

Refutation of Cantor's original continuum hypothesis via injection and binary trees

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From: Pindsle, C. (2018). "The continuum hypothesis". vixra.org/pdf/1803.0088v1.pdf

Note: Because of no email contact disclosed at that venue, that author's name is likely a pseudonym.

"[With representation using binary trees: the intention was] to prove the hypothesis in its original form as proposed by Georg Cantor in 1878: Any uncountable set of real numbers is equinumerous with \mathbb{R} . Since there is a bijection between the open interval (0,1) and the set of all the real numbers, there is a bijection between any subset of (0,1) and a subset of \mathbb{R} . Therefore it is sufficient to prove: Any uncountable subset of (0,1) is equinumerous with \mathbb{R} ."

$\phi : RJ \mapsto RJT$ is bijective: It is injective because:
 $\phi(r1) = \phi(r2) \Rightarrow (\phi(r1) \succ \phi(r2) \text{ and } \phi(r2) \succ \phi(r1)) \Rightarrow (r1 \succ r2 \text{ and } r2 \succ r1) \Rightarrow r1 = r2$ (3.5.1.)

Because the intention of the proof is to show $\phi(r1) = \phi(r2) \Rightarrow \dots \Rightarrow r1 = r2$, we rewrite Eq. 3.5.1.

$$\phi(r1) = \phi(r2) \Rightarrow r1 = r2 \tag{3.5.1.1}$$

We assume the apparatus and method of Meth8/VL4 with designated *proof* value \mathbb{T} , and contradiction value \mathbb{F} . The 16-valued result table is row-major and presented horizontally.

LET p q r: ϕ , lc_phi ; r1; r2; & And; \succ Imply, \succ , \Rightarrow ; = Equivalent to.

$$((p \& q) = (p \& r)) \succ (q = r); \quad \mathbb{T}\mathbb{T}\mathbb{F}\mathbb{T} \quad \mathbb{F}\mathbb{T}\mathbb{T}\mathbb{T} \quad \mathbb{T}\mathbb{T}\mathbb{F}\mathbb{T} \quad \mathbb{F}\mathbb{T}\mathbb{T}\mathbb{T} \tag{3.5.1.2}$$

Eq. 3.5.1.2 as rendered is *not* tautologous. Hence, the hypothesis as Eq. 3.5.1.1 fails.

This is the briefest known such refutation of Cantor's continuum conjecture.

Remark: To coerce Eq. 3.5.1.2 into tautology, we weaken the argument by replacing the Equivalent connective with the Imply connective.

$$((p \& q) \succ (p \& r)) \succ (q \succ r); \quad \mathbb{T}\mathbb{T}\mathbb{F}\mathbb{T} \quad \mathbb{T}\mathbb{T}\mathbb{T}\mathbb{T} \quad \mathbb{T}\mathbb{T}\mathbb{F}\mathbb{T} \quad \mathbb{T}\mathbb{T}\mathbb{T}\mathbb{T} \tag{3.5.1.3}$$

Eq. 3.5.1.3 does come closer to tautology with two less contradiction \mathbb{F} values, but to no avail.