Harmonic Oscillation in Special Relativity

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A physical system of a mechanical spring is chosen to manifest a physics law, conservation of momentum, in Special Relativity. Two identical objects are attached to the ends of this mechanical spring. The single force between two identical objects demands that total momentum of this physical system remains constant in any inertial reference frame in which the center of mass moves at a constant velocity. The calculation of total momentum in two different inertial reference frames shows that the momentum expression from Special Relativity violates conservation of momentum.

I. INTRODUCTION

In 20th century, the theory of Special Relativity[1] proposed a new definition of kinetic energy. This resulted in new definitions of both momentum and force.

However, the physics law, conservation of momentum, remains intact. Any definition of kinetic energy is expected to generate a force that results in the conservation of momentum.

This paper examines the new expression of momentum from Special Relativity in an isolated mechanical system which consists of two identical objects attached to the ends of a mechanical spring. The force of the mechanical spring causes both objects to oscillate. The total momentum is calculated for this physical system in two different inertial reference frames in which the center of mass moves at a constant velocity.

The single force in this isolated system demands conservation of momentum in both reference frames.

The concept of relativistic mass becomes less popular in modern physics. Relativistic force and relativistic momentum do not share the same relativistic mass. The momentum of an object is represented by either $\gamma(v) * m(0) * v$ or $m(v) * v$. Both representations are equivalent to each other mathematically. In this paper, $\gamma(v) * m * v$ is chosen to emphasize Lorentz Factor, $\gamma(v)$, from Lorentz Transformation.

$$\frac{dm}{dv} = \frac{dm(0)}{dv} = 0 \tag{1}$$

II. PROOF

Consider two-dimensional motion.

A. Kinetic Energy and Momentum

In Special Relativity, kinetic energy K is defined as

$$K = (\gamma(v) - 1) * m * C^2 \tag{2}$$

$$\gamma(v) = \frac{1}{\sqrt{1 - \frac{v^2}{C^2}}} \tag{3}$$

The derivation of momentum from kinetic energy remains intact. Kinetic Energy K is defined as integration of force over distance.

$$K = \int F\,dx \tag{4}$$

Momentum P is defined as integration of force over time.

$$P = \int F\,dt \tag{5}$$

$$\frac{dP}{dt} = F \tag{6}$$

In Newtonian Mechanics,

$$K = \frac{1}{2} * m * v^2 \tag{7}$$

$$P = m * v \tag{8}$$

In Special Relativity,

$$K = (\gamma(v) - 1) * m * C^2 \tag{9}$$

$$P = \gamma(v) * m * v \tag{10}$$

The difference in two expressions indicates that only one expression of momentum can be correct By applying conservation of memntum to both expressions of momentum in a physical system such as a mechanical spring, the correct expression can be distinguished.

B. Mechanical Spring

Two identical objects move along y axis under a mechanical force between them along y axis. One object is attached to each end of a mechanical spring. In order to simplify calculation, the mass of the mechanical spring is chosen to be infinitely small compared to the mass of each object. The contribution from the spring to total momentum is ignored. The single force of this isolated physical system demands that total momentum P should remain constant.

$$\frac{dP}{dt} = 0 \tag{11}$$

C. Reference Frame

Let the center of mass be stationary in a reference frame F_1. Both objects move at the same speed u' but in opposite direction along y axis in F_1.

$$\frac{du'}{dt'} = a'(t') \tag{12}$$

$a'(t')$ is the acceleration and varies with time.

TABLE I. Velocity and Momentum in F_1

object	**Value**
Velocity of object 1 O_1	is $(0, u')$
Velocity of object 2 O_2	is $(0, -u')$
Newtonian momentum of O_1	is $m * (0, u')$
Newtonian momentum of O_2	is $m * (0, -u')$
Relativistic momentum of O_1	is $\gamma(u') * m * (0, u')$
Relativistic momentum of O_2	is $\gamma(-u') * m * (0, -u')$

Let another reference frame F_2 move at a constant velocity of -V relatively to F_1 along x axis. In F_2, both objects acquire a new velocity V in x direction.

$$\frac{dV}{dt} = 0 \tag{13}$$

Their velocity in y direction become u and -u. The mechanical force oscillates both objects toward and away from each other.

$$\frac{du}{dt} = a(t) \tag{14}$$

$a(t)$ is the acceleration and varies with time.

O_1 moves at the speed v_1 in F_2.

$$v_1 = \sqrt{V^2 + u^2} \tag{15}$$

O_2 moves at the speed v_2 in F_2.

$$v_2 = \sqrt{V^2 + (-u)^2} = v_1 \tag{16}$$

TABLE II. Velocity and Momentum in F_2

object	**Value**
Velocity of O_1	is (V, u)
Velocity of O_2	is $(V, -u)$
Newtonian momentum of O_1	is $m * (V, u)$
Newtonian momentum of O_2	is $m * (V, -u)$
Relativistic momentum of O_1	is $\gamma(v_1) * m * (V, u)$
Relativistic momentum of O_2	is $\gamma(v_2) * m * (V, -u)$

D. Conservation of Momentum

The single force in this isolated gravitational system demands conservation of momentum in both F_1 and F_2.

In F_1, total momentum is zero in both Newtonian Mechanics and Special Relativity.

In Newtonian Mechanics, total momentum in F_2 is P_n. P_n remains constant.

$$P_n = m * (V, u) + m * (V, -u) = 2 * m * (V, 0) \tag{17}$$

$$\frac{dP_n}{dt} = 2 * m * (\frac{dV}{dt}, 0) = (0, 0) \tag{18}$$

In Special Relativity, total momentum in F_2 is P_r. P_r varies with time.

$$P_r = \gamma(v_1) * m * (V, u) + \gamma(v_2) * m * (V, -u) \tag{19}$$

$$= 2 * \gamma(v_1) * m * (V, 0) \tag{20}$$

$$\frac{dP_r}{dt} = 2 * m * (V, 0) * \frac{d\gamma(v_1)}{dt} \tag{21}$$

$$= 2 * m * (V, 0) * \gamma(v_1)^3 * \frac{v_1}{C^2} * \frac{dv_1}{dt} \tag{22}$$

$$= 2 * m * (V, 0) * \gamma(v_1)^3 * \frac{u}{C^2} * a(t) \tag{23}$$

Total momentum remains constant in Newtonian Mechanics but not in Special Relativity.

III. CONCLUSION

Special Relativity violates conservation of momentum in an isolated mechanical system.

Conservation of momentum fails to hold if momentum is defined as $\gamma(v) * m * v$. The failure of this physics law is due to the introduction of Lorentz factor, $\gamma(v)$, from Lorentz Transformation[8][11].

Lorentz Transformation was proposed on the assumption that the speed of light is independent of inertial reference frame.

As the result of this incorrect assumption[3], Lorentz Transformation violates Translation Symmetry[4] and Conservation of Momentum[10] in physics. Translation Symmetry requires conservation of simultaneity[5], conservation of distance[6], and conservation of time[7]. All three conservation properties are broken by Lorentz Transformation.

Therefore, Lorentz Transformation is an invalid transformation in physics. Consequently, any theory based on Lorentz Transformation is incorrect in physics. For example, Special Relativity.

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