ON EPR PARADOX AND MATTERWAVE IN EUCLIDEAN RELATIVITY

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Abstract: In our previous works, we showed that the Einstein-Podolsky-Rosen (EPR) paradox could be resolved by constructing a Euclidean relativity that not only leads to the same results obtained from Einstein general relativity but also permits an instantaneous transmission of interaction. However, there still remains the question about the nature of these physical fields and their mathematical formulations if they exist. In this work we show that it is possible to formulate Euclidean relativistic field equations similar Dirac equation from a general system of linear first order partial differential equation. Since the speeds of the Euclidean relativistic fields have no upper values, they can be used to rectify the quantum entanglement in quantum mechanics.

In quantum mechanics, EPR paradox is an argument against the completeness of the quantum theory which, as required in order to be complete, must be supplemented with additional or hidden variables. And Bell's theorem is a mathematical formulation of the EPR paradox. A physical theory is complete only when it satisfies the criteria that require that a certainly predicted value of the theory must be corresponded to an element of physical reality and the theory must also satisfy the locality requirement [1,2]. However, recent results from experiments that are performed to test the Bell's inequalities seem to support the Bell's theorem which rules out local hidden variable theories with the conclusion that a theory that complies with quantum mechanics could not be Lorentz invariant [3]. In spite of the important role played by the Lorentz invariance in the current theories of mainstream physics, we noted in our previous works that Bell's conclusion in itself does not provide the ultimate answer to resolve the conceptual difficulties but instead raises the question of whether Lorentz invariance is in fact the cause of the conceptual conflict between classical and quantum mechanics [4]. In fact we showed that a special theory of relativity with a Euclidean metric that allows not only local interactions but also instantaneous interactions can be constructed. In particular, such special relativity can also be generalised to formulate a general theory of relativity that leads to the same experimental results as Einstein theory of general relativity. In order to construct a special relativistic transformation that endows spacetime with a Euclidean metric rather than a pseudo-Euclidean metric as in the case of the Lorentz transformation, we considered the following Euclidean Lorentz transformation [5]

$$x' = \gamma_E(x - \beta ct) \quad (1)$$

$$y' = y \quad (2)$$

$$z' = z \tag{3}$$

$$ct' = \gamma_E(\beta x + ct) \tag{4}$$

where $\beta = v/c$ and γ_E will be determined from the principle of relativity and the postulate of a universal speed. Instead of assuming the invariance of the Minkowski spacetime interval, if we now assume the invariance of the Euclidean interval $c^2t^2 + x^2 + y^2 + z^2$ then from the Euclidean Lorentz transformation given in Equations (1-4), we obtain the following expression for γ_E

$$\gamma_E = \frac{1}{\sqrt{1+\beta^2}} \tag{5}$$

It is seen from the expression of γ_E given in Equation (5) that there is no upper limit in the relative speed v between inertial frames. In particular we also obtained the following Euclidean relativistic energy-momentum relationship

$$E^2 = (m_0c^2)^2 - (pc)^2 \tag{6}$$

With the permission of an instantaneous transmission of interaction from the Euclidean relativity, an obvious question that arises is about the nature of these physical fields if they exist and their corresponding mathematical formulations. In this work we will assume that Euclidean relativistic physical fields exist and formulate their corresponding field equations. Even though it is possible to simply adopt Dirac method and derive Euclidean relativistic field equations based on the energy-momentum relationship given in Equation (6), in the following we will formulate Euclidean relativistic physical fields from a general system of linear first order partial differential equations. For simplicity we will also use the units in which $\hbar = 1$ and $c = 1$. A general system of linear first order partial differential equation can be written as [6,7]

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^{r}\frac{\partial\psi_i}{\partial x_j} = k_1\sum_{l=1}^{n} b_l^r\,\psi_l + k_2c^r, \qquad r = 1, 2, \dots, n \tag{7}$$

The system of equations given in Equation (7) can be rewritten in a matrix form as

$$\left(\sum_{i=1}^{n} A_i\frac{\partial}{\partial x_i}\right)\psi = k_1\sigma\psi + k_2J \tag{8}$$

where $\psi = (\psi_1, \psi_2, \dots, \psi_n)^T$, $\partial\psi/\partial x_i = (\partial\psi_1/\partial x_i, \partial\psi_2/\partial x_i, \dots, \partial\psi_n/\partial x_i)^T$, A_i, σ and J are matrices representing the quantities a_{ij}^k, b_l^r and c^r, and k_1 and k_2 are undetermined constants. Now, if we apply the operator $\sum_{i=1}^{n} A_i\frac{\partial}{\partial x_i}$ on the left on both sides of Equation (8) then we have

$$\left(\sum_{i=1}^{n} A_i \frac{\partial}{\partial x_i}\right)\left(\sum_{j=1}^{n} A_j \frac{\partial}{\partial x_j}\right)\psi = \left(\sum_{i=1}^{n} A_i \frac{\partial}{\partial x_i}\right)(k_1\sigma\psi + k_2 J) \tag{9}$$

If we assume further that the coefficients a_{ij}^k and b_l^r are constants and $A_i\sigma = \sigma A_i$, then Equation (9) can be rewritten in the following form

$$\left(\sum_{i=1}^{n} {A_i}^2 \frac{\partial^2}{\partial x_i^2} + \sum_{i=1}^{n}\sum_{j>i}^{n}\left(A_iA_j + A_jA_i\right)\frac{\partial^2}{\partial x_i \partial x_j}\right)\psi = k_1^2\sigma^2\psi + k_1k_2\sigma J + k_2\sum_{i=1}^{n} A_i \frac{\partial J}{\partial x_i} \tag{10}$$

In order for the above systems of partial differential equations to be used to describe physical phenomena, the matrices A_i must be determined. We have shown that, as in the case of Dirac and Maxwell field equations, the matrices A_i must take a form so that Equation (10) reduces to the following equation

$$\left(\sum_{i=1}^{n} {A_i}^2 \frac{\partial^2}{\partial x_i^2}\right)\psi = k_1^2\sigma^2\psi + k_1k_2\sigma J + k_2\sum_{i=1}^{n} A_i \frac{\partial J}{\partial x_i} \tag{11}$$

From the expression for the energy-momentum relationship given in Equation (6) for Euclidean relativity it is reasonable to suggest that the required field equations should be formulated so that the operators A_i satisfy the following commutation relations

$$A_iA_j + A_jA_i = 0 \qquad \text{for } i \neq j \tag{12}$$

$$A_i^2 = 1 \tag{13}$$

As shown in our previous works, the operators A_i that satisfy the commutation relations given in Equations (12) and (13) for the case $n = 4$ can be derived from the general matrix form without relying on Pauli matrices. Some of the possible forms that the operators A_i can take are Dirac matrices which are obtained from Pauli matrices [8]

$$A_0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad A_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \quad A_3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \tag{14}$$

In the following we will discuss the case in which $k_1 = m$, $k_2 = \mu$ and either $\sigma = 1$ or $\sigma = i$. If the energy-momentum relationship in Euclidean relativity given in Equation (6) is imposed then $\sigma = 1$ and in this case the system of linear first order partial differential equations given in Equation reduces to the field equations written in the matrix form

$$\left(A_0\frac{\partial}{\partial t} + A_1\frac{\partial}{\partial x} + A_2\frac{\partial}{\partial y} + A_3\frac{\partial}{\partial z}\right)\psi = m\psi + \mu J \tag{15}$$

Equation (15) written in expanded form becomes

$$\frac{\partial\psi_1}{\partial t}+\frac{\partial\psi_4}{\partial x}-i\frac{\partial\psi_4}{\partial y}+\frac{\partial\psi_3}{\partial z}=m\psi_1+\mu j_1 \tag{16}$$

$$\frac{\partial\psi_2}{\partial t}+\frac{\partial\psi_3}{\partial x}+i\frac{\partial\psi_3}{\partial y}-\frac{\partial\psi_4}{\partial z}=m\psi_2+\mu j_2 \tag{17}$$

$$-\frac{\partial\psi_3}{\partial t}+\frac{\partial\psi_2}{\partial x}-i\frac{\partial\psi_2}{\partial y}+\frac{\partial\psi_1}{\partial z}=m\psi_3+\mu j_3 \tag{18}$$

$$-\frac{\partial\psi_4}{\partial t}+\frac{\partial\psi_1}{\partial x}+i\frac{\partial\psi_1}{\partial y}-\frac{\partial\psi_2}{\partial z}=m\psi_4+\mu j_4 \tag{19}$$

It is worth discussing the appearance of the quantity J in our formulation of field equations from a general system of linear first order partial differential equations. In the case of Maxwell field equations of the electromagnetic field the quantity J is the electric current density which is set to zero for free fields. In the case of Dirac field equations for free matter waves we also have $J=0$. By comparison, it is possible to suggest that for a more complete Dirac equation that is used to describe matter wave we should add the current density J into the original Dirac equation. Therefore, there might be a current flowing inside matter wave, such as the electron, and this current may be related to the intrinsic spin angular momentum of the electron as discussed in our work on the spin angular momentum of elementary particles [9]. In the following, however, for simplicity we also assume $J=0$. In this case the wave components satisfy the following equation

$$\frac{\partial^2\psi_i}{\partial t^2}+\frac{\partial^2\psi_i}{\partial x^2}+\frac{\partial^2\psi_i}{\partial y^2}+\frac{\partial^2\psi_i}{\partial z^2}=m^2\psi_i \quad for \quad i=1,2,3,4 \tag{20}$$

Solutions to this equation can be expressed in the form

$$\psi_i=e^{-i\alpha t}\chi_i(x,y,z) \tag{21}$$

With the form of solutions given in Equation (21), Equation (20) reduces to

$$\nabla^2\chi_i-R^2\chi_i=0 \tag{22}$$

where $R^2=m^2+\alpha^2$. In this case the functions χ_i can be viewed as static Yukawa potentials

$$\chi_i(r)=\frac{g}{4\pi r}e^{-r/R} \tag{23}$$

where g is an undetermined dimensional constant [10]. It seems there is no reason why we cannot suggest that the Yukawa potential given in Equation (23) has nothing to do with the strong interaction between nuclear particles.

Now we would like to discuss further the possibility to formulate Euclidean relativistic field equations by considering the case when the system of equations given in Equations (16-19) is assumed to be derived only from the general system of linear first order partial differential equations given in Equation (8) without the need to impose the energy-momentum relationship given in Equation (6). In this case we can choose $\sigma = i$ and the system of linear first order partial differential equations given in Equation (8) reduces to the field equations written in the following matrix form

$$\left(A_0\frac{\partial}{\partial t} + A_1\frac{\partial}{\partial x} + A_2\frac{\partial}{\partial y} + A_3\frac{\partial}{\partial z}\right)\psi = im\psi + \mu J \tag{24}$$

Equation (24) written in expanded form now becomes

$$\frac{\partial\psi_1}{\partial t} + \frac{\partial\psi_4}{\partial x} - i\frac{\partial\psi_4}{\partial y} + \frac{\partial\psi_3}{\partial z} = im\psi_1 + \mu j_1 \tag{25}$$

$$\frac{\partial\psi_2}{\partial t} + \frac{\partial\psi_3}{\partial x} + i\frac{\partial\psi_3}{\partial y} - \frac{\partial\psi_4}{\partial z} = im\psi_2 + \mu j_2 \tag{26}$$

$$-\frac{\partial\psi_3}{\partial t} + \frac{\partial\psi_2}{\partial x} - i\frac{\partial\psi_2}{\partial y} + \frac{\partial\psi_1}{\partial z} = im\psi_3 + \mu j_3 \tag{27}$$

$$-\frac{\partial\psi_4}{\partial t} + \frac{\partial\psi_1}{\partial x} + i\frac{\partial\psi_1}{\partial y} - \frac{\partial\psi_2}{\partial z} = im\psi_4 + \mu j_4 \tag{28}$$

If we also assume $\mu = 0$ then the wave components ψ_i satisfy the following equation

$$\frac{\partial^2\psi_i}{\partial t^2} + \frac{\partial^2\psi_i}{\partial x^2} + \frac{\partial^2\psi_i}{\partial y^2} + \frac{\partial^2\psi_i}{\partial z^2} = -m^2\psi_i \quad for \quad i = 1, 2, 3, 4 \tag{29}$$

Solutions to this equation can be expressed in the form given in Equation (21). With this form of solutions Equation (29) reduces to

$$\nabla^2\chi_i + (m^2 - \alpha^2)\chi_i = 0 \tag{30}$$

There are three different cases that follow from Equation (30).

- The case when $m^2 - \alpha^2 = 0$

In this case Equation (30) reduces to Laplace equation $\nabla^2\chi_i = 0$ and the functions χ_i can be viewed as a static Coulomb potential given by

$$\chi_i(r) = \frac{k}{r} \tag{31}$$

- The case when $m^2 - \alpha^2 < 0$

In this case the functions χ_i can be viewed as static Yukawa potentials as discussed above for the situation when the energy-momentum relationship for Euclidean relativity is applied.

- The case when $m^2 - \alpha^2 > 0$

In this case the functions χ_i can be viewed as a stationary wave equation $\nabla^2\chi_i + k^2\chi_i = 0$, where $k^2 = m^2 - \alpha^2$. This equation is similar to Schrödinger wave equation for a free particle whose eigenfunctions in spherical polar coordinates are given in terms of Bessel and harmonic functions as $\chi_i(\mathbf{r}) = Cj_l(kr)Y_{lm}(\theta,\phi)$ [11]. These wave functions are spherical waves. It should be emphasised here that the speeds of matter waves in Euclidean relativity have no upper limits even though, as in the case of Schrödinger and Dirac formulations of wave mechanics, the speeds themselves cannot be determined. It is also worth noting the following. The system of linear first order partial differential equations given in Equations (25-28) can be rewritten in the following form

$$-\frac{\partial\psi_1}{\partial t} + im\psi_1 + \mu j_1 = \left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\psi_4 + \frac{\partial\psi_3}{\partial z} \tag{32}$$

$$-\frac{\partial\psi_2}{\partial t} + im\psi_2 + \mu j_2 = \left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\psi_3 - \frac{\partial\psi_4}{\partial z} \tag{33}$$

$$\frac{\partial\psi_3}{\partial t} + im\psi_3 + \mu j_3 = \left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)\psi_2 + \frac{\partial\psi_1}{\partial z} \tag{34}$$

$$\frac{\partial\psi_4}{\partial t} + im\psi_4 + \mu j_4 = \left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)\psi_1 - \frac{\partial\psi_2}{\partial z} \tag{35}$$

With the form of the field equations given in Equations (32-35), we may interpret that the change of the field (ψ_1,ψ_2) with respect to time generates the field (ψ_3,ψ_4), similar to the case of Maxwell field equations where the change of the electric field generates the magnetic field. With this observation it may be suggested that, like the Maxwell electromagnetic field which is composed of two essentially different physical fields, Euclidean fields of massive particles may also be viewed as being composed of two different physical fields, namely the field (ψ_1,ψ_2), which plays the role of the electric field in Maxwell field equations, and the field (ψ_1,ψ_2), which plays the role of the magnetic field. Furthermore, as shown in our previous work, the complex field equations given in Equations (32-35) can also be reformulated to form real field equations if we equate the real parts with the real parts and the imaginary parts with the imaginary parts. Like vector calculus, complex mathematics may simply be a convenient mathematical method and this procedure could be avoided if we knew in advance how to impose conditions so that real equations for massive physical fields could be derived directly from a general system of linear first order partial differential equations.

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