Refutation of tensor product and Bernstein-Vazirani algorithm

We assume the apparatus and method of Meth8/VŁ4. The designated *proof* value is T. Result tables are in row-major and presented horizontally.

1. We initially evaluate the tensor product operation.

From en.wikipedia.org/wiki/Kronecker_product (relation to the abstract tensor product),

v, w, x, y are vector spaces; linear transformations are s=(v>x) and t=(w>y); and the tensor product symbol ⊗ is taken as @, to mean Not Equivalent, the XOR operator.

The abstract tensor product of linear maps is:

((s=(v>x))&(t=(w>y)))>((s@t)=((v@w)>(x@y))) ;
repeating tables as: `TTTT TTTT TTTT TTTT, ... , TTTT TTTT FFFF FFFF` (1.2)

By substitution for s and t, we rewrite Eq. 1.2.

(((v>x)@(w>y))>(((v@w)>(x@y)) ; (2.2)

We cast Eq. 2.2 into the four variable version of Meth8/VL4 for the brevity of 16-valued result tables.

LET p q r s: v, w, x, y
((p>r)&(q>s))=((p@q)>(r@s)) ; `TTTF TTFF TFTF TFFT` (3.2)

From Eq. 3.2 as rendered, the tensor product operation is *not* tautologous. This was expected because vector spaces are not bivalent but probabilistic.

2. We next evaluate the Bernstein-Vazirani algorithm in two variables.

From: Krishna, R.; Makwanay, V.; Suresh, A. (2016). "A generalization of Bernstein-Vazirani algorithm to qudit systems". arxiv.org/pdf/1609.03185.pdf

"in a tensor product of two quantum states we are free to associate the sign with whichever state we choose to. $|u\rangle \otimes(-|v\rangle) = -(|u\rangle \otimes |v\rangle) = (-|u\rangle) \otimes |v\rangle$ (4.1)

LET p q: |u⟩ ; |v⟩ ; = Equivalent; @ Not Equivalent; ~ Not

(p@~q) = (~(p@q) = (~p@q)) ; `TFFT TFFT TFFT TFFT` (4.2)

Eq. 4.2 as rendered is *not* tautologous, hence Bernstein-Vazirani is refuted.

Remark: Eq. 4.2 coerces a tautology with the Imply connective: (p@~q)>(~(p@q)=(~p@q)). However, that violates the strength of the Bernstein-Vazirani algorithm as based on the Equivalent connective. The other replacement of the Imply connective does *not* coerce a tautology: ((p@~q)=~(p@q))>(~p@q), with the result table of Eq. 4.2.