

(Lothar) Collatz conjecture in one variable confirmed as tautologous

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From: blogs.ams.org/matheducation/2015/05/01/famous-unsolved-math-problems-as-homework/

Given a positive integer n , if it is even, calculate $n/2$, otherwise if it is odd then calculate $3n+1$; repeat this process with the resulting value. (1.1)

We assume the apparatus and method of Meth8/VL4 modal logic model checker. Meth8 allows to mix four logical values with four analytical values. The designated *proof* value is T.

Definition	Axiom	Symbol	Name	Meaning	2-tuple	Ordinal
1	$p=p$	T	Tautology	proof	11	3
2	$p@p$	F	Contradiction	absurdum	00	0
3	$\%p>\#p$	N	Non-contingency	truthity	01	1
4	$\%p<\#p$	C	Contingency	falsity	10	2

LET: ~ Not; + Or, addition; - Not Or, subtraction; & And, multiplication; \ Not And, division; > Imply greater than; < Not Imply, less than; = Equivalent; @ Not Equivalent; # necessity, for all; % possibility, for one or some.

The proof table in 16-values in row-major and presented horizontally.

$$(p > (p @ p)) > (((p = (\%p < \#p)) > (p \ (\%p < \#p))) + ((p = (\%p > \#p)) > ((p \& (p = p)) + (\%p > \#p)))));$$

TTTT TTTT TTTT TTTT (1.2)

A more elaborate alternative proof uses two ordinal values as {0,1} to derive the four {3,0,1,2} and explicitly tests arity results based on 0 for even or 1 for odd, with iteration specified by the universal operator as applied to the single variable p in the antecedent.

$$p > \text{zero, as tautologous: } (p > ((\%p > \#p) - (\%p > \#p))) = (r=r);$$

TCTC TCTC TCTC TCTC (1.3.1)

$$p = \text{even, as tautologous: } (((p - (p \ (\%p < \#p))) \& (\%p < \#p)) = ((\%p > \#p) - (\%p > \#p))) > (p = (p \ (\%p < \#p))) = (r=r);$$

CTCT CTCT CTCT CTCT (1.3.2)

$$p = \text{odd, as tautologous: } (\sim(((p - (p \ (\%p < \#p))) \& (\%p < \#p)) = ((\%p > \#p) - (\%p > \#p)))) > (p = ((\%p > \#p) + (p \& ((\%p > \#p) + (\%p < \#p)))));$$

TTTT TTTT TTTT TTTT (1.3.3)

$$\text{all instances of } p > \text{zero: } \#(p > ((\%p > \#p) - (\%p > \#p))) > ((((p - (p \ (\%p < \#p))) \& (\%p < \#p)) = ((\%p > \#p) - (\%p > \#p))) > (p = (p \ (\%p < \#p))) + (\sim(((p - (p \ (\%p < \#p))) \& (\%p < \#p)) = ((\%p > \#p) - (\%p > \#p))) > (p = ((\%p > \#p) + (p \& ((\%p > \#p) + (\%p < \#p)))));$$

TTTT TTTT TTTT TTTT (1.3.4)

Eq. 1.2 or 1.3.5 as rendered show the conjecture is confirmed as tautologous.

This is the briefest known such proof.