Twin-Body Orbital Motion in Special Relativity

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A stellar system of two identical stars in orbital motion is chosen to manifest a physics law, conservation of momentum, in Special Relativity. Both stars move around each other in a non-circular orbit. The single gravitational force between two stars demands that total momentum of this stellar system remains constant in any inertial reference frame in which the center of mass moves at a constant velocity. The calculation of total momentum in two different inertial reference frames shows that the momentum expression from Special Relativity violates conservation of momentum.

I. INTRODUCTION

In 17th century, Issac Newton proposed a definition of force, $F = m * a$. From this definition, both kinetic energy and momentum can be derived.

In 20th century, the theory of Special Relativity[1] proposed a new definition of kinetic energy. This resulted in new definitions of momentum and force.

However, the physics law, conservation of momentum, remains intact. Any definition of kinetic energy is expected to generate a force that results in the conservation of momentum.

This paper examines the new expression of momentum from Special Relativity in an isolated stellar system which consists of two identical stars in motion. The gravitational force between stars causes both stars to move around each other in non-circular orbit. The total momentum is calculated for this stellar system in two different inertial reference frames in which the center of mass moves at a constant velocity.

The single force in this isolated system demands conservation of momentum in both reference frames.

The concept of relativistic mass becomes less popular in modern physics. Relativistic force and relativistic momentum do not share the same relativistic mass. The momentum of an object is represented by either $\gamma(v) * m(0) * v$ or $m(v) * v$. Both representations are equivalent to each other mathematically. In this paper, $\gamma(v) * m * v$ is chosen to emphasize Lorentz Factor, $\gamma(v)$, from Lorentz Transformation.

$$\frac{dm}{dv} = \frac{dm(0)}{dv} = 0 \tag{1}$$

II. PROOF

Consider three-dimensional motion.

A. Kinetic Energy and Momentum

In Special Relativity, kinetic energy K is defined as

$$K = (\gamma(v) - 1) * m * C^2 \tag{2}$$

$$\gamma(v) = \frac{1}{\sqrt{1 - \frac{v^2}{C^2}}} \tag{3}$$

The derivation of momentum from kinetic energy remains intact. Kinetic Energy K is defined as integration of force over distance.

$$K = \int F\, dx \tag{4}$$

Momentum P is defined as integration of force over time.

$$P = \int F\, dt \tag{5}$$

$$\frac{dP}{dt} = F \tag{6}$$

In Newtonian Mechanics,

$$K = \frac{1}{2} * m * v^2 \tag{7}$$

$$P = m * v \tag{8}$$

In Special Relativity,

$$K = (\gamma(v) - 1) * m * C^2 \tag{9}$$

$$P = \gamma(v) * m * v \tag{10}$$

The difference in two expressions indicates that only one expression of momentum can be correct By applying conservation of memntum to both expressions of momentum in a twin-body stellar system, the correct expression can be distinguished.

B. Twin-Body Stellar System

Two identical stars move around each other on x-y plane under a gravitational force between them. The orbit of each star is typically non-circular. Instead of making head-on collision, both stars accelerate toward and pass by each other. The single force of this isolated stellar system demands that total momentum P should remain constant.

$$\frac{dP}{dt} = 0 \tag{11}$$

C. Inertial Reference Frame

Let the center of mass be stationary in a reference frame F_1. Both stars move at the same speed but in opposite direction in F_1.

TABLE I. Velocity and Momentum in F_1

Star	Value
Velocity of star 1 S_1	is $(w_x, w_y, 0)$
Velocity of star 2 S_2	is $(-w_x, -w_y, 0)$
Newtonian momentum of S_1	is $m*(w_x, w_y, 0)$
Newtonian momentum of S_2	is $m*(-w_x, -w_y, 0)$
Relativistic momentum of S_1	is $\gamma(w)*m*(w_x, w_y, 0)$
Relativistic momentum of S_2	is $\gamma(w)*m*(-w_x, -w_y, 0)$

$$w = \sqrt{w_x^2 + w_y^2} \tag{12}$$

Let another reference frame F_2 move at a constant velocity of -V relatively to F_1 along z axis. In F_2, both stars acquire a new velocity V in z direction.

$$\frac{dV}{dt} = 0 \tag{13}$$

Their velocities become (u_x, u_y, V) and $(-u_x, -u_y, V)$ in F_2. Both u_x and u_y vary with time.

S_1 moves at the speed v_1 in F_2.

$$v_1 = \sqrt{u_x^2 + u_y^2 + V^2} \tag{14}$$

S_2 moves at the speed v_2 in F_2.

$$v_2 = \sqrt{(-u_x)^2 + (-u_y)^2 + V^2} = v_1 \tag{15}$$

v_1 varies with time since w varies with time in a non-circular orbit. For most of the time in the orbit,

$$\frac{dv_1}{dt} \neq 0 \tag{16}$$

TABLE II. Velocity and Momentum in F_2

Star	Value
Velocity of S_1	is (u_x, u_y, V)
Velocity of S_2	is $(-u_x, -u_y, V)$
Newtonian momentum of S_1	is $m*(u_x, u_y, V)$
Newtonian momentum of S_2	is $m*(-u_x, -u_y, V)$
Relativistic momentum of S_1	is $\gamma(v_1)*m*(u_x, u_y, V)$
Relativistic momentum of S_2	is $\gamma(v_2)*m*(-u_x, -u_y, V)$

D. Conservation of Momentum

The single force in this isolated stellar system demands conservation of momentum in both F_1 and F_2.

Total momentum in F_1 is zero in both Newtonian Mechanics and Special Relativity.

In Newtonian Mechanics, total momentum in F_2 is P_n. P_n remains constant.

$$P_n = m*(u_x, u_y, V) + m*(-u_x, -u_y, V) \tag{17}$$

$$= 2*m*(0, 0, V) \tag{18}$$

$$\frac{dP_n}{dt} = 2*m*(0, 0, \frac{dV}{dt}) = (0, 0, 0) \tag{19}$$

In Special Relativity, total momentum in F_2 is P_r. P_r varies with time.

$$P_r = \gamma(v_1)*m*(u_x, u_y, V) \tag{20}$$

$$+\gamma(v_2)*m*(-u_x, -u_y, V) \tag{21}$$

$$= 2*\gamma(v_1)*m*(0, 0, V) \tag{22}$$

$$\frac{dP_r}{dt} = 2*m*(0, 0, V)*\frac{d\gamma(v_1)}{dt} \tag{23}$$

$$= 2*m*(0, 0, V)*\gamma(v_1)^3*\frac{v_1}{C^2}*\frac{dv_1}{dt} \tag{24}$$

Total momentum remains constant in Newtonian Mechanics but not in Special Relativity.

III. CONCLUSION

Special Relativity violates conservation of momentum in an isolated stellar system.

Conservation of momentum fails to hold if momentum is defined as $\gamma(v)*m*v$. The failure of this physics law is due to the introduction of Lorentz factor, $\gamma(v)$, from Lorentz Transformation[8][11].

Lorentz Transformation was proposed on the assumption that the speed of light is independent of inertial reference frame.

As the result of this incorrect assumption[3], Lorentz Transformation violates Translation Symmetry[4] and Conservation of Momentum[10] in physics. Translation Symmetry requires conservation of simultaneity[5], conservation of distance[6], and conservation of time[7]. All three conservation properties are broken by Lorentz Transformation.

Therefore, Lorentz Transformation is an invalid transformation in physics. Consequently, any theory based on Lorentz Transformation is incorrect in physics. For example, Special Relativity[12][13][14].

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