta^a_a\ [\text{ K }]_{\text{plane } a-b} = 2\ [\text{ K }]_{\text{plane } a-b}$ (74)

We can see that this expression would also be useful for more dimensions **as long as we have a diagonal metric** and add all curvatures

$$\text{g} = \begin{pmatrix} g_{11} & 0 & 0 & 0 \\ 0 & g_{22} & 0 & 0 \\ 0 & 0 & g_{33} & 0 \\ 0 & 0 & 0 & g_{44} \end{pmatrix} \qquad (75)$$

$R = 2\ [\text{ K }]_{\text{plane } 1-2} + 2\ [\text{ K }]_{\text{plane } 1-3} + 2\ [\text{ K }]_{\text{plane } 1-4} + 2\ [\text{ K }]_{\text{plane } 2-3} + 2\ [\text{ K }]_{\text{plane } 2-4} + 2\ [\text{ K }]_{\text{plane } 3-4}$ (76)

12. Einstein Tensor & Gauss curvature

The Einstein tensor definition it is [23]:

$G_{\text{ab}} = R_{\text{ab}} - \frac{1}{2} g_{\text{ab}} R$ (77)

Therefore, (70) (76) always **as long as we have a diagonal metric**

$$\mathbf{g} = \begin{pmatrix} g_{11} & 0 & 0 & 0 \\ 0 & g_{22} & 0 & 0 \\ 0 & 0 & g_{33} & 0 \\ 0 & 0 & 0 & g_{44} \end{pmatrix} \qquad (78)$$

$G_{\text{ab}} = R_{\text{ab}} - \frac{1}{2} g_{\text{ab}} R = g_{\text{ba}} \sum [\text{ K }]_{\text{plane } a-b} - g_{\text{ab}} \sum [\text{ K }]_{\text{all planes}} = - g_{\text{ab}} \sum [\text{ K }]_{\text{planes except plane } a-b}$ (79)

$G_{11} = R_{11} - \frac{1}{2} g_{11} R = - g_{11}$ ($[\text{ K }]_{\text{plane } 2-3} + [\text{ K }]_{\text{plane } 2-4} + [\text{ K }]_{\text{plane } 4-3}$)

$G_{22} = R_{22} - \frac{1}{2}\, g_{22}\, R = -\, g_{22}\, (\, [\text{ K }]_{\text{plane }1-3} + [\text{ K }]_{\text{plane }1-4} + [\text{ K }]_{\text{plane }4-3}\,)$

$G_{33} = R_{33} - \frac{1}{2}\, g_{33}\, R = -\, g_{33}\, (\, [\text{ K }]_{\text{plane }2-1} + [\text{ K }]_{\text{plane }2-4} + [\text{ K }]_{\text{plane }4-2}\,)$

$G_{44} = R_{44} - \frac{1}{2}\, g_{44}\, R = -\, g_{44}\, (\, [\text{ K }]_{\text{plane }2-3} + [\text{ K }]_{\text{plane }2-1} + [\text{ K }]_{\text{plane }1-3}\,)$ (80)

13. Zoom universe model & Gauss curvature

Now first let's calculate how would be the curvature of each plane if we do the zoom level fixed (R) we would have a hypersphere with 3 dimensions on its surface (α, θ, ϕ) and a radius R.

But for each bi-dimensional plane (plane α-θ) (plane α-ϕ) (plane ϕ-θ) we have a bi-dimensional sphere with the same radius R and the same spherical Gauss curvature $\frac{1}{R^2}$.

$[\text{ K }]_{\text{plane }\alpha-\theta}$ or $[\text{ K }]_{\text{plane }\alpha-\phi}$ or $[\text{ K }]_{\text{plane }\phi-\theta} = \frac{1}{R^2}$ (81)

We know for [6. Mass-energy equivalence] that: U = - F R (potential energy, work done on an object is found by multiplying force and distance)

$[\text{ K }]_{\text{plane }\alpha-\theta}$ or $[\text{ K }]_{\text{plane }\alpha-\phi}$ or $[\text{ K }]_{\text{plane }\phi-\theta} = \frac{1}{R^2} = \frac{\text{FF}}{\text{UU}}$ (82)

Multiplying both sides for Area and radius (A R):

$[\text{ K }]_{\text{plane }\alpha-\theta}$ or $[\text{ K }]_{\text{plane }\alpha-\phi}$ or $[\text{ K }]_{\text{plane }\phi-\theta} = \frac{1}{R^2} = \frac{\text{FF}}{\text{UU}} = \frac{\text{FFAR}}{\text{UUAR}}$ (83)

Now we know for [6. Mass-energy equivalence] that U = - F R = mc^2, we know that the sphere area it's A = $4\pi R^2$, we know that the Pressure it's P = $\frac{F}{A} = \frac{F\,R}{A\,R} = \frac{-U}{\text{Vol}}$, (Vol = volume) and in one volume we can have n particles P = $\frac{F}{A} = \frac{F\,R}{A\,R} = \frac{-U}{\text{Vol}} = -\frac{n\,\text{mc}^2}{\text{Vol}}$, therefore:

$[\text{ K }]_{\text{plane }\alpha-\theta}$ or $[\text{ K }]_{\text{plane }\alpha-\phi}$ or $[\text{ K }]_{\text{plane }\phi-\theta} = \frac{1}{R^2} = \frac{\text{FF}}{\text{UU}} = \frac{\text{FFAR}}{\text{UUAR}} = -\frac{\text{mc}^2\, 4\,\pi\, R^2\, n\,\text{mc}^2}{U^2\, R\,\text{Vol}}$ (84)

We know that the mass density $\rho_m = \frac{n\,m}{\text{Vol}}$, we know the light escape velocity $c^2 = \frac{2\,G\,m}{R}$, (G it is the gravitational constant) and we know for (31) that: v = $\frac{c}{\gamma}$

$[\text{ K }]_{\text{plane }\alpha-\theta}$ or $[\text{ K }]_{\text{plane }\alpha-\phi}$ or $[\text{ K }]_{\text{plane }\phi-\theta} = \frac{1}{R^2} = \frac{\text{FF}}{\text{UU}} = \frac{\text{FFAR}}{\text{UUAR}} = -\frac{\text{mc}^2\, 4\,\pi\, R^2\, n\,\text{mc}^2}{U^2\, R\,\text{Vol}} = -\frac{m\,2\,G\,m\,4\,\pi\,R^2\,\rho_m\,c^2}{m^2\,c^4\,R\,R} = -\frac{8\,\pi\,G}{c^4}\,\rho_m\,\gamma^2 v^2$ (85)

We can see what all mass and mass density in this equation it's refer to all mass and mass density into all universe plane sphere

14. The rest of planes in Zoom universe model & Gauss curvature

To calculate the rest of planes, we can do one of this dimensions fixed (α, θ, ϕ) and r would be part of the surface:

with α fixed (A) $\longrightarrow$ $[\text{ K }]_{\text{plane }r-\theta}$, $[\text{ K }]_{\text{plane }r-\phi}$

with θ fixed (Θ) $\longrightarrow$ $[\text{ K }]_{\text{plane }r-\alpha}$, $[\text{ K }]_{\text{plane }r-\phi}$

with ϕ fixed (Φ) $\longrightarrow$ $[\text{ K }]_{\text{plane }r-\theta}$, $[\text{ K }]_{\text{plane }r-\alpha}$ (86)

All Gauss curvature will be $\frac{1}{(\text{radius})^2} = \frac{1}{\text{A}^2}$ or $\frac{1}{\Theta^2}$ or $\frac{1}{\Phi^2}$ and all plane-sphere contains all universe plane-sphere mass and mass density regardless of the dimension used as a radio.

$[\text{ K }]_{\text{rest of planes}} = \frac{1}{(\text{radius})^2} = \frac{1}{\text{A}^2}$ or $\frac{1}{\Theta^2}$ or $\frac{1}{\Phi^2} = \frac{\text{FF}}{\text{UU}} = \frac{\text{FFA(radius)}}{\text{UUA(radius)}}$ (87)

Now we can observe that the energy used to be in a position on the surface of the sphere it is the Kinetic energy (K),

$U = -F\,(\text{radius}) = K = \frac{mv}{2}$, and the Pressure $P = \frac{F}{A} = \frac{F\,(\text{radius})}{A\,(\text{radius})} = \frac{-K}{\text{Vol}} = -\frac{n\,\text{mv}}{2\,\text{Vol}}$ (88)

$[\,K\,]_{\text{rest of planes}} = \frac{1}{(\text{radius})^2} = \frac{1}{A^2}$ or $\frac{1}{\Theta^2}$ or $\frac{1}{\Phi^2} = \frac{\text{FF}}{\text{UU}} = \frac{\text{FFA(radius)}}{\text{KKA(radius)}} = -\frac{\text{mv}^2\,\pi\,(\text{radius})^2\,n\,\text{mcv}^2}{K^2(\text{radius})\,\text{Vol}}$ (89)

We know that the mass density $\rho_m = \frac{n\,m}{\text{Vol}}$, we know the escape velocity $v^2 = \frac{2\,G\,m}{R}$, (G it is the gravitational constant) and we know for (31) $v = \frac{c}{\gamma}$

$[\,K\,]_{\text{rest of planes}} = \frac{1}{(\text{radius})^2} = \frac{1}{A^2}$ or $\frac{1}{\Theta^2}$ or $\frac{1}{\Phi^2} = \frac{\text{FF}}{\text{UU}} = \frac{\text{FFA(radius)}}{\text{KKA(radius)}} = -\frac{\text{mv}^2\,\pi\,(\text{radius})^2\,n\,\text{mcv}^2}{K^2(\text{radius})\,\text{Vol}} = -\frac{4\,m\,2\,G\,m\,\pi\,(\text{radius})^2\,\rho_m\,v^2}{m^2\,v^4(\text{radius})\,(\text{radius})} = -\frac{8\,\pi\,G\,\rho_m\,v^2\,\gamma^4}{m^2\,c^4} = -\frac{8\,\pi\,G}{c^4}\,\rho_m\,\gamma^2 c^2$

(90)

We can see that the velocity of r dimension either as a radius or as a surface should be c, that agrees with the exposed in [7. Light transmission medium]

15. Join Einstein Tensor & Zoom universe model

Now if we join all equations (79) (90) (the sphere have a diagonal metric tensor) (index 1 = r, index 2 = α, index 3 = θ, index 4 = ϕ):

$G_{ab} = R_{ab} - \frac{1}{2}\,g_{ab}\,R = -\,g_{ab}\,\sum[\,K\,]_{\text{planes except plane } a-b} = g_{ab}\,\frac{8\,\pi\,G}{c^4}\,\rho_m\,\gamma^2\left(\sum^2 \text{velocity planes}\right)$ (91)

We can see that the sum of 3 bi-dimensional velocity components squared gives us a velocity squared in 3 dimensions

$G_{11} = R_{11} - \frac{1}{2}\,g_{11}\,R = \frac{8\,\pi\,G}{c^4}\,\rho_m\,\gamma^2\,g_{11}\left(v^2_{\text{plane }2-3} + v^2_{\text{plane }2-4} + v^2_{\text{plane }3-4}\right) = \frac{8\,\pi\,G}{c^4}\,\rho_m\,\gamma^2\,g_{11}\,v^2_{\text{cube planes }2-3-4}$

$G_{22} = R_{22} - \frac{1}{2}\,g_{22}\,R = \frac{8\,\pi\,G}{c^4}\,\rho_m\,\gamma^2\,g_{22}\left(c^2_{\text{plane }1-3} + c^2_{\text{plane }1-4} + v^2_{\text{plane }3-4}\right) = \frac{8\,\pi\,G}{c^4}\,\rho_m\,\gamma^2\,g_{22}\,v^2_{\text{cube planes }1-3-4}$

$G_{33} = R_{33} - \frac{1}{2}\,g_{33}\,R = \frac{8\,\pi\,G}{c^4}\,\rho_m\,\gamma^2\,g_{33}\left(c^2_{\text{plane }2-1} + c^2_{\text{plane }1-4} + v^2_{\text{plane }2-4}\right) = \frac{8\,\pi\,G}{c^4}\,\rho_m\,\gamma^2\,g_{33}\,v^2_{\text{cube planes }1-2-4}$

$G_{44} = R_{44} - \frac{1}{2}\,g_{44}\,R = \frac{8\,\pi\,G}{c^4}\,\rho_m\,\gamma^2\,g_{44}\left(v^2_{\text{plane }2-3} + c^2_{\text{plane }2-1} + c^2_{\text{plane }3-1}\right) = \frac{8\,\pi\,G}{c^4}\,\rho_m\,\gamma^2\,g_{44}\,v^2_{\text{cube planes }1-2-3}$ (92)

16. Join Metric Tensor & velocity cube planes squared

The velocity squared can be expressed in terms of the first fundamental form [16]:

$\vec{e}_1\bullet\vec{e}_1(du^1)^2 + \vec{e}_2\bullet\vec{e}_2(du^2)^2 + \vec{e}_3\bullet\vec{e}_3(du^3)^2 + 2\,\vec{e}_1\bullet\vec{e}_2(du^1)(du^2) + 2\,\vec{e}_1\bullet\vec{e}_3(du^1)(du^3) + 2\,\vec{e}_2\bullet\vec{e}_3(du^2)(du^3) =$
$g_{11}(du^1)^2 + g_{22}(du^2)^2 + g_{33}(du^3)^2 + 2\,g_{12}(du^1)(du^2) + 2\,g_{13}(du^1)(du^3) + 2\,g_{23}(du^2)(du^3) =$
$g_{cd}(du^c)(du^d)$ (93)

$v^2_{\text{cube planes }1-2-3} = g_{cd}(du^c)\left(du^d\right)$, with index c = 1, 2, 3, with index d = 1, 2, 3
$v^2_{\text{cube planes }2-3-4} = g_{cd}(du^c)\left(du^d\right)$, with index c = 2, 3, 4, with index d = 2, 3, 4
$v^2_{\text{cube planes }1-3-4} = g_{cd}(du^c)\left(du^d\right)$, with index c = 1, 3, 4, with index d = 1, 3, 4
$v^2_{\text{cube planes }1-2-4} = g_{cd}(du^c)\left(du^d\right)$, with index c = 1, 2, 4, with index d = 1, 2, 4 (94)

Now let's multiply those metric tensors of 3 dimensions (g_{cd}) with the metric tensor of 4 dimensions (g_{ab}):

(We must bear in mind that the missing dimension in g_{cd} will always be in g_{cd} so, finally we have 2 metric tensors of 4 dimensions, I develop a case for better understood):

$g_{ab}\,g_{cd}(du^c)(du^d) = (\,\vec{e}_a\bullet\vec{e}_b)\,(\,\vec{e}_c\bullet\vec{e}_d)\,(du^c)(du^d) = (\,\vec{e}_a\bullet\vec{e}_d)\,(\,\vec{e}_c\bullet\vec{e}_b)\,(du^c)(du^d) = g_{ad}\,g_{cb}\,(du^c)(du^d)$ (95)

$g_{11}\,v^2_{\text{cube planes }2-3-4} = g_{11}\,g_{cd}(du^c)(du^d)$, with index c = 2, 3, 4, with index d = 2, 3, 4 =

$g_{11}\,g_{22}(du^2)(du^2) + g_{11}\,g_{23}(du^2)(du^3) + g_{11}\,g_{24}(du^2)(du^4) + g_{11}\,g_{32}(du^3)(du^2) + g_{11}\,g_{33}(du^3)(du^3) + g_{11}\,g_{34}(du^3)(du^4) + g_{11}\,g_{42}(du^4)(du^2) +$

$g_{11}\ g_{43}(\mathrm{d}u^4)(\mathrm{d}u^3) + g_{11}\ g_{44}(\mathrm{d}u^4)(\mathrm{d}u^4) =$

$g_{12}\ g_{12}(\mathrm{d}u^2)(\mathrm{d}u^2) + g_{12}\ g_{13}(\mathrm{d}u^2)(\mathrm{d}u^3) + g_{12}\ g_{14}(\mathrm{d}u^2)(\mathrm{d}u^4) + g_{13}\ g_{12}(\mathrm{d}u^3)(\mathrm{d}u^2) + g_{13}\ g_{13}(\mathrm{d}u^3)(\mathrm{d}u^3) + g_{13}\ g_{14}(\mathrm{d}u^3)(\mathrm{d}u^4) + g_{14}\ g_{12}(\mathrm{d}u^4)(\mathrm{d}u^2) + g_{14}\ g_{13}(\mathrm{d}u^4)(\mathrm{d}u^3) + g_{14}\ g_{14}(\mathrm{d}u^4)(\mathrm{d}u^4)$

Therefore (94) (95):

$g_{\mathrm{ab}}\left(\sum^2 \text{velocity planes}\right) = g_{\mathrm{ad}}\ g_{\mathrm{cb}}\ (\mathrm{d}u^c)(\mathrm{d}u^d)$, with (index c and d) ≠ (index a and b) (96)

And we have (91) (96):

$R_{\mathrm{ab}} - \frac{1}{2}\ g_{\mathrm{ab}}\ R\ =\ \frac{8\pi G}{c^4}\ \rho_m\ \gamma^2\ g_{\mathrm{ad}}\ g_{\mathrm{cb}}\ (\mathrm{d}u^c)(\mathrm{d}u^d)$, with (index c and d) ≠ (index a and b) (97)

17. Finally some tensor algebra

Multiplying both sides (97) by g^{bi} [23] [25]:

$R_a^i - \frac{1}{2}\ \delta_a^i\ R\ =\ \frac{8\pi G}{c^4}\ \rho_m\ \gamma^2\ g_{\mathrm{ad}}\ \delta_c^i\ (\mathrm{d}u^c)(\mathrm{d}u^d)$ (98)

Multiplying both sides by g^{aj}:

$R^{\mathrm{ij}} - \frac{1}{2}\ g^{\mathrm{ij}}\ R\ =\ \frac{8\pi G}{c^4}\ \rho_m\ \gamma^2\ \delta_d^j\ \delta_c^i\ (\mathrm{d}u^c)(\mathrm{d}u^d)$ (99)

$R^{\mathrm{ij}} - \frac{1}{2}\ g^{\mathrm{ij}}\ R\ =\ \frac{8\pi G}{c^4}\ \rho_m\ \gamma^2\ (\mathrm{d}u^i)(\mathrm{d}u^j)$ (100)

The derivative of a coordinate is the velocity component in that coordinate

(Remember that the velocity in coordinate 1 (r) is c [7. Light transmission medium]):

$R^{\mathrm{ij}} - \frac{1}{2}\ g^{\mathrm{ij}}\ R\ =\ \frac{8\pi G}{c^4}\ \rho_m\ \gamma^2\ v^i\ v^j$ (101)

This expression $\rho_m\ \gamma^2\ v^i\ v^j$ it is the stress–energy tensor T^{ij} therefore, we have the Einstein field equations [23]:

$R^{\mathrm{ij}} - \frac{1}{2}\ g^{\mathrm{ij}}\ R\ =\ \frac{8\pi G}{c^4}\ T^{\mathrm{ij}}$ (102)

18. Gravity

We can observe that the previous equation (101) was obtained with spheres in one point that includes the whole universe, with all its radius and all its mass-energy density (because mass and energy are related by E = mc^2).
This mass-energy density it cause the whole universe spherical-plane curvature form.
Obviously if we have a smaller mass-energy, we will have a smaller sphere with a smaller radius.
And the curvature of this bi-dimensional plane we can calculate it with the radius of that’s smaller sphere, similar to how we calculate the curvature of a 1 dimension line using the radius of circles.

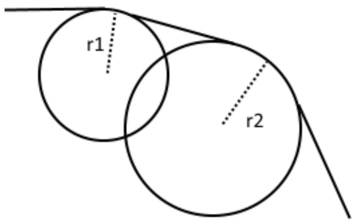

Fig. 6. Curvature in line caused for circles

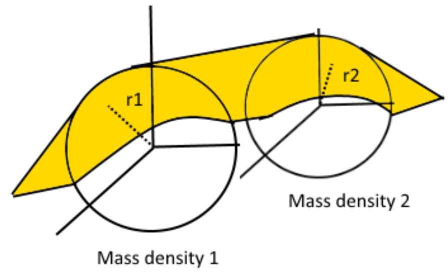

Fig. 7. Curvature in plane caused for spheres

It was already known that gravity was caused by the curvature of the 4 dimensions of space-time, but with this zoom universe model and its 4 dimensions of space-zoom it can be understood better.

An object with constant velocity causes a centrifugal acceleration as i have shown in [6. Mass-energy equivalence] its velocity will be perpendicular to r, and the centrifugal acceleration it travels by the radius dimension (r). There is no curvature.

But if that object has an acceleration, that acceleration will be added to the centrifugal acceleration. Therefore the radius will bend and how the velocity is perpendicular, we can see how the acceleration of an object curves the space-zoom.

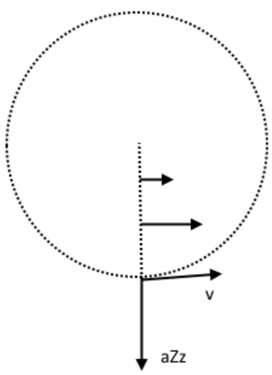

Fig 8. Object with velocity in hypersphere zoom universe

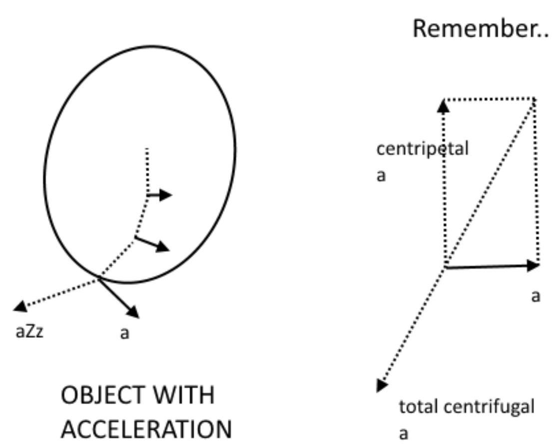

Fig 9. Object with acceleration in hypersphere zoom universe

And if an acceleration causes a curvature, then a curvature then causes an acceleration.

That’s the gravity

Appendix 1. Sphere example (drawn with one dimension less) (without 3d perspective)

First we draw a circle with yours polar coordinates (x, y) → (r, ϕ):

$x = r\cos\phi$

$y = r\sin\phi$

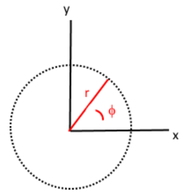

Fig 10. 2D circle

Now we take the radius dimension and we replace it with another circle perpendicular to the rest of the dimensions

$$\mathrm{r} \quad \longrightarrow \quad \begin{aligned} z_x &= \mathrm{r}\cos\theta \\ z_y &= \mathrm{r}\sin\theta \end{aligned}$$

We can see that the z_y is a new dimension and the z_x matches with the radius, therefore:

$z = r\sin\theta$

$x = r\cos\theta\cos\phi$

$y = r\cos\theta\sin\phi$

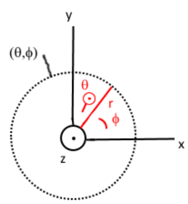

Fig. 11. 3D sphere

But if we want that the last angle begins with the last dimension:

$z = r\cos\theta$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

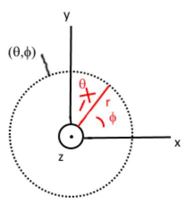

Fig. 12. 3D sphere

We have the spherical coordinates (x, y, z) → (r, θ, ϕ)

$\mathrm{ds}^2 = \mathrm{dr}^2 + r^2\,\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2$

$\vec{N} = \frac{(x,y,z)}{r} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$

1.1 If we make constant r → dr = 0

$$\mathrm{g} = \begin{pmatrix} g_{22} & g_{23} \\ g_{32} & g_{33} \end{pmatrix} = \begin{pmatrix} g_{\theta\theta} & g_{\theta\phi} \\ g_{\phi\theta} & g_{\phi\phi} \end{pmatrix} = \begin{pmatrix} r^2 & 0 \\ 0 & r^2\sin^2\theta \end{pmatrix}$$

$$\mathrm{l} = \begin{pmatrix} l_{22} & l_{23} \\ l_{32} & l_{33} \end{pmatrix} = \begin{pmatrix} l_{\theta\theta} & l_{\theta\phi} \\ l_{\phi\theta} & l_{\phi\phi} \end{pmatrix} = \begin{pmatrix} -r & 0 \\ 0 & -r\sin^2\theta \end{pmatrix}$$

$K_{\text{plane }\theta-\phi} = \frac{1}{r^2}$, $K_{\text{plane }r-\phi} = 0$, $K_{\text{plane }\theta-r} = 0$

$$\text{Ricci Tensor} = \begin{pmatrix} R_{22} & R_{23} \\ R_{32} & R_{33} \end{pmatrix} = \begin{pmatrix} R_{\theta\theta} & R_{\theta\phi} \\ R_{\phi\theta} & R_{\phi\phi} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \sin^2\theta \end{pmatrix}, \quad \text{Ricci Scalar} = \frac{2}{r^2}$$

$$\text{Einstein Tensor} = \begin{pmatrix} G_{22} & G_{23} \\ G_{32} & G_{33} \end{pmatrix} = \begin{pmatrix} G_{\theta\theta} & G_{\theta\phi} \\ G_{\phi\theta} & G_{\phi\phi} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

We observed that the flat coordinate and the curved coordinates could be chosen differently:

1.2 If we make constant θ $\to$ dθ = 0 and:

$z = \theta \cos r$

$x = \theta \sin r \cos \phi$

$y = \theta \sin r \sin \phi$

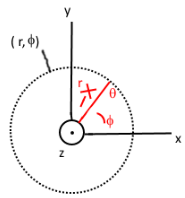

Fig. 13. 3D sphere

$K_{\text{plane }\theta-\phi} = 0$, $K_{\text{plane }r-\phi} = \frac{1}{\theta^2}$,$K_{\text{plane }\theta-r} = 0$

1.3 If we make constant ϕ $\to$ dϕ = 0 and:

$z = \phi \cos \theta$

$x = \phi \sin \theta \cos r$

$y = \phi \sin \theta \sin r$

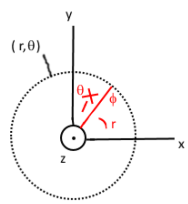

Fig. 14. 3D sphere

$K_{\text{plane }\theta-\phi} = 0$, $K_{\text{plane }r-\phi} = 0$, $K_{\text{plane }\theta-r} = \frac{1}{\phi^2}$

And we can use that's spheres to determine the curvature of any pseudo-spherical object

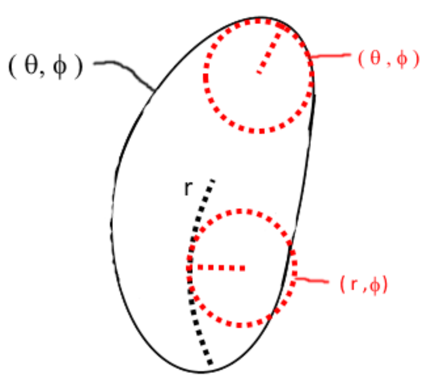

Fig. 15. Curvature 3D pseudo-spherical object

Appendix 2. Hypersphere example (drawn with two dimension less) (without 3d perspective) (without 4d perspective)

First we draw a sphere with ours spherical coordinates (x, y, z) → (r, θ, ϕ):

$z = r\cos\theta$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

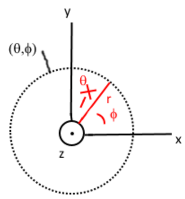

Fig. 16. 3D sphere

Now we take the radius dimension and we replace it with another circle perpendicular to the rest of the dimensions:

r $\longrightarrow$ z_y = r cos α

z_z = r sin α

We can see that the z_z is a new dimension and the z_y matches with the radius, therefore:

$z_z = r\sin\alpha$

$z = r\cos\alpha\cos\theta$

$x = r\cos\alpha\sin\theta\cos\phi$

$y = r\cos\alpha\sin\theta\sin\phi$

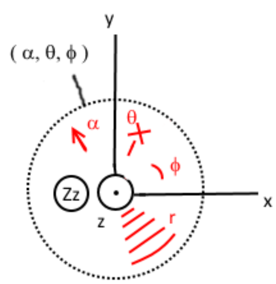

Fig. 17. 4D hypersphere

But if we want the last angle begins with the last dimension:

$z_z = r\cos\alpha$

$z = r\sin\alpha\cos\theta$

$x = r\sin\alpha\sin\theta\cos\phi$

$y = r\sin\alpha\sin\theta\sin\phi$

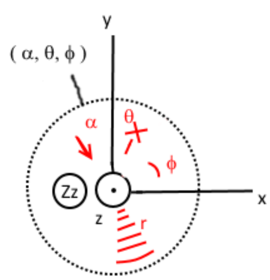

Fig. 18. 4D hypersphere

We have the hyperspherical coordinates (x, y, z, z_z) → (r, α, θ, ϕ)

$ds^2 = dr^2 + r^2\, d\alpha^2 + r^2 \sin^2\alpha\, d\theta^2 + r^2 \sin^2\alpha \sin^2\theta\, d\phi^2$

$\vec{N} = \frac{(x, y, z, z_z)}{r} = (\sin\alpha\sin\theta\cos\phi, \sin\alpha\sin\theta\sin\phi, \sin\alpha\cos\theta, \cos\alpha)$

2.1 If we make constant r → dr = 0

$$g = \begin{pmatrix} g_{22} & g_{23} & g_{24} \\ g_{32} & g_{33} & g_{34} \\ g_{42} & g_{43} & g_{44} \end{pmatrix} = \begin{pmatrix} g_{\alpha\alpha} & g_{\alpha\theta} & g_{\alpha\phi} \\ g_{\theta\alpha} & g_{\theta\theta} & g_{\theta\phi} \\ g_{\phi\alpha} & g_{\phi\theta} & g_{\phi\phi} \end{pmatrix} = \begin{pmatrix} r^2 & 0 & 0 \\ 0 & r^2 \sin^2\alpha & 0 \\ 0 & 0 & r^2 \sin^2\alpha \sin^2\theta \end{pmatrix}$$

$$l = \begin{pmatrix} l_{22} & l_{23} & l_{24} \\ l_{32} & l_{33} & l_{34} \\ l_{42} & l_{43} & l_{44} \end{pmatrix} = \begin{pmatrix} l_{\alpha\alpha} & l_{\alpha\theta} & l_{\alpha\phi} \\ l_{\theta\alpha} & l_{\theta\theta} & l_{\theta\phi} \\ l_{\phi\alpha} & l_{\phi\theta} & l_{\phi\phi} \end{pmatrix} = \begin{pmatrix} -r & 0 & 0 \\ 0 & -r\sin^2\alpha & 0 \\ 0 & 0 & -r\sin^2\alpha\sin^2\theta \end{pmatrix}$$

$K_{\text{plane }\alpha-\theta} = \frac{1}{r^2}$, $K_{\text{plane }\alpha-\phi} = \frac{1}{r^2}$, $K_{\text{plane }\theta-\phi} = \frac{1}{r^2}$

$K_{\text{plane }r-\alpha} = 0$, $K_{\text{plane }r-\theta} = 0$, $K_{\text{plane }r-\phi} = 0$

$$\text{Ricci Tensor} = \begin{pmatrix} R_{22} & R_{23} & R_{24} \\ R_{32} & R_{33} & R_{34} \\ R_{42} & R_{43} & R_{44} \end{pmatrix} = \begin{pmatrix} R_{\alpha\alpha} & R_{\alpha\theta} & R_{\alpha\phi} \\ R_{\theta\alpha} & R_{\theta\theta} & R_{\theta\phi} \\ R_{\phi\alpha} & R_{\phi\theta} & R_{\phi\phi} \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2\sin^2\alpha & 0 \\ 0 & 0 & 2\sin^2\alpha\sin^2\theta \end{pmatrix}, \qquad \text{Ricci Scalar} = \frac{6}{r^2}$$

$$\text{Einstein Tensor} = \begin{pmatrix} G_{22} & G_{23} & G_{24} \\ G_{32} & G_{33} & G_{34} \\ G_{42} & G_{43} & G_{44} \end{pmatrix} = \begin{pmatrix} G_{\alpha\alpha} & G_{\alpha\theta} & G_{\alpha\phi} \\ G_{\theta\alpha} & G_{\theta\theta} & G_{\theta\phi} \\ G_{\phi\alpha} & G_{\phi\theta} & G_{\phi\phi} \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -\sin^2\alpha & 0 \\ 0 & 0 & -\sin^2\alpha\sin^2\theta \end{pmatrix}$$

We observed that the flat coordinate and the curved coordinates could be chosen differently, as in the example of the sphere.

And as in the example of the sphere we can use spheres to determine the curvature of any pseudo-hyperspherical object, as the zoom-universe model.

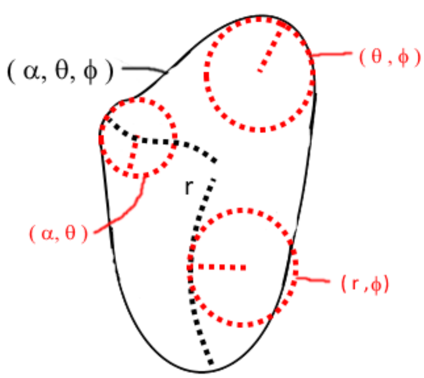

Fig. 19 Curvature 4D pseudo-hyperspherical object

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