An autotautological description of the world

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We report the discovery of a tautological construction that is autological to the world. The autological property allows the derivation of the laws of physics from first principles as properties of the construction. As such, we conclude that the world admits a necessarily implied and self-explaining description of itself. We first explain and derive the construction, then we explicitly recover the major laws of physics as properties of the construction, including: special relativity, general relativity, dark energy, the Schrödinger equation, the Dirac equation, the arrow of time, quantum field theory, the speed of light as a maximal speed and the space-time background.

1 *Introduction*

The characteristics of *self-explanation* and *necessary implication* are logically supported respectively by the autological and the tautological properties of the construction. We may call a description of the world having both properties; an autotautological theory. Let us first define the word autological.

Definition 1.1 (Autological)**.** *From linguistics, we have the following preexisting definitions:*

- *(Linguistics) Of a word, especially an adjective: having or representing the property it denotes. Opposed to "heterological".*[1]
- *(Linguistics) An autological word (also called homological word or autonym) is a word that expresses a property that it also possesses (e.g. the word "short" is short, "noun" is a noun, "English" is English, "pentasyllabic" has five syllables, "word" is a word). The opposite is a heterological word, one that does not apply to itself (e.g. "long" is not long, "monosyllabic" has five syllables).*[2]

These definitions of autological are used in logic and philosophy and they apply to linguistics. Applied to a description of a physical system, we would say that it is autological if:

- *(Physics) The description exhibits the same properties (e.g. limits, symmetries, etc.) as the system it describes.*

For a description applicable to the world itself, the laws of physics will be shown to emerge as a consequence of its own description.

[1] –Source: `https://en.oxforddictionaries.com/definition/autological`

[2] –Source: `https://en.wikipedia.org/wiki/Autological_word`

This is as fundamental as it gets. In a loose epistemological sense, "*the description is reality* and *reality is the description*". Thus, it is only natural that the laws of physics are emergent from the description as they would be from reality.

This remarkable property is the first of two core requirements of a description of the world that is necessarily implied and self-explaining. The second requirement explains why the description is applicable to the whole world. The construction is also tautological and universal[3]. With these two properties, the full circle is completed. The question: "Why is the world what it is?" has for short answer: its laws are the autological consequence of its tautological and universal description.

[3] in the Turing sense of universal

We will review these terms in great detail. However, before we do so, let us first review the prior art, then show how the autotautological method differs from it.

2 *Prior art*

The construction of a modern physical theory usually goes as follows. First, a background to reality is assumed. Usually this is taken to be space-time; one dimension of time and three or more dimensions of space. Then, symmetries are associated to this space-time. As per Noether's theorem, each symmetry gives rise to a conserved quantity which becomes a law of physics. These symmetries, along with potentially complicated initial conditions, are responsible for describing the state of the universe over time (deciding the facts).

Figure 1: The logical flowchart of a modern theory of physics.

$$\begin{bmatrix} \text{Background } \textit{(Space-time)} \\ \Downarrow \\ \text{Symmetries } \textit{(Lorentz, Poincaré, Gauge, ...)} \\ \Downarrow \\ \text{Laws } \textit{(Special relativity, general relativity, ...)} \end{bmatrix} \tag{2.1}$$

$$+$$

$$\text{Initial conditions} \tag{2.2}$$

$$\Downarrow$$

$$\text{Facts } \textit{(There is milk in the fridge)} \tag{2.3}$$

Here, the background is taken as axiomatic (assumed to be true but not proven from first principles). The background leads to symmetries (some of which are taken as axiomatic) and those symmetries leads to laws. The laws plus the initial conditions decide the facts of the universe over time.

As appealing as this picture might be, it is quite difficult to probe the universe on all interesting scales so as to identify all symmetries. We have found that it is ultimately *significantly easier* to rethink the process via an autological description. Let us introduce the starting point of the autotautological theory, then we will rearrange the logical flowchart appropriately.

3 *Starting point*

We consider an *idealist observer* pondering about the world. For short, let's call this observer *Alice*. We ask: "given no experimental knowledge of any kind, can Alice correctly describe the world that embeds her?" To illustrate, we suppose that Alice believes that her physical senses (eyes, ears, touch, etc.) are unreliable. As such, she would not trust the process of falsification. Thus, acting rationally, she should only believe in logical conclusions that are entirely irrefutable.

To answer the question rigorously, we will consider the problem only in the context of *formal systems*. Let us review the definition of a formal system.

Definition 3.1 (Formal system)**.** *A formal system is any well-defined system of abstract thought based on the model of mathematics.*[4] *In formal logic, a formal system consists of a formal language and a set of inference rules, used to derive (to conclude) an expression from one or more other premises that are antecedently supposed (axioms) or derived (theorems). The axioms and rules may be called a deductive apparatus. A formal system may be formulated and studied for its intrinsic properties, or it may be intended as a description (i.e. a model) of external phenomena.*[5]

[4] –Source: `https://en.wikipedia.org/wiki/Formal_system`

[5] –Source: `https://math.stackexchange.com/questions/16917/formal-system-and-formal-logical-system`

Within these restrictions, Alice can still find a correct description of the world by producing a purely mathematical construction that has the autotautological property. Indeed, autotautology is quite possibly her only way to do so. To introduce the autotautological construction, let us compare it to its scientific analogue:

Scientific	*Autotautological*	
Experimental evidence	Tautological statements	(3.2)
$\Downarrow$	$\Downarrow$	
Falsifiable theory explaining all known evidence	Autotautological theory resulting from a universal group of tautological statements	(3.3)
	$\downarrow$	
	Optionally, falsifiability is recovered by corroborating the theory to experimental evidence	(3.4)

The correctness of the autotautological derivation most directly depends upon the completeness of the group of tautological statements. The theory is autological only if the group of tautologies is universal. Essentially, we have found a way to construct a minimalist formal system which is both tautological and universal. The formal system is obtained by stripping first order logic of its deductive apparatus. We named the formal system *Miniversal logic* and we give its construction in section 4.3. The following tautological statements:

$$tautology_1 := [(deductive\text{-}apparatus)_1 \implies fact_1] \tag{3.5}$$

$$tautology_2 := [(deductive\text{-}apparatus)_2 \implies fact_2] \tag{3.6}$$

$$\vdots$$

$$tautology_n := [(deductive\text{-}apparatus)_n \implies fact_n] \tag{3.7}$$

$$\vdots$$

are its theorems. Each tautology is simply a claim that a certain deductive apparatus is able to prove a certain fact. As the list contains all facts for all deductive apparatus, the formal system of Miniversal logic is actually a universal function in disguise.

Special care must be taken when constructing the system and when listing the tautologies. Notably, we will have to sacrifice the philosophical notion of a *synthetic statement* and thus, must take the position that all facts are provable within and up to some deductive apparatus (e.i. are analytical). We discuss this in section 4.7. We can now adopt a working definition of a *possible world*.

Definition 3.8 (Possible world)**.** *A possible world is a world in which each of its fact is an element of the set of the tautologies of Miniversal logic. Thus, the tautologies of Miniversal logic restrict what the facts of any possible world can be; they are the analytical facts (the permissible facts).*

Which world then is the actual world? By using notions of statistical physics, we found that the actual world corresponds to a statistically emergent behavior over the ensemble of all analytical facts. In section 5 we explicitly derive the actual world by constructing such a statistical ensemble. Then in part II, we show how the laws of physics are emergent from the ensemble. Since the result is statistical, we expect fluctuations around the average. These fluctuations connect to the quantum laws of physics. Indeed, this is how we recover the Schrödinger equation, the Dirac equation and the Feynman path integral formulation of quantum field theory. We adopt the definition of the actual word as:

Definition 3.9 (Actual world)**.** *The actual world is the result of a statistically emergent behavior over the ensemble of all analytical facts. The construction of an ensemble of analytical facts using the tools of statistical physics produces a partition function from which the laws of physics are emergent.*

The general interpretation of the derivation is explained in the context of statistical physics. Let's take the typical example of statistical physics; the ideal gas. In the ideal gas scenario, we construct a partition function describing the energy of the molecules of a gas. Then, when the number of such particles is very large (a.k.a in the thermodynamic limit), we recover the ideal gas as a macroscopic law emergent from the microscopic behavior of the molecules.

The interpretation presented here is similar. We consider all possible tautologies of Miniversal logic as the possible facts of the world. Here, these analytical facts take the role of the "microscopic" description of the world. Then, using statistical physics, we recover the "macroscopic" laws emergent from the properties of these analytical facts. As the list of fact applies to the entire world, the laws that emerge also apply to the whole universe.

The derivation of the laws of physics is not a mere coincidence but a direct result of the autological property of the construction (part 4 and 6). Thus, the main premise of the paper is to show the following:

$$\begin{aligned}\left(\textit{tautological} \land \textit{universal}\right) \implies \textit{autotautological} \implies \textit{statistical-physics} \implies \big(&\textit{special-relativity} \\ &\land \textit{general-relativity} \land \textit{dark-energy} \\ &\land \textit{arrow-of-time} \land \textit{Schrödinger-equation} \\ &\land \textit{Dirac-equation} \land \textit{quantum-field-theory} \\ &\land \cdots\big)\end{aligned} \tag{3.10}$$

Part I

Proof

The main steps of the proof are:

- Step 1 - From formal systems to all analytical facts (section 4)

  Specifically;

  1. We will derive Miniversal logic.
  2. We will list all analytical facts as tautologies of Miniversal logic.
  3. We will explain in more detail the relation

$$\left(\textit{tautological} \wedge \textit{universal} \right) \implies \textit{autotautological} \tag{3.11}$$

- Step 2 - Properties emergent from analytical facts (section 6)
  Specifically:

  1. We will derive *feasible mathematics* as the connection between the logical verification of analytical facts and statistical physics.
  2. We will pose a mapping between the quantities of the statistical ensemble and the physical quantities.

Furthermore, we are also looking for the precise mathematical formulation of the autotautological physical theory. We will find that the function Z,

$$Z = \sum_{q \in \mathcal{Q}} e^{-Fx(q) - Wt(q)} \tag{3.12}$$

obtained as the conclusion to this proof, is the mathematical expression of the autotautological construction. It describes a statistical ensemble of analytical facts q from within the set $\mathcal{Q}$ of all possible analytical facts. Each possible analytical fact is a micro-state of the statistical ensemble. Furthermore, each is statistically weighted by the length of its proof $t(q)$ and by the length of its description $x(q)$. The function

$$p(q) = \frac{1}{Z} e^{-Fx(q) - Wt(q)} \tag{3.13}$$

is the probability that the analytical fact $q \in \mathcal{Q}$ is actual in the universe. We will show that this construction is autological and tautologically-implied, and thus, as we will argue, it provides a self-explaining and necessarily implied description of the world. In part II, we will derive the laws of physics from it.

Remark: To model a world with more than one fact, as Z is a micro-canonical ensemble, it suffices to consider N systems described by Z - one for each fact of the world. Alternatively, it can be extended to a Grand canonical ensemble by adding the usual μN thermodynamic conjugate-pair. In this case, we obtain a world with a variable amount of analytical facts for a cost per fact of μ. We found however that the micro-canonical representation was sufficient to recover the laws of physics. This is simply because, as implied by Miniversal logic all of its facts are logically independent, suggesting that $\mu = 0$. Thus, in this manuscript, we will only study Z as micro-canonical whilst keeping in mind that it can be extended to N independent systems without changing the laws derived from it. For $N = 1$, we interpret each emergent law as a limit applicable to a fact $q \in \mathcal{Q}$. Since all facts will be in $\mathcal{Q}$, the emergent laws applies to all facts.

4 Step 1 - From formal systems to all analytical facts

In step 1;

1. We will derive Miniversal logic.
2. We will list all analytical facts as tautologies of Miniversal logic.
3. We will explain in more detail the relation

$$\big(\textit{tautological} \wedge \textit{universal}\big) \implies \textit{autotautological} \tag{4.1}$$

4.1 Facts and definitions

Consider a

Definition 4.2 (Brute Fact)**.** *A statement of a language which is true without formal justification. In physics, the axioms of the eventual theory of everything along with any possible initial conditions would generally be considered to be the brute facts of reality. "Brute facts are not justified by any other more fundamental principles."*

versus a

Definition 4.3 (Logically verifiable Fact)**.** *A statement of a language which is true within a formal theory because it is a theorem of said theory.*

Finding the brute facts of the universe is generally thought to be the end goal of physics. However, it is not the end goal of philosophy. Indeed, if reality is contingent upon brute facts, then an irreducible philosophical gap appears. As it cannot explain its brute facts, such a reality cannot explain why it exists the way it does (versus any other alternative) or why it exists over nothing.

Of course and as a result, an autotautological physical theory cannot be contingent upon brute facts, or it would cease to be tautological. The key to successfully construct an autotautological theory is to avoid importing any brute facts while maintaining the universality of the theory. This can be done by importing all theorems for all formal theories as tautologies of a minimalistic formal theory. We will perform a preliminary philosophical exercise to solidify the intuition, then we will show how it can be done rigorously.

4.2 *The universal doubt method of Descartes*

We will recall the philosophy of René Descartes (1596-1650), the famous 17th century French philosopher most well-known for his derivation of *cogito ergo sum* - I think, therefore I am. As we will soon see, we can guarantee the elimination of all brute facts when we modernize his universal doubt method into a formal logic system such as first order logic. But first, let us recall what the universal doubt method is.

Descartes' main idea was to come up with a test that every statement must pass before it will be accepted as true. The test will be the strictest test imaginable. Any reason to doubt a statement will be a sufficient reason to reject it. Then, any statement which survives the test will be considered irrefutable.

Using this test, and for a few years, Descartes rejected every statement he considered. The laws and customs of society, as they have no logical justifications, are obviously the first to be rejected. Then, he rejects any information that he collects with his senses; vision, taste, hearing, etc, on the grounds that a "demon" (think hallucinogens) could trick his senses without him knowing. He also rejects the theorems of mathematics on the grounds that axioms are required to derive them, and such axioms could be wrong. For a while, his efforts were fruitless and he doubted if he would ever find an irrefutable statement.

But, eureka! He finally found one which he published in 1641. He doubts of things! The logic goes that if he doubts of everything, then it must be true that he doubts. Furthermore, to doubt he must think and to think, he must exist (at least as a thinking being). Hence, cogito ergo sum, or I think, therefore I am.

4.3 *Miniversal logic*

We now refocus our interest to Descartes' universal doubt method itself and not so much in the cogito. To identify the theorems of the universe, we will repeat Descartes' universal doubt method within the context of a formal logic system. The method will produce a *minimal* set of rules whose theorems are the theorems of the *universe* - hence the name *Miniversal logic*.

Miniversal logic is, in many ways, similar to the constructivism project in mathematics but taken to the extreme. We select first-order logic as our starting point. Then, as we do not know which axioms are the true axioms of the universe, we remove all formal axioms from first-order logic on the ground that they carry doubt. Then, we further remove all *rules of inference* with the exception of the *rule of deduction*. This method parallels Descartes' universal doubt method within first order logic. The main argument is that if we remove from first order logic all formal axioms and all rules of inference which could potentially be controversial, then whatever theorem is left will surely be irrefutable. The result is a system of logic which, essentially, does not deceive its user.

Like Descartes with the cogito, we will also obtain statements that cannot be doubted of, but since we have formalized Descartes' method within first-order logic, the irrefutable statements obtained will be logic statements and are therefore mathematically usable. Specifically, the irrefutable statements obtained are the theorems of Miniversal logic.

To write sentences in a clear and unambiguous manner, Miniversal logic preserves the syntax of first order logic but does not retain its rules of inference (with the exception of the rule of deduction). As only the *rule of deduction* remains, let us recall its definition.

Definition 4.4 (Rule of deduction)**.** *The rule of deduction formalize the idea that proving a theorem using a set of assumptions is valid within these assumptions. It shows that if by assuming A one can show that $A \vdash B$, then $A \to B$ is a theorem of the logical system. It is often considered one of the most obvious rule of inference of logic, as without it we cannot extend a logical system with new axioms/assumptions. Using the rule of deduction, we can start from seemingly nothing and rebuild any of the familiar logic systems such as Peano's axioms (PA) or set theory (ZFC) by assuming their axioms.*

Why keep the rule of deduction? For the simple reason that using it does not introduce doubt but removing it would. It is the only standard rule that has this property. For example lets consider another rule, say the rule of excluded middle. Adopting this rule in the foundation of the theory would increase doubt as it is impossible to determine a-priori if this is a valid rule of the universe or not. However, introducing it by first appealing to the rule of deduction would be fine. Indeed, in the latter case we would say "if we *assume the rule of excluded middle* (via the rule of deduction), then we can prove by contradiction, for example, that $\sqrt{2}$ is irrational". It only affects the branch of the tree under which it is assumed and not the whole system.

In Miniversal logic, no theorem stands on its own. Any theorem must include, within its description, the list of assumptions that are required to prove it. The user of Miniversal logic is always reminded that the theorems that he proves are of the form 'Assume A, then A proves C'. Hence, by the rule of deduction, $A \to C$ is a theorem, but C by itself never is. Miniversal logic can be interpreted as the starting point of all logical work - it is the state of mind a logician is in before having morning coffee and selecting a specific system of axioms to work with. As a result and compared to other logic systems, it more accurately represents reality as it reflects the full freedom available to the logician to select any set of axioms prior to working.

4.4 Discussion on metaphysics

To solidify the intuition, let us have a small conversation on metaphysics. The goal of this section is to construct a bridge from metaphysics to physics. The completion of the exercise will identify all theorems of the universe. To iron out the subtleties we will present, in the long standing tradition of philosophy, an hypothetical dialogue about the thesis. The dialogue is based on a number of real conversations[6] which has been edited and combined to remove repetitions, to accelerate the flow and to help illustrate the points being made.

[6] Specifically, when Bob's dialogue is taken verbatim from a conversation with Toid Boigler, it will be side-noted with the initials TB.

> Bob: - *I believe in empiricism. To derive the laws of physics, one must make observations. Without these observations, there is no way to know which of many possible worlds is the actual world. For example, is the geometry of the universe euclidean or hyperbolic? Is the speed of light maximal? Does the microscopic world obeys the Schrödinger equation? Etc. Pure reason alone cannot prove these to be actual. Only continual observations followed by refinements or falsifications can improve our degree of confidence in a scientific theory.*

I understand your point of view, but I believe I have found a bridge between metaphysics and physics that allows one to obtain irrefutable knowledge about the universe. I will try to explain the bridge from the following angle. First, lets assume that the cogito is true: *I think, therefore I exist*. Do you agree with the cogito?

> Bob: - *Yes.*

Then, for I to exist, the universe must be *restricted* in some way. At the very least, it must be such that it does not contradict the existence of thought. We have now essentially reformulated the anthropic principle as an extension to the cogito. I think, therefore I exist - *and to exist, I must actually exist in a universe capable of supporting such existence.* Would you agree that this argument rules out some universes?

Bob: - *Fair enough, yes - it rules out the [...] universes incompatible with the existence of thought.[...]*[7]

[7] TB

OK. From that, we already have a slight connection between metaphysics and physics. An argument from pure reason, the cogito extended with the anthropic principle, can be used to place restrictions on what the universe can be. As it contains very little information, the restriction is very loose, but it is nonetheless a restriction.

Bob: - *I agree that the anthropic principle rules out universes that are not capable of producing an observer. But, a scientific theory should make precise and falsifiable predictions and the anthropic principle is not sufficiently specific for that.*

Now we enter the core of the argument. We will use Miniversal logic to improve the specificity of the anthropic principle. Each theorem of Miniversal logic that we can provide will serve to further restrict what the universe is. For example, using my mind I can prove the sentence "PA implies that two plus two equals four", and since my mind is in the universe, then the sentence must be a theorem of both my mind and the universe. This is how we find the theorems applicable to the universe. We have now restricted the universe by two statements instead of one. So the previously poorly defined bound is now slightly better defined. Agree, or disagree?

Bob: *Well, you want the phrase "theorem of the universe" to be telling us something about the physical world; to put it in your own words, "...this is how we bridge metaphysics to physics." But how does this work? If "true in the universe" just means provable in PA or ZFC or whatever (as you seem to have just said), how does this provide any link with physical reality at all?* [8]

[8] TB

Hold on, it appears that you have missed a subtlety. "Provable in the universe" means provable in the Miniversal logic system I defined earlier. If you use another logic system than Miniversal logic (such as PA) then the argument does not work. If you use PA or ZFC, then the theorems rely on the axioms PA or ZFC. As the universe might have other axioms than PA or ZFC, we cannot prove that PA's or ZFC's theorems are indeed the theorems of the universe. However, Miniversal logic teaches us that the theorems of the universe are not "two plus two equals four", they are "Assume PA, then two plus two equals four". The "Assume PA" prefix is what the subtlety is all about. "Assume PA, then 2+2=4" is a theorem of the universe because, it is true that in the universe, if you assume PA you can prove (within PA) that 2+2=4. You can easily do the exercise in your head to prove that it can be done in the universe.

- Bob: *OK, so you want to think of all mathematical proofs as conditional - if certain axioms hold, then certain consequences follow. Fine. How does that provide any connection with physics or the physical world?*[9]

[9] TB

Well yes, mathematical proofs that are explicitly conditional on assumptions derived exclusively from the rule of deduction are theorems of the universe. Whereas those that do not meet this condition are theorems of their respective logic system. For example, "2+2=4" is a theorem of Peano's axioms. But, "Assume PA, then 2+2=4" is a theorem of the universe. So all worlds where "Assume PA, then 2+2=4" is not true are ruled out.

> - Bob: *This is one point where I am a little confused. Pure logic (call it [Miniversal] logic if you want) guarantees that PA implies 2+2=4. So it's hard to see what worlds it rules out - unless you mean worlds in which there is a mind, but that [this] mind is too [primitive] to realize that PA implies 2+2=4. Is that the kind of world that you take to have been ruled out? If so, I am OK with what you have said.*[10]

[10] TB

Yes - that is part of what I am ruling out. Generally speaking, I am ruling out any world which does not embed *universal reason*. I also rule out worlds for which logic would be incomplete and worlds which would contradict logic by say, letting you prove any theorems regardless of the axioms that you assume.

Since our mind is able, in principle, to explore all branches of Miniversal logic and since the universe must embed our mind, we can precisely identify all the theorems of the universe: The ultimate theory which describes the universe must have, as its theorems, all theorems of Miniversal logic.

> Bob: *Here I really don't know what you mean, unless you are just saying that there are no 'violations' of [Miniversal] logic in the world. If that's what you mean, I'm happy with that claim.*[11]

[11] TB

I am indeed claiming that there are no violations of Miniversal logic in the universe, but I am also claiming something additional. What I am claiming is that we can use Miniversal logic and the anthropic principle to completely restrict reality to a single solution. Think of it as the anthropic principle on steroids.

Consider the following; each theorem of Miniversal logic that we supply can be used to restrict the universe further. In principle, we can supply arbitrarily many theorems. PA has "2+2=4" as a theorem, but it also has "2+3=5", etc. Then, ZFC also has infinitely many theorems as well. If we keep supplying theorems, we will eventually supply all theorems for all branches of Miniversal logic[12]. Furthermore as Miniversal logic is universal, all possible theorems for all possible sets of assumptions will eventually be supplied. No patches of theorems will be left out by the process.

[12] To avoid hanging on non-provable sentences, we will have to work in dovetail.

As a result, we will have maximally restricted what the universe can be. Indeed, the universe cannot be simpler than Miniversal logic because that would mean leaving a theorem out (but we already said the work will eventually supply all possible theorems so none can be left out). What about complexity - can the universe be more complex then Miniversal logic? The universe cannot be more complex than Miniversal logic either because that would mean the universe has theorems that Miniversal logic hasn't (but this cannot be the case because Miniversal logic already embeds all possible theorems within its branches).

Therefore, as the universe is restricted both from the perspective of increasing its complexity as well as reducing it, the bound cannot be improved furthermore. The method herein described fully restricts the universe to a single solution.

> Bob: *I am not sure [I see where you are going with this]. I'm happy to say that the universe must allow for the possibility of a mind that, in principle, can verify all the theorems of [Miniversal] logic. But what follows from that?*[13]

[13] TB

Usually a theory is first defined by a set of axioms, then the theorems follows from them. In our present situation, we have a list of theorems but we do not have the theory which neatly explains such theorems. We will use a meta-theory such as first-order arithmetic or set theory to study Miniversal logic. The process is somewhat reminiscent of how we have invented and are now using mathematics to describe the physical universe we live in.

> Bob: *[What I mean is that] I don't understand [the connection to physics] at all. What we have now are all the tautologies of [Miniversal] logic. What connection is there between that and a physical theory?*[14]

[14] TB

The connection is that, for the reasons stated, the theorems of Miniversal logic are the theorems of the universe. Hence Miniversal logic, as its theorems are identical to those of the universe, must fully describe the universe.

> Bob: *You say "The theorems of [Miniversal] logic are theorems of the universe.". If by this you just mean that the universe obeys the laws of [Miniversal] logic, then yes, I agree. Then you say "Hence Miniversal logic, as its theorems are identical to those of the universe, must fully describe the universe." This seems clearly wrong. It is true in the universe that there [is the law of gravity]. That there [is the law of gravity] is, however, not a theorem of [Miniversal] logic. Thus, the theorems of [Miniversal] logic do not fully describe the universe.*[15]

[15] TB

There is a misunderstanding. I am not claiming that the laws of the universe can be found within Miniversal logic under a certain set of assumptions. What I am claiming is that Miniversal logic is autological; e.g. it possess the properties of reality for the following reason:

$$\textit{(tautological)} \wedge \textit{(universal)} \implies \textit{(autological)} \tag{4.5}$$

Thus, studying Miniversal logic with a meta-theory is equivalent to trying to make sense of the universe using mathematics. Except, as the facts of Miniversal logic are precisely defined and listed, correctly deriving the proper laws and symmetries from them will be easier then it is in the physical case.

> Bob: *Can you spell out the [connection] you have in mind [between the tautologies of Miniversal logic and the laws of physics]?*[16]

[16] TB

Yes, let us explicitly enumerate the facts of reality, then we will be ready for step 2.

4.5 Enumeration of all facts

We can make this rigorous using the standard tools of formal logic. Let us do a recap of formal logic.

Definition 4.6 (Symbol). *For our purposes, a symbol is a reproducible mark or shape that can be distinguished from other marks or shapes. For example* 0 *and* 1 *are the symbols of the binary language.*

Symbols can be grouped as

Definition 4.7 (Sentences). *A sentence is a group of symbols written one after the other. For completeness, we consider groups of one symbol to also be sentences. The absence of symbols will be the empty sentence ϵ. A sentence is of finite length. For example* 000110 *and* 111 *are sentences of the binary language.*

With these primitive notions defined, a notion of truth can be imported into a formal theory. For this, a list of rules is defined:

Definition 4.8 (Rules of inference). *Typically, a rule of inference will be truth-preserving. A rule of inference takes sentences as input, then produces a conclusion. For example, $(p \wedge q) \implies p$ is a rule of inference in propositional logic.*

Definition 4.9 (Axioms). *A sentence that is considered true by definition is an axiom.*

Definition 4.10 (Theorem). *A sentence that is proven to be true as a result of a valid proof is a theorem. A valid proof is obtained by applying in succession either an axiom or a rule of inference to the previous line until the sentence is recovered.*

Definition 4.11 (Logical truth)**.** *A sentence that is proven to be true as a result of the application of the rules of inference of the formal theory, and with no application of the axioms. Thus, a logical truth is a specific kind of theorem. In philosophy, logical truths are also called analytic truth and are valid for all worlds. By contrast, axiom-dependent theorems are only true for a certain world (e.i. for a certain set of axioms).*

Within the context of formal logic, we can think of the previously introduced categories of facts as follows:

$$\textit{Brute-facts} \Leftrightarrow \textit{Axioms} \tag{4.12}$$

$$\textit{Logically-verifiable-facts} \Leftrightarrow \textit{Theorems} \tag{4.13}$$

We can then convert theorems to logical truths using Miniversal logic as follows;

$$\textit{logical-truth}_1 := [\{\textit{rules-of-inference}\}_1 \cup \{\textit{axioms}\}_1 \implies \textit{theorem}_1] \tag{4.14}$$

$$\textit{logical-truth}_2 := [\{\textit{rules-of-inference}\}_2 \cup \{\textit{axioms}\}_2 \implies \textit{theorem}_2] \tag{4.15}$$

$$\textit{logical-truth}_3 := [\{\textit{rules-of-inference}\}_3 \cup \{\textit{axioms}\}_3 \implies \textit{theorem}_3] \tag{4.16}$$

$$\vdots$$

$$\textit{logical-truth}_n := [\{\textit{rules-of-inference}\}_n \cup \{\textit{axioms}\}_n \implies \textit{theorem}_n] \tag{4.17}$$

$$\vdots$$

We note that each logical truth so defined now embeds the set of axioms and the rules of inference which permits the verification of the theorem it pertains to.

We now consider that we can enumerate all sentences of a language. Then, as logical truths are sentences of a language, they too can be enumerated. For the proof, we select the binary language with symbol 0 and 1. As every language can be encoded in binary, the choice of language has no impact on the generality of the derivation. We can enumerate all sentences of the binary language in short-lex (sorted by length and then alphabetically) as:

$$0 \tag{4.18}$$
$$1 \tag{4.19}$$
$$00 \tag{4.20}$$
$$01 \tag{4.21}$$
$$10 \tag{4.22}$$
$$11 \tag{4.23}$$
$$000 \tag{4.24}$$
$$\vdots$$

Most of these sentences will be nonsensical, but once in a while a sentence does make sense within the chosen formal theory - in our case Miniversal logic. We can see this more clearly if we consider listing all sentences of the English language including special characters. We list them as $a, b, c, ..., aa, ab, ..., ba, bb, ...$ Eventually, all sentences, all books and all manuscripts will be enumerated. Then, as the English language can be encoded in binary, the binary language is equivalent in scope to the English language.

The next step will be to extract from the enumeration only the facts, and to eliminate non-facts and nonsensical sentences. To do so, we will have to think of facts as computer programs.

4.6 *Facts as computer programs*

To distinguish the logical truths from the invalid or false statements, we define a function $F : S \to \mathbb{Z}_2$. The function returns 1 if the sentence S is verifiable and 0 otherwise. This function is known as the universal function. Gregory Chaitin[17] has shown that for a universal system, such a function exists but is non-computable.

[17] Gregory J. Chaitin. A theory of program size formally identical to information theory. J. ACM, 22(3): 329–340, July 1975. ISSN 0004-5411. DOI: 10.1145/321892.321894. URL http://doi.acm.org/10.1145/321892.321894

S	$F(S)$ *(example values)*	
0	0	(4.25)
1	0	(4.26)
00	1	(4.27)
01	0	(4.28)
$\vdots$	$\vdots$	

The logical truth of Miniversal logic can be interpreted formally as computer programs running on a universal Turing machine. Indeed, it suffices to consider that the program script is the set axioms and the program input is the theorem. Then the universal Turing machine takes as input the concatenation of the program script (axioms) with the program input (theorem) and outputs the proof. It halts once the proof is complete (the theorem is verified from the axioms) or runs forever (the theorem is not provable from the axioms).

As Miniversal logic describes a universal system, the function $F(s)$ associated with it is non-computable. Non-computable systems do have a computable sub-domain. To properly study this sub-domain, we will introduce feasible mathematics and use it in step 2 of the proof. Before we do that, let us have a small discussion on the nature of facts.

4.7 *Discussion on the nature of facts*

What then is a fact in this context? This construction is likely to be at least somewhat controversial amongst philosophers because it eliminates a commonly used class of facts from our definition of physical reality. Specifically, philosophy considers the existence of synthetic facts. These are facts related to the current world. For example, "Julius Caesar was the emperor of Rome" relates to the present world as it has unfolded historically, but is unlikely to be true for all possible world. Thus, it is a synthetic truth but not a logical truth.

In the present construction of Miniversal logic, all such synthetic statements are converted to analytic statements. Thus, there is no place left for synthetic statement as carriers of truth. Indeed, our understanding of any facts is now always contingent on a set of assumptions. The validity of the fact can be verified based on the assumptions, but cannot be proven to stand on its own as a brute fact. The claim of the existence of Julius Caesar must be corrected to include the assumption which permits the deduction; somewhat informally we can say "If we find evidence of an emperor named Julius Caesar, such as signed parchments and historical records, and that we assume that these records are valid and representative, then Julius Caesar was the emperor of Rome". The fact "Julius Caesar was the emperor of Rome" is no more certain than the assumptions the claim depends upon.

Indeed, I would argue that this representation is more legitimate than the synthetic representation which ignores (or obscures) the underlying assumptions. We might one day find that the historical record has been tempered with (or that perhaps Caesar was just the pawn of a shadow government); thus this construction can account for the discovery of future tampering of the evidence (by switching the assumptions). The possibility of tampering the historical record (or even of the senses) is suggestive that all synthetic statements are no more than logical truths in disguise and contingent upon some set of assumptions.

Do not fear the loss of synthetic statements; as by removing them we will be rewarded with an autotautological physical theory.

Bob: *How does an actual world arises purely from analytic truths?*

That is a great question but you have asked it too soon into the manuscript. If you must know now; I can provide you with a tentative answer. We will first need a theory by which the laws of physics emerges from the set of analytical truths. This will be introduced in the next section as *feasible mathematics*. Then, we can introduce the idea of a world which is actual as follows: The world that we perceive as actual will be the statistical average associated with the greatest amount of equivalent arrangements of analytical truths that produces the same "macroscopic" description in the sense given by feasible mathematics. The absence of a precisely defined actual world is the reason why our universe is fundamentally quantum mechanical. Indeed, fluctuations around the average explicitly connect to quantum fluctuations. In the last chapter we recover the Feynman path integral formulation of quantum field theory from fluctuations over analytical truths arrangements, as well as the Schrödinger and the Dirac equations. What we think of as the actual world are average properties emergent from the set of all analytical truths.

5 *Feasible mathematics*

Some research has been done in the area of feasible numbers. Perhaps the most promising is from Vladimir Yu. Sazonov's paper on feasible numbers[18]. He suggest that feasible numbers are intuitively the set of numbers F which satisfies $0 \in F$, $F+1 \subseteq F$ and $2^{1000} \notin F$. Then, he goes on to investigate various constructions which would allow the consistent treatment of such a set.

[18] Vladimir Yu Sazonov. On feasible numbers. In Logic and computational complexity, pages 30–51. Springer, 1995

Here, we take a different approach. We recognize that 2^{1000} is a large number but nonetheless, it can be compressed to a short representation. Thus, we accept that theorems featuring this number can be proven even in the context of limited resources. Hence, a more general approach to feasibility is required. We propose a method to treat feasibility as a limit applicable to the complexity of the proofs themselves.

Mathematical proofs come in various sizes and have various indicators of complexity. By bounding proofs based on such indicators, the proof landscape available to a mathematician with limited resources is reduced (made feasible). We believe that a representation of mathematical feasibility based on limited proof complexity more accurately describe the intuitive notion of feasibility. After-all, a theorem whose shortest proof requires 2^{1000} bits will surely never be proven in our lifetime, but the number 2^{1000} is easily representable even in simple proofs.

What is feasible mathematics? Feasible mathematics is an alternative to (and in some permissive sense, a generalization of) computational complexity theory (CT). Let us first see what CT is. CT is the study of the inherent difficulty of computational problems; as such, CT classifies problems by the increase in difficulty associated with an increase in input size. For example, a binary search algorithm will have a difficulty of $O(\log n)$, and thus it has a logarithmic complexity. In this example, the number of steps required to find an item amongst n sorted items grows proportionally to the logarithm of n.

Why bother with an alternative? The problem is that CT does not correctly distinguish between all indicators of complexity. As an example, the difficulty between, say, an exponential problem with a small multiplication constant $0.0001 * O(2^n)$ and a polynomial problem with a large multiplication constant $10^{99999} * O(n^2)$ is incorrectly classified. As far as CT is concerned, the latter problem is much simpler than the first as it only grows in n^2 versus 2^n. However, in practice, the latter might never be solved because there might not be enough resources in the observable universe to do so (even for n=1). Thus, to truly represent reality, something is missing from CT.

This is where feasible mathematics comes in. Feasible mathematics treats computational complexity according to the absolute difficulty of problems. Thus, it is not "fooled" by, say, a mere multiplication constant. Its domain is defined as an restriction on the domain of the halting probability Ω of computer science. Using a similar construction, we define a probability Z that represents the probability that a random program will halt within some available computing resources. These resources can be time, memory, clock speed, etc. Z does for feasible mathematics what Ω does for "abstract mathematics"; it can decide all the programs that halt within the available computing resources. When the limits are made to vanish, Z converges to Ω.

As the construction of Z is meta-logically applicable to an arbitrary set of formal axioms, we introduce a distinction between *feasible mathematics* and *universal mathematics*. Universal mathematics is made feasible when, intuitively, the proof landscape of the mathematician is bounded by computational limits. In this sense, all practical work in mathematics is feasible.

To formalize feasible mathematics, we will consider mathematical proofs as computer programs that are executed on a self-delimiting universal Turing machine. We will then construct a statistical ensemble able to decide feasible mathematics.

5.1 Main problem

Suppose a research group with access to a supercomputer. Bob has been granted a fixed amount of computing resources to use on the supercomputer. She has further been instructed to run a program q_A. With no prior knowledge of q_A, what is the probability that the program will halt within the allocated resources?

Answering this question will require notions of *algorithmic thermodynamics* and *statistical physics*.

5.2 Statistical physics

We will provide a brief recap of statistical physics. In statistical physics, we are interested in the distribution that maximizes entropy,

$$S = -k_B \sum_{x \in X} p(x) \ln p(x) \tag{5.1}$$

subject to the fixed macroscopic quantities. The solution for this is the Gibbs ensemble. Typical thermodynamic quantities are:

quantity	*name*	*units*	*type*	
$T = 1/(k_B \beta)$	temperature	K	intensive	(5.2)
E	energy	J	extensive	(5.3)
$p = \gamma/\beta$	pressure	J/m^3	intensive	(5.4)
V	volume	m^3	extensive	(5.5)
$\mu = \delta/\beta$	chemical potential	J/kg	intensive	(5.6)
N	number of particles	kg	extensive	(5.7)

Taking these quantities as examples, the partition function becomes:

$$Z = \sum_{x \in X} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{5.8}$$

The probability of occupation of a micro-state is:

$$p(x) = \frac{1}{Z} e^{-\beta E(x) - \gamma V(x) - \delta N(x)} \tag{5.9}$$

The average values and their variance for the quantities are:

$$\overline{E} = \sum_{x \in X} p(x)E(x) \qquad \overline{E} = \frac{-\partial \ln Z}{\partial \beta} \qquad \overline{(\Delta E)^2} = \frac{\partial^2 \ln Z}{\partial \beta^2} \tag{5.10}$$

$$\overline{V} = \sum_{x \in X} p(x)V(x) \qquad \overline{V} = \frac{-\partial \ln Z}{\partial \gamma} \qquad \overline{(\Delta V)^2} = \frac{\partial^2 \ln Z}{\partial \gamma^2} \tag{5.11}$$

$$\overline{N} = \sum_{x \in X} p(x)N(x) \qquad \overline{N} = \frac{-\partial \ln Z}{\partial \delta} \qquad \overline{(\Delta N)^2} = \frac{\partial^2 \ln Z}{\partial \delta^2} \tag{5.12}$$

The laws of thermodynamics can be recovered by taking the following derivatives

$$\left.\frac{\partial S}{\partial \overline{E}}\right|_{V,N} = \frac{1}{T} \qquad \left.\frac{\partial S}{\partial \overline{V}}\right|_{E,N} = \frac{p}{T} \qquad \left.\frac{\partial S}{\partial \overline{N}}\right|_{E,V} = -\frac{\mu}{T} \tag{5.13}$$

, which can be summarized as

$$d\overline{E} = TdS - pd\overline{V} + \mu d\overline{N} \tag{5.14}$$

This is known as the equation of state of the thermodynamic system. The entropy can be recovered from the partition function and is given by:

$$S = k_B \left(\ln Z + \beta \overline{E} + \gamma \overline{V} + \delta \overline{N} \right) \tag{5.15}$$

5.3 *Algorithmic thermodynamics*

Many authors (Bennett et al., 1998, Chaitin, 1975, Fredkin and Toffoli, 1982, Kolmogorov, 1965, Zvonkin and Levin, 1970, Solomonoff, 1964, Szilard, 1964, Tadaki, 2002, 2008) have discussed the similarity between physical entropy $S = -k_B \sum p_i \ln p_i$ and the entropy in information theory $S = -\sum p_i \log_2 p_i$. Furthermore, the similarity between the halting probability Ω and the Gibbs ensemble of statistical physics has also been studied[19]. Tadaki suggests to augment Ω with a multiplication constant D which acts as a decompression term on Ω.

$$\text{Chaitin construction} \qquad\qquad \text{Tadaki ensemble} \tag{5.16}$$

$$\Omega = \sum_{q \in \text{halts}} 2^{-|q|} \quad \Rightarrow \quad \Omega_D = \sum_{q \in \text{halts}} 2^{-D|q|} \tag{5.17}$$

With this change, the Gibbs ensemble compares to the Tadaki ensemble as follows;

[19] Ming Li and Paul M.B. Vitanyi. An Introduction to Kolmogorov Complexity and Its Applications. Springer Publishing Company, Incorporated, 3 edition, 2008. ISBN 0387339981, 9780387339986; Cristian S. Calude and Michael A. Stay. Natural halting probabilities, partial randomness, and zeta functions. Inf. Comput., 204(11): 1718–1739, November 2006. ISSN 0890-5401. DOI: 10.1016/j.jc.2006.07.003. URL `http://dx.doi.org/10.1016/j.jc.2006.07.003`; John Baez and Mike Stay. Algorithmic thermodynamics. Mathematical. Structures in Comp. Sci., 22(5):771–787, September 2012. ISSN 0960-1295. DOI: 10.1017/S0960129511000521. URL `http://dx.doi.org/10.1017/S0960129511000521`; and Kohtaro Tadaki. A generalization of chaitin's halting probability omega and halting self-similar sets. Hokkaido Math. J., 31(1):219–253, 02 2002. DOI: 10.14492/hokmj/1350911778. URL `http://dx.doi.org/10.14492/hokmj/1350911778`

$$\begin{array}{ll} \text{Gibbs ensemble} & \text{Tadaki ensemble} \\ Z = \sum_{x \in X} e^{-\beta E(x)} & \Omega_D = \sum_{q \in \text{halts}} 2^{-D|q|} \end{array} \tag{5.18}$$

Interpreted as a Gibbs ensemble, the Tadaki construction forms a statistical ensemble where each program corresponds to one of its micro-state. The Tadaki ensemble admits a single quantity; the prefix code length $|q|$ conjugated with D. As a result, it describes the partition function of a system which maximizes the entropy subject to the constraint that the average length of the codes is some constant $\overline{|q|}$;

$$\overline{|q|} = \sum_{q \in \text{halts}} |q| 2^{-|q|} \qquad \text{from 5.10} \tag{5.19}$$

The entropy of the Tadaki ensemble corresponds to the average length of prefix-free codes available to encode programs.

$$S = k_B \left(\ln \Omega + D\overline{|q|} \ln 2 \right) \qquad \text{from 5.15} \tag{5.20}$$

The constant $\ln 2$ comes from the base 2 of the halting probability function instead of base e of the Gibbs ensemble.

John C. Baez and Mike Stay[20] take the analogy further by suggesting an interpretation of algorithmic information theory based on thermodynamics, where the characteristics of programs are considered to be thermodynamic quantities. Starting from Gregory Chaitin's Ω number, the Chaitin construction

[20] John Baez and Mike Stay. Algorithmic thermodynamics. Mathematical. Structures in Comp. Sci., 22(5):771–787, September 2012. ISSN 0960-1295. DOI: 10.1017/S0960129511000521. URL http://dx.doi.org/10.1017/S0960129511000521

$$\Omega = \sum_{q \in \text{halts}} 2^{-|q|} \tag{5.21}$$

is extended with algorithmic quantities to obtain

$$\begin{array}{ll} \text{Gibbs ensemble} & \text{Baez-Stay ensemble} \end{array} \tag{5.22}$$

$$Z = \sum_{x \in X} e^{-\beta E(x) \gamma V(x) - \mu N(x)} \qquad \Omega' = \sum_{q \in \text{halts}} 2^{-\beta E(q) - \gamma V(q) - \delta N(q)} \tag{5.23}$$

Noting the similarity between the Gibbs ensemble of statistical physics (5.8) and (5.23), these authors suggest an interpretation where E is the expected value of the logarithm of the program's runtime, V is the expected value of the length of the program and N is the expected value of the program's output. Furthermore, they interpret the conjugate variables as (quoted verbatim from their paper);

"

1. $T = 1/\beta$ is the *algorithmic temperature* (analogous to temperature). Roughly speaking, this counts how many times you must double the runtime in order to double the number of programs in the ensemble while holding their mean length and output fixed.
2. $p = \gamma/\beta$ is the *algorithmic pressure* (analogous to pressure). This measures the trade-off between runtime and length. Roughly speaking, it counts how much you need to decrease the mean length to increase the mean log runtime by a specified amount, while holding the number of programs in the ensemble and their mean output fixed.
3. $\mu = -\delta/\beta$ is the *algorithmic potential* (analogous to chemical potential). Roughly speaking, this counts how much the mean log runtime increases when you increase the mean output while holding the number of programs in the ensemble and their mean length fixed.

"

–John C. Baez and Mike Stay

From equation (5.23), they derive analogues of Maxwell's relations and consider thermodynamic cycles, such as the Carnot cycle or Stoddard cycle. For this, they introduce the concepts of *algorithmic heat* and *algorithmic work*.

Other authors have suggested other alternative mappings in other but related context[21].

5.4 *Derivation of feasible mathematics*

1. We start from the standard Chaitin construction applicable to a self-delimiting universal Turing machine[22].

$$\Omega = \sum_{q \in \text{halts}} 2^{-|q|} \tag{5.24}$$

, where

$$\Omega \in (\mathbb{R} \cap [0,1]) \quad \text{numerical value of the sum} \tag{5.25}$$

$$q \in \Sigma_b \quad \text{binary program (encoded as a prefix-free code)} \tag{5.26}$$

$$|q| : \Sigma_b \to \mathbb{N} \quad \text{length of the program's code} \tag{5.27}$$

2. We augment Ω with a multiplication constant D; we obtain the Tadaki ensemble[23].

$$\Omega_D = \sum_{q \in \text{halts}} 2^{-D|q|} \tag{5.28}$$

[21] Ming Li and Paul M.B. Vitanyi. An Introduction to Kolmogorov Complexity and Its Applications. Springer Publishing Company, Incorporated, 3 edition, 2008. ISBN 0387339981, 9780387339986; and Kohtaro Tadaki. A statistical mechanical interpretation of algorithmic information theory. In Local Proceedings of the Computability in Europe 2008 (CiE 2008), pages 425–434. University of Athens, Greece, Jun 2008. URL http://arxiv.org/abs/0801.4194

[22] Gregory J. Chaitin. A theory of program size formally identical to information theory. J. ACM, 22(3): 329–340, July 1975. ISSN 0004-5411. DOI: 10.1145/321892.321894. URL http://doi.acm.org/10.1145/321892.321894

[23] Kohtaro Tadaki. A generalization of chaitin's halting probability omega and halting self-similar sets. Hokkaido Math. J., 31(1):219–253, 02 2002. DOI: 10.14492/hokmj/1350911778. URL http://dx.doi.org/10.14492/hokmj/1350911778

, where

$$\Omega_D \in (\mathbb{R} \cap [0,1]) \qquad \text{numerical value of the sum} \tag{5.29}$$

$$D \in \mathbb{R} \qquad \text{Conjugate to program length} \tag{5.30}$$

3. With this addition, Ω_D has the same mathematical structure as a Gibbs ensemble of statistical physics[24].

$$\begin{array}{ll} \text{Gibbs ensemble} & \text{Tadaki ensemble} \\ G = \sum_{x \in X} e^{-\beta E(x)} & \Omega_D = \sum_{q \in \text{halts}} 2^{-D|q|} \end{array} \tag{5.31}$$

, where

$$\left(2^{-D|q|}\right) \qquad \text{micro-state representing a program} \tag{5.32}$$

$$D \in \mathbb{R} \qquad \text{algorithmic decompression} \tag{5.33}$$

4. We interpret the Tadaki ensemble within the context of algorithmic thermodynamics[25]. We can introduce a probability distribution for Ω and Ω_D that maximizes the entropy of the system.

$$\begin{array}{ll} \text{Halting probability} & \text{Halting probability with fixable } \overline{|q|} \\ p(q) = \frac{1}{\Omega} 2^{-|q|} & p(q, D) = \frac{1}{\Omega_D} 2^{-D|q|} \end{array} \tag{5.34}$$

In the case of Ω_D, D is a Lagrange multiplier and $p(q, D)$ is the probability measure that maximizes the entropy subject to the constraint that the average program length is $\overline{|q|}$.

$$\overline{|q|} = \sum_{q \in \text{halts}} p(x)|q| \tag{5.35}$$

, where

$$\overline{|q|} \in \mathbb{R}_{\geq 0} \qquad \text{average program length} \tag{5.36}$$

5. Finally, to obtain feasible mathematics, we introduce into Ω_D a new quantity $t(q)$ the runtime of program q and pair it with its conjugate W. We obtain the construction Z.

$$Z = \sum_{q \in \text{halts}} 2^{-Wt(q) - D|q|} \tag{5.37}$$

$$\tag{5.38}$$

, where

$$Z \in \mathbb{R}_{\geq 0} \qquad \text{numerical value of the sum} \tag{5.39}$$

$$t(q) : q \to \mathbb{N} \qquad \text{number of iterations required for } q \text{ to halt} \tag{5.40}$$

$$W \in \mathbb{R} \qquad \text{conjugate to } t(q) \text{ in units of } (iterations)^{-1} \tag{5.41}$$

$$|q| : q \to \mathbb{N} \qquad \text{number of bits of program } q \tag{5.42}$$

$$D \in \mathbb{R} \qquad \text{conjugate to } |q| \text{ in units of } (bits)^{-1} \tag{5.43}$$

[24] Kohtaro Tadaki. A statistical mechanical interpretation of algorithmic information theory. In Local Proceedings of the Computability in Europe 2008 (CiE 2008), pages 425–434. University of Athens, Greece, Jun 2008. URL http://arxiv.org/abs/0801.4194; and John Baez and Mike Stay. Algorithmic thermodynamics. Mathematical. Structures in Comp. Sci., 22(5):771–787, September 2012. ISSN 0960-1295. DOI: 10.1017/S0960129511000521. URL http://dx.doi.org/10.1017/S0960129511000521

[25] John Baez and Mike Stay. Algorithmic thermodynamics. Mathematical. Structures in Comp. Sci., 22(5):771–787, September 2012. ISSN 0960-1295. DOI: 10.1017/S0960129511000521. URL http://dx.doi.org/10.1017/S0960129511000521

The corresponding probability measure is:

$$p(q, W, D) = \frac{1}{Z} 2^{-Wt(q) - D|q|} \tag{5.44}$$

It maximizes the entropy subject to the following constraints:

$$\overline{|q|} = \sum_{q \in \text{halts}} p(q, W, D)|q| \qquad \text{average program length } \overline{|q|} \tag{5.45}$$

$$\bar{t} = \sum_{p \in \text{halts}} p(q, W, D)t(q) \qquad \text{average program runtime } \bar{t} \tag{5.46}$$

Let us now study this equation in more detail in the following section.

5.5 *Results*

We interpret the supercomputing research group as taking a similar role to the role taken by the various baths in thermodynamics (heat bath, particle bath). For example, in thermodynamics we would say that a system which can exchange energy with its environment is in a heat bath. Its temperature will be constant but its total energy would fluctuate as it is exchanged with the bath. By analogy, in feasible mathematics, we would imagine that a computation occurs in a supercomputer which schedule priority, assigns memory, etc. so has to maintain various computing resources fixed during the calculation. This is analogous the role of the thermodynamic baths.

To make this more precise, let us define what we mean by fixed resources.

5.6 *Fixed resources*

Each Lagrange multiplier of the partition function Z is a computing resource fixed by the supercomputer. In the provided definition of Z, there are two such constants : W and D. They can be interpreted as follows:

$$\text{Halting-power} \qquad \mathcal{P} = \frac{1}{W} \tag{5.47}$$

$$\text{Halting-force} \qquad \mathcal{F} = \frac{D}{W} \tag{5.48}$$

- The halting-power counts how much the runtime must be doubled in order to double the entropy of the ensemble while holding the mean length fixed.
- The halting-force counts how much the average length must be decreased to increase the average runtime by a specified amount, while holding the entropy in the ensemble fixed.

By adjusting the halting-power and the halting-force, the supercomputer is able to control the value of the constraints of the system $\bar{t}$ and $\overline{|p|}$. Thus, the halting probability of Bob's program q_A depends on the halting-power allocated by the supercomputer. In the supercomputer analogy, the halting-power can be understood as related to the clock speed of the processor(s), and the halting-force as a compression algorithm applied to input memory.

5.7 *Alternative formulations*

There exists alternative constructions of Z such that other resources are fixed by the supercomputer.

Action-frequency formulation:

$$Z' = \sum_{q \in \text{halts}} 2^{-\mathcal{A} f(q) - D|q|} \tag{5.49}$$

The supercomputer must fix

$$\text{Halting-action} \qquad \mathcal{S} = \frac{1}{\mathcal{A}} \tag{5.50}$$

- The halting-action counts how much the action must be doubled in order to double the entropy of the ensemble while holding the mean length fixed.

Time-power formulation:

$$Z'' = \sum_{q \in \text{halts}} 2^{-tP(q) - D|q|} \tag{5.51}$$

The supercomputer must fix

$$\text{Halting-time} \qquad t \tag{5.52}$$

- The halting-time counts how much the time must be doubled in order to double the entropy in the ensemble while holding the mean length fixed.

This formulation does not describe a time cutoff (see next formulation). Rather, it describes a system where all programs halt at the same time. To guarantee that the work on each program terminates simultaneously (e.g. there are no partial executions), the supercomputer must adjust the computation power on a per program basis.

Time-cutoff formulation:

$$Z''' = \sum_{q \in \text{halts}; t(q) \leq k} 2^{-D|q|} \tag{5.53}$$

The sum Z''' is performed only on programs that halt within a time cutoff k. Thus, Z''' contains no halting information and is computable. Ω is recovered in the limit when $k \to \infty$.

Size-cutoff formulation:

$$Z'''' = \sum_{q \in \text{halts}; |q| \leq k} 2^{-D|q|} \tag{5.54}$$

The sum Z'''' is performed only on programs with sizes less or equal to k. Ω is recovered in the limit when $k \to \infty$. Z'''' represents the first n bits Ω up to the cutoff k.

5.8 *Relation to* Ω

Theorem 5.55. *$Z \to \Omega_D$ as the amount of available resources is increased arbitrarily.*

$$\lim_{\mathcal{P} \to \infty} Z \to \Omega_D \tag{5.56}$$

Proof. First, we rewrite Ω_D as:

$$\Omega_D = \sum_{i=1}^{\infty} 2^{-H(q_i) - D|q_i|} \qquad \text{where } H(q_i) := \begin{cases} 0 & q_i \text{ halts} \\ \infty & \text{otherwise} \end{cases} \tag{5.57}$$

Second, we note that the runtime $t(q_i)$ of a program q_i will be finite if it halts and infinite otherwise.

$$t(q_i) = \begin{cases} t_i \in \mathbb{R}_{\geq 0} & q_i \text{ halts} \\ \infty & \text{otherwise} \end{cases} \tag{5.58}$$

Then taking the limit of Z,

$$\lim_{\mathcal{P} \to \infty} \frac{1}{\mathcal{P}} t(q_i) = \begin{cases} 0 & q_i \text{ halts} \\ \infty & \text{otherwise} \end{cases} \tag{5.59}$$

This is the definition of $H(q_i)$. Therefore,

$$\lim_{\mathcal{P} \to \infty} \frac{1}{P} t(q_i) \to H(q_i) \tag{5.60}$$

Thus,

$$\lim_{\mathcal{P}\to\infty} Z \to \Omega_D \tag{5.61}$$

□

Theorem 5.62. *Z monotonically converges towards Ω_D as the available resources are increased.*

Proof. Without loss of generality, let us now expand Z explicitly with an example. Assume a system comprised of three micro-states with prefix code-length $|q_1| = 1$, $|q_2| = 2$ and $|q_3| = 3$ and with the *run-times* $t_1 = 5$, $t_2 = \infty$ and $t_3 = 5$. In this example, q_1 and q_2 halt and q_3 does not. For the purposes of simplicity we can assume that all other programs do not halt. In this case the system is not universal but let us nonetheless use it as a simplified numerical example. The sum Z becomes;

$$Z(W) = 2^{-1-5W} + 2^{-2-\infty W} + 2^{-3-5W} \tag{5.63}$$

We will now produce a series of numerical calculations with progressively smaller values of W and we will look at the evolution of the error rate $\xi(W) = \Omega - Z(W)$. For this system, $\Omega = 0.101\bar{0}_b$.

W	$Z(W)$	$\xi(W)$	error	
∞	0	Ω	max	(5.64)
1	$0.000000101..._b$	0.10011011_b	$\approx 2^{-1}$	(5.65)
0.1	$0.011100010..._b$	$0.00101110..._b$	$\approx 2^{-3}$	(5.66)
0.01	$0.100110101..._b$	$0.00000010..._b$	$\approx 2^{-6}$	(5.67)
0.001	$0.011100010..._b$	$0.00000000..._b$	$\approx 2^{-9}$	(5.68)
⋮	⋮	⋮	⋮	
0	Ω	0	none	(5.69)

As we can see, increasing the halting-power ($\mathcal{P} = 1/W$) causes the value Z to monotonically converges towards Ω. The error rate decreases as more valid bits of Ω are obtained. □

Theorem 5.70. *An observer knowing n bits of Z will be able to decide at most 2^N programs.*

Proof. We consider a numerical value for Z whose first k bits corresponds to the bits of Ω. We look at two cases: 1) For the first k bits,

Z (as with Ω) can decide 2^N programs per bit. 2) For the bits after k, the situation is a bit more complex:

To recover the feasible programs beyond k, an observer can execute programs on a universal Turing machine in dovetail. As they halt, the observer adds their contribution to Z. Once the value of Z is recovered, then all programs taking longer to halt are beyond the feasible bound, regardless of whether they ultimately halt or not.

□

6 Step 2 - Properties emergent from analytical facts

Now armed with feasible mathematics, the derivation of Z is quite direct. The main result from feasible mathematics is that if we look at the properties of facts objectively: e.i. we only look at those properties which are measurable such as the length of the sentence corresponding to the fact, or the length of proof verifying the fact, and avoid "poetic" properties such as how meaningful, interesting or how "deep" a fact is, then we can construct such an ensemble. This is the "all facts are purely analytical" interpretation.

Each fact has a cost. The cost is intrinsic and irreducible. The world, as it is fundamentally describable by facts, is not immune to this cost. Thus, it must bare the cost of each fact that comprises it. For each fact, its irreducible costs are;

irreducible cost 1: *length (in bits) required to describe the fact* (6.1)

irreducible cost 2: *length (in bits) required to proof the fact* (6.2)

6.1 Main result of feasible mathematics

The main result of feasible mathematics is that the ensemble of all analytical facts, organized by proof length and description length will produce, when all other things are equal, two equilibrium quantities that are constant throughout the system; namely the halting-density $\mathcal{F}$ and the halting-power $\mathcal{P}$. If the universe is indeed built out of analytical facts, evidence for these two quantities should empirically be plentiful.

The function Z of feasible mathematics

$$Z = \sum_{q \in \mathcal{Q}} e^{-Fx(q) - Wt(q)} \tag{6.3}$$

describes a statistical ensemble of analytical fact q from within the set $\mathcal{Q}$ of all such facts. Each micro-state is statistically weighted by the length of its proof $t(q)$ and by the length of its description $x(q)$. Furthermore, the function

$$p(q) = \frac{1}{Z}e^{-Fx(q)-Wt(q)} \tag{6.4}$$

is the probability that a fact $q \in \mathcal{Q}$ is actual in the universe.

6.2 *Enumerating all analytical facts at the limits of computation*

Ω is an oracle for the halting problem of a universal Turing machine. Thus, knowing Ω is equivalent to knowing the function $F : S \to \mathbb{Z}_2$ used to list all analytical facts. The statistical ensemble connects to Ω when the feasible bound is removed.

6.3 *Soft versus hard limits*

Assuming the shortest proof, the length of the proof of an analytical fact is irreducible. Indeed, reducing its size can only be done by taking steps out (and then its no longer a valid proof). The length of its description is also irreducible for the reason that changing it will transform it to another fact. Thus, the limits associated to these quantities will be what we would call hard limits. Physically, we can think of hard limits as the speed of light or horizons applicable to the observable universe - all of which are inviolable.

Other properties of facts and proofs may, I hypothesize, give rise to softer limits. For example, a solution requiring lots of memory and time would require the contribution of a large amount of physical substrate (many molecules working together) for a long time. Thus, the computation of such fact is likely to be destroyed by fluctuations of the environment. This makes it unlikely that such high memory long running statement be verified early in the history of the universe. These types of limits are statistical and yield soft laws and can be investigated by usual complexity class theory. As a concrete example we can think of the evolution of life and its many near extinction events.

However interesting soft limits may be, in this manuscript, we will only be concerned with deriving the hard limits.

6.4 *Mapping to the space-time background*

The function Z applies to all worlds which are exclusively comprised of verifiable facts and that are of sufficient generality to embed all deductive apparatuses - as required for self explanation via the autological property.

We will show in the next part that the laws implied by Z correspond to the familiar laws of physics. Specifically, we will derive the following laws: special relativity, general relativity, dark energy, the arrow of time, the second law of thermodynamics, the Schrödinger equation, the Dirac equation and quantum field theory from Z.

How do these laws come out of Z? The first step will be to connect the quantities of Z to corresponding physical quantities[26]. Our strategy to do so will be be borrowed from introductory statistical physics. We will adopt the same line of reasoning which allows the Lagrange multiplier β of statistical physics to be connected to the notion of a physical temperature. As you may recall, in introductory statistical physics;

[26] Alexandre Harvey-Tremblay. A derivation of the laws of physics from pure information. `http://vixra.org/abs/1705.0274`, 2017

1. The Gibbs ensemble is first derived from statistical arguments as the ensemble which maximizes the entropy subject to fixed quantities. The process introduces a multiplication constant known as the Lagrange multiplier and is designated by β.

2. From the Gibbs ensemble, a relation between β, energy and entropy is obtained: $\beta dE = dS$.

3. Then, it is shown that this relation recovers a well known and empirically-uncontested law; such that the two are exact replicas if and only if β is defined using the temperature T. In this case, $S = \ln \Omega$, connect β to T via $\beta = 1/(k_B T)$.

4. Thus, we conclude that the Gibbs ensemble is a description of a physical system involving energy, entropy and temperature.

We adopt the same line of reasoning for the derivation of the space-time background from Z. Our goal is to derive as many laws of physics as we can from Z so as to show the extent of the physical connection. Specifically, we will show that a certain quantity of Z corresponds to the time in the equations for Special relativity, the Dirac equation, etc. and that another quantity of Z corresponds to space in those same equations.

The universal validity of the mapping between the quantities of Z and the physical notion of space-time is ultimately the conclusion of this manuscript and rests on deriving overwhelmingly many known laws of physics from Z and to a degree such that it exceeds that which would be expected from a mere coincidence.

For pedagogical reasons, we explicitly state the mapping obtained as the conclusion of this manuscript. Each law of physics that we derive from Z adds weight to the mapping.

Property	Variable	Mapping	Units	
proof-length	$t(q)$	$t(q) \to time$	*seconds*	(6.5)
description-length	$x(q)$	$x(q) \to space$	*meters*	(6.6)

Part II

The laws of physics

Using the language of physics, we consider an interpretation of Z isomorphic to the analytic-fact interpretation. Consistent with Z, we inject as thermodynamic conjugate-pairs the two quantities that are mapped to time and space, respectively proof-length and description-length. Interpreted as time and space and to recover the units of energy (appropriate for statistical physics), time must be multiplied by a power and space (e.g. a length) must be multiplied by a force; thus, the partition function describes arbitrary micro-states in terms of both space and time. Our goal will be to show that this mapping is valid by deriving a plurality of well known laws of physics from Z. Due to the simplicity and generality of the construction, it is perhaps reassuring that the relations of space-time; special relativity, general relativity, and dark energy are provable solutions of its equation of state. Furthermore, thermal fluctuations along the time and space quantities produce the Schrödinger and Dirac equations as thermo-statistical extensions to classical analogues. The notion of temperature is recovered simply as the proportion between entropy and energy as per the standard definition. The construction suggests that both general relativity and the quantum world emerge from a more fundamental thermo-statistical world which is isomorphic to analytic-truths.

7 *First proposed partition function: Time and Space*

Consistent with the function Z applicable to analytical truth and to the physical mapping, we propose the following partition function, constructed as a Gibbs ensemble:

$$Z(\beta, P, F) = \sum_{q \in \mathcal{Q}} e^{-\beta[Fx(q) - Pt(q)]} \tag{7.1}$$

where

quantity	*name*	*units*	*type*	
P	entropic power	J/s	intensive	(7.2)
F	entropic force	J/m	intensive	(7.3)
$t(q)$	thermal time	s	extensive	(7.4)
$x(q)$	thermal space	m	extensive	(7.5)

The introduction of the term β is done to cancel out whatever units we ascribed to $Fx(q)$ and $Pt(q)$ by the mapping - in this case energy in Joules.

The partition function includes the familiar entropic force and the unfamiliar entropic power. Its equation of state is:

$$TdS = -Pd\overline{t} + Fd\overline{x} \tag{7.6}$$

We can convert it to an equivalent representation by converting the time to a frequency and the power to an action. Let us do that now.

$$TdS = -Pd\overline{t} + Fd\overline{x} \tag{7.7}$$
$$TdS = -Pd(\overline{f}^{-1}) + Fd\overline{x} \qquad [t := 1/f] \tag{7.8}$$
$$TdS = P\overline{f}^{-2}d\overline{f} + Fd\overline{x} \qquad \left[d(f^{-1}) = -f^{-2}df\right] \tag{7.9}$$
$$TdS = \mathcal{S}d\overline{f} + Fd\overline{x} \qquad \left[\mathcal{S} := Pf^{-2}\right] \tag{7.10}$$

This representation introduces two new quantities, defined as:

quantity	*name*	*units*	*type*	
$\mathcal{S}$	entropic action	Js	intensive	(7.11)
$f(q)$	thermal frequency	s	extensive	(7.12)

Thus, the equation of state admits these two formulations:

$$TdS = \mathcal{S}d\overline{f} + Fd\overline{x} \qquad \text{action-frequency formulation} \tag{7.13}$$
$$TdS = -Pd\overline{t} + Fd\overline{x} \qquad \text{power-time formulation} \tag{7.14}$$

which we will refer to throughout this part.

Note that this "physical" Z is of the same mathematical structure as the one obtained from feasible mathematics. Hence, they describe the same mathematical object, precisely as we would expect from the autological property.

$$\begin{array}{ll} \text{Physical} & \text{Analytic-truths} \\ Z(\beta, P, F) = \sum_{q \in \mathcal{Q}} e^{-\beta[Fx(q)-Pt(q)]} & Z(P, F) = \sum_{q \in \mathcal{Q}} e^{-Fx(q)+Pt(q)} \end{array} \tag{7.15}$$

The only difference is the negligible introduction of a multiplication constant β to cancel out the units introduced by the physical mapping.

7.1 *Regimes and cycles*

We will derive the familiar laws of physics by studying the equation of state in terms of its regimes. To do so, we will fix some derivatives (e.g. $dS = 0$) and analyze what happens when we let the others vary.

7.2 *Special relativity*

Here, we use the power-time formulation and pose $dS = 0$. We obtain the fundamental relation of special relativity linking space to time.

$$0 = -Pd\bar{t} + Fd\bar{x} \tag{7.16}$$

$$Fd\bar{x} = Pd\bar{t} \tag{7.17}$$

$$d\bar{x} = \frac{P}{F}d\bar{t} \tag{7.18}$$

As the power P and the force F are Lagrange multipliers of the partition function, they are constant throughout the system. Therefore, their quotient is also a constant.

$$c := \frac{P}{F} \tag{7.19}$$

Therefore,

$$d\bar{x} = cd\bar{t} \tag{7.20}$$

As the units of P/F are meters per second, c will be our working definition of the speed of light.

Remark: When P is the Planck power and F is the Planck force, we do indeed recover the speed of light:

$$P\left(\frac{1}{F}\right) = \frac{c^5}{G}\left(\frac{G}{c^4}\right) = c \tag{7.21}$$

7.3 *Light cones as thermodynamic cycles*

In this section, we look at the thermodynamic cycle of the system transiting through time and space starting at O to A to B and back to O, as illustrated on Figure 2. During the transitions and to keep the energy constant, trade-offs must be made between time, distance and entropy. This cycle is reminiscent of other thermodynamic cycles, such as those involving pressure and volume. Interestingly, the cycles can also be interpreted as light cones.

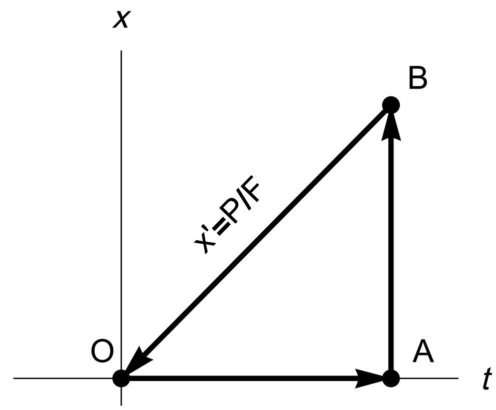

Figure 2: A "thermodynamic cycle" through space and time.

O to A: As O is translated forward in time to A while keeping the distance constant ($d\bar{x} = 0$), the entropy decreases over time.

$$\left(TdS = Fd\bar{x} - Pd\bar{t}\right|_{d\bar{x}=0} \tag{7.22}$$

$$\implies \frac{dS}{dt} = -\frac{P}{T} \tag{7.23}$$

A to B: As A is translated forward in space to B while keeping the time constant ($d\bar{t} = 0$), the entropy increases over the distance.

$$\left(TdS = Fd\bar{x} - Pd\bar{t}\right|_{d\bar{t}=0} \tag{7.24}$$

$$\implies \frac{dS}{dx} = \frac{F}{T} \tag{7.25}$$

O to B: As O is translated forward both in time and in space to B while keeping the entropy constant ($dS = 0$), the system has a speed of c.

$$\left(TdS = Fd\bar{x} - Pd\bar{t}\right|_{dS=0} \tag{7.26}$$

$$\implies \frac{dx}{dt} = \frac{P}{F} = c \tag{7.27}$$

We conclude that an object traveling at speed c is neither encouraged nor discouraged by entropy. The speed of light represents an inflexion point in the rate of entropy production over time. We will return to that notion in the section on the arrow of time.

7.4 *Lorentz's transformation*

To recover the Lorentz's factor γ, let us consider figure 3. Two observers start at the origin S and travel in space-time respectively to O and O'. We regard O' as traveling at speed $|v|$ in the reference frame of O. From standard trigonometry, we derive the following values for the length of the segment;

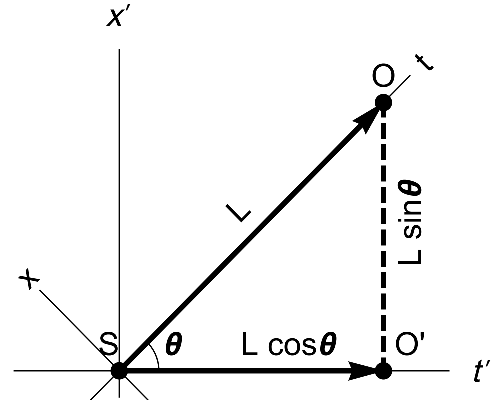

Figure 3: The space-time intervals between two observers. Here O' travels at speed $|v|$ in O's reference frame.

Segment	Length	
$\lvert\overline{SO}\rvert$	L	(7.28)
$\lvert\overline{SO'}\rvert$	$L\cos\theta$	(7.29)
$\lvert\overline{O'O}\rvert$	$L\sin\theta$	(7.30)

From the Pythagorean theorem and solving for $\cos\theta$, we obtain:

$$|\overline{SO}|^2 = |\overline{SO'}|^2 + |\overline{O'O}|^2 \tag{7.31}$$

$$L^2 = (L\cos\theta)^2 + (L\sin\theta)^2 \tag{7.32}$$

$$1 = (\cos\theta)^2 + (\sin\theta)^2 \tag{7.33}$$

$$\sqrt{1-(\sin\theta)^2} = \cos\theta \tag{7.34}$$

We consider that the distance between two observers moving at constant speed is simply vt. Hence, $|\overline{O'O}| = vt$. Solving for $\sin\theta$, we obtain:

$$|\overline{O'O}| = vt = L\sin\theta \tag{7.35}$$

$$\implies \sin\theta = \frac{vt}{L} \tag{7.36}$$

From equation (7.34) and (7.36), we get the reciprocal of the Lorentz factor:

$$\sqrt{1-\frac{v^2t^2}{L^2}} = \cos\theta = \gamma^{-1} \tag{7.37}$$

$$\implies \gamma = \frac{1}{\sqrt{1-\frac{v^2t^2}{L^2}}} \tag{7.38}$$

Finally, we consider that L is the distance traveled by O in the reference frame of O' such that the entropy of O is constant over time. According to the relation $dx = cdt$, for this to be the case, the speed of O must be c. Thus, the distance traveled by O during time t is $L = ct$. We obtain:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{7.39}$$

which is the well-known Lorentz factor and is the multiplication constant connecting $|\overline{SO}|$ to $|\overline{SO'}|$.

7.5 Connection to analytic truths

The conclusion, from the perspective of analytic-truths, is that any observer who perceives reality exclusively in terms of verifiable facts will perceive a ratio between the length of the description of their facts, and the length of their proofs. As the universe is comprised of facts, this ratio is necessarily also found in the universe. The ratio depends of P and F, two Lagrange multiplier that are constant in the system. Thus, all observers should agree on the value of the ratio. Furthermore, via the physical mapping, the ratio has the mathematical structure of a speed.

The reader will surely notice that we come just short of claiming that this speed is maximal. This will be proved explicitly in section 9.5 on limiting relations. We reserve the proof for later because we also show along it that other quantities are also maximal; namely, the viscosity of space-time and the maximum volumetric flow rate of space.

7.6 Inertial mass

In this section, we will need to use the Unruh temperature[27]. As can be reviewed in the citations provided, the Unruh temperature is an exact result obtained from special relativity. The Unruh effect is the prediction that an accelerating observer will observe blackbody radiation (at the Unruh temperature), whereas an inertial observer would observe none. The Unruh temperature is:

$$T = \frac{\hbar a}{2\pi c k_B} \tag{7.40}$$

The Unruh temperature connects acceleration to the temperature. We will use it here to convert an entropic force expressed in terms of a temperature to an entropic force expressed in terms of acceleration.

Furthermore, we start from the power-time formulation and pose $d\bar{t} = 0$. As originally done by Erik Verlinde[28], from these starting points, we can derive $F = ma$ as follows:

$$TdS = Fd\overline{x} \tag{7.41}$$

$$F = T\frac{dS}{dx} \tag{7.42}$$

$$F = \left(\frac{\hbar a}{2\pi c k_B}\right)\frac{dS}{dx} \tag{7.43}$$

$$F = \left(\frac{\hbar}{2\pi c k_B}\frac{dS}{dx}\right) a \tag{7.44}$$

[27] Stephen A. Fulling. Nonuniqueness of canonical field quantization in riemannian space-time. Phys. Rev. D, 7:2850–2862, May 1973. DOI: 10.1103/PhysRevD.7.2850. URL https://link.aps.org/doi/10.1103/PhysRevD.7.2850; P C W Davies. Scalar production in schwarzschild and rindler metrics. Journal of Physics A: Mathematical and General, 8(4): 609, 1975. URL http://stacks.iop.org/0305-4470/8/i=4/a=022; W. G. Unruh. Notes on black-hole evaporation. Phys. Rev. D, 14:870–892, Aug 1976. DOI: 10.1103/PhysRevD.14.870. URL https://link.aps.org/doi/10.1103/PhysRevD.14.870; and Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. DOI: 10.1007/JHEP04(2011)029. URL https://doi.org/10.1007/JHEP04(2011)029

[28] Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. DOI: 10.1007/JHEP04(2011)029. URL https://doi.org/10.1007/JHEP04(2011)029

This equation corresponds to $F = ma$ provided that $\left(\frac{\hbar}{2\pi c k_B}\frac{dS}{dx}\right) = m$. How reasonable is that? Well, for it to be the mass, it suffices that dS/dx is the inverse of the reduced Compton wavelength multiplied by a constant. Recall that the reduced Compton wavelength is $\hbar/(mc)$. Let us investigate:

$$\frac{\hbar}{2\pi c k_B}\frac{dS}{dx} = m \implies \frac{dS}{dx} = 2\pi k_B\left(\frac{mc}{\hbar}\right) \tag{7.45}$$

We obtain a relation between entropy and x. What could this mean? It means two things.

1. The further away an object is from the origin, the higher its positional entropy.
2. The more massive an object is, the higher its positional entropy.

Why then the factor 2π? The presence of π suggest a connection between a line and a circle. Therefore, a possible interpretation is that the entropy associated with positional entropy is scaled proportionally to the curvature of a circle (we can think of it as a one-dimensional case of the holographic principle). Then, as an object with a small Compton wavelength that can be more finely located, it requires more positional entropy to describe its position than an object with a large Compton wavelength. Why then the factor k_B? The factor k_B converts the reduced Compton wavelength to the units of entropy/length (joules per kelvin per meter).

8 *Second proposed partition function: Time and generalized length*

The first partition function we proposed was constructed with an entropic-force conjugated with a thermal-length. The length was, of course, linear and expressed by x. In this section, however, we extend the representation to consider a thermal-length described by an arbitrary function; after all, the structure of the universe is not linearly distributed with clockwork precision. To achieve this, we consider an arbitrary function $l(q) : q \to \mathbb{R}$ used to express the lengths of the micro-states. We will study such function via a Taylor expansion. A Taylor expansion requires that $\mathcal{Q}$ as in $q \in \mathcal{Q}$ be uncountable. As $l(q)$ is an arbitrary length with meter units, it will still be conjugated with the entropic-force.

The Taylor expansion of $Fl(q)$ is:

$$Fl(q) = Fl(0) + Fl'(0)q + \frac{Fl''(0)}{2}q^2 + \frac{Fl'''(0)}{6}q^3 + O(q^4) \tag{8.1}$$

and its derivative with respect to q is:

$$Fdl(q) = Fl'(0)dq + Fl''(0)qdq + \frac{Fl'''(0)}{2}q^2dq + 4O(q^3)dq \qquad (8.2)$$

As the micro-states $q \in \mathcal{Q}$ must be uncountable for the Taylor expansion of $l(q)$ to be well defined, the partition function must be continuous. Therefore, it becomes:

$$Z = \frac{1}{h}\int e^{-\beta[Fl(q)-Pt(q)]}dq \qquad (8.3)$$

and is integrated over $\mathcal{Q}$. Likewise, its equation of state is

$$TdS = \mathcal{S}d\bar{f} + Fd\bar{l} \qquad \text{action-frequency formulation} \qquad (8.4)$$

$$TdS = -Pd\bar{t} + Fd\bar{l} \qquad \text{power-time formulation} \qquad (8.5)$$

Note on sentences: We recall that n bits can encode 2^n sentences. Thus, the growth of sentences over length is not a linear function. Further requirements on the function required for a convergent definition of Ω such as the prefix-free property might also be required (not discussed). For these reasons and because an approximate real continuation of x nats to encode e^x sentences lends itself to a Taylor expansion analysis.

8.1 *Discussion on the smoothness continuation*

Equations such as general relativity assumes that space is continuous. Thus, it would be naive to think that general relativity could be recovered without smoothing out the discrete partition function Z to a continuous reciprocal. The theory, once so approximated as smooth, becomes an effective theory able to describe reality within a certain limit (as opposed to being applicable on all scales). In this case, it is applicable in the limit far away from the Planck scales. Thus, the results obtained in this section are to be taken as laws emergent in the limit.

The price to pay for this approximation is that the space-time events described by the discrete Z will manifest themselves "awkwardly" in the continuous regime. Perhaps this can be connected to the quantum measurement. Consider the case if we were to interpreted the quantum measurement as a tool to keep the smooth approximation in-sync with discrete events. In this case the system would have a unitary evolution in-between events, and such unitary evolution would be violated as the events occur. Furthermore, as the events occurs at the limit of computability, they would likely appear algorithmically random to any observer with limited computational resources (compared to the universe). The permissible measurement outcomes are the micro-states of the partition function; the facts. The behavior is quite reminiscent of the quantum measurement.

The smoothness continuation is best interpreted as an interpolation technique between discrete events in space-time. Below the Planck limit, the entropy of the smoothed partition function becomes less than one. Thus, below the Planck scale, it would describes partial facts (if such a thing were to exist). Making $\mathcal{Q}$ uncountable does not imply that we pack infinitely many facts in every point of space. It simply means that a single fact is describe by a section of area under the curve of smoothed Z. Thus, the smoothness approximation is a continuation for the region in-between events.

8.2 *Taylor expansion of $d\bar{l}$*

We convert the term $d\bar{l}$ of the power-time formulation into its Taylor expansion. The first change we will do is rename $q := x$. The multiplication term 4 in $4O(x^3)$ can be absorbed in to $O(x^3)$.

$$Fdl(x) = Fl'(0)dx + Fl''(0)xdx + \frac{Fl'''(0)}{2}x^2dx + O(x^3)dx \tag{8.6}$$

Then, injecting it into the power-time formulation, we obtain:

$$TdS = -Pd\bar{t} + Fd\bar{l} \tag{8.7}$$

$$TdS = -Pd\bar{t} + Fl'(0)d\overline{x} + Fl''(0)\overline{x}d\overline{x} + \frac{Fl'''(0)}{2}\overline{x}^2d\overline{x} + O(\overline{x}^3)d\overline{x} \tag{8.8}$$

Something interesting appends with the units of the Taylor expansion. Let us investigate:

Taylor term	*quantity*	*units*	
$Fl'(0)d\overline{x}$	F	N	(8.9)
$''$	$l'(0)$	$\nexists$	(8.10)
$''$	$d\overline{x}$	m	(8.11)
$Fl''(0)xd\overline{x}$	F	N	(8.12)
$''$	$l''(0)$	$1/m$	(8.13)
$''$	$xd\overline{x}$	m^2	(8.14)
$Fl'''(0)x^2d\overline{x}$	F	N	(8.15)
$''$	$l'''(0)$	$1/m^2$	(8.16)
$''$	$x^2d\overline{x}$	m^3	(8.17)
$\vdots$	$\vdots$	$\vdots$	

Since $xd\overline{x}$ has units m^2 and $x^2d\overline{x}$ has units m^3, we pose $\gamma d\overline{A} := xd\overline{x}$ and $\alpha d\overline{V} := x^2 d\overline{x}$. Furthermore, as $l'(0)$ has no units, we define it as the baseline $l'(0) := 1$ and we define $l''(0) := l_A/L$ and $l'''(0) := l_V/A$ as they respectively have units m^{-1} and m^{-2}. For empirical reasons (e.g., the observable universe is a sphere), we consider that $\gamma d\overline{A}$ describes the surface of a sphere and that $\alpha d\overline{V}$ describes the volume of a sphere. Therefore, to properly link $\gamma d\overline{A}$ to $xd\overline{x}$, the factor γ must be $1/(4\pi)$ and the factor α must be $3/(4\pi)$. Introducing these replacements, the equation of state becomes:

$$TdS = -Pd\overline{t} + Fd\overline{x} + l_A \frac{F}{4\pi L} d\overline{A} + l_V \frac{3F}{8\pi A} d\overline{V} + O(\overline{x}^3)d\overline{x} \tag{8.18}$$

where l_A and l_V are leftovers of the Taylor coefficients. We can recover three relations by varying the intensity of the Taylor approximation.

$$TdS = -Pd\overline{t} + Fd\overline{x} + O(\overline{x})d\overline{x} \tag{8.19}$$

$$TdS = -Pd\overline{t} + Fd\overline{x} + l_A \frac{F}{4\pi L} d\overline{A} + O(\overline{x}^2)d\overline{x} \tag{8.20}$$

$$TdS = -Pd\overline{t} + Fd\overline{x} + l_A \frac{F}{4\pi L} d\overline{A} + l_V \frac{3F}{8\pi A} d\overline{V} + O(\overline{x}^3)d\overline{x} \tag{8.21}$$

With the first relation, and by posing $O(\overline{x})d\overline{x} \to 0$, we recover the first proposed partition function:

$$TdS = -Pd\overline{t} + Fd\overline{x} \tag{8.22}$$

Thus, the results derived with the previous partition function are importable into this more general equation of state.

8.3 Gravitational constant

To find a suitable definition for G, we must derive Newton's law of gravitation from the equation of states. A derivation of Newton's law of gravitation from the entropic perspective has been done before[29] by Erik Verlinde. His derivation can be imported into our equation of states. To obtain G, we start from the power-time formulation expanded with two Taylor terms:

[29] Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. DOI: 10.1007/JHEP04(2011)029. URL https://doi.org/10.1007/JHEP04(2011)029

$$TdS = -Pd\overline{t} + Fd\overline{x} + l_A \frac{F}{4\pi L} d\overline{A} + O(\overline{x}^2)d\overline{x} \tag{8.23}$$

Then, we pose $d\overline{t} = 0$ and $O(\overline{x}^2)d\overline{x} \to 0$. We obtain:

$$TdS = Fd\overline{x} + l_A \frac{F}{4\pi L} d\overline{A} \tag{8.24}$$

We notice that the term $d\overline{x}$ grows linearly as the term $d\overline{A}$ grows quadratically. Thus, as $\overline{x}$ is increased, there will be a point where $d\overline{A} \gg d\overline{x}$ (recall that $d\overline{A} = \overline{x}d\overline{x}$). The approximation yields:

$$TdS = l_A \frac{F}{4\pi L} d\overline{A} \tag{8.25}$$

This regime contains the holographic principle and, as a result, the entropy of the system grows proportional to $\overline{x}^2$, an area law. To recover Newton's law of gravity, and consistent with the holographic principle, we further pose the assumption that an entropy is associated to this area law and is given by bits occupying a small area L^2 on the surface of a sphere. In this case, the total number of bits on the surface is given by:

$$N = \frac{4\pi x^2}{L^2} \qquad \text{holographic assumption} \tag{8.26}$$

The term $\overline{x}d\overline{x}$ of the equation of state is associated to $x^2/2$ in the partition function. As a result of the equipartition theorem, which applies to quadratic energy terms, the average energy will be $\overline{E} = k_B T/2$. Multiplying $\overline{E}$ by N, we get the total energy associated with $\overline{x}d\overline{x}$:

$$E = \frac{1}{2}\left(\frac{4\pi x^2}{L^2}\right) k_B T \tag{8.27}$$

$$\implies T = \frac{L^2}{2\pi k_B}\frac{E}{x^2} \tag{8.28}$$

Consistent with thermodynamic equilibrium, we obtain a temperature T. As our goal is to recover the gravitational force, we inject this temperature in the entropic force relation.

$$Fd\overline{x} = TdS \qquad \text{entropic force} \tag{8.29}$$

$$Fd\overline{x} = \left(\frac{L^2}{2\pi k_B}\frac{E}{x^2}\right) dS \qquad \text{derived temperature} \tag{8.30}$$

$$F = \left(\frac{L^2}{2\pi k_B}\frac{E}{x^2}\right) \frac{dS}{d\overline{x}} \tag{8.31}$$

What then is $dS/d\overline{x}$? Recall equation 7.45; the connection between the reduced Compton wavelength and the distance entropy.

$$F = \left(\frac{L^2}{2\pi k_B}\frac{E}{x^2}\right)\left(2\pi k_B \frac{mc}{\hbar}\right) \qquad \text{Compton wavelength} \tag{8.32}$$

$$F = \left(\frac{L^2 c}{\hbar}\right)\frac{Em}{x^2} \qquad \text{clean up} \tag{8.33}$$

We then convert E to its rest mass via $E = mc^2$.

$$F = \left(\frac{L^2c^3}{\hbar}\right)\frac{Mm}{x^2} \tag{8.34}$$

We obtain the Newton's law of gravitation along with a definition for G.

$$F = G\frac{Mm}{x^2} \tag{8.35}$$

$$\implies G := \frac{L^2c^3}{\hbar} \tag{8.36}$$

which further implies that

$$L = \sqrt{\frac{\hbar G}{c^3}} \qquad \text{Planck's length} \tag{8.37}$$

8.4 *Energy-to-frequency equation*

We have previously shown that $0 = Fd\overline{x} - Pd\overline{t}$ implies that all observers agree on a certain speed c. We now ask; what is the energy relation of such a system with respect to frequency? To derive the result, we assume that the system has access to a pool of energy. The question, then becomes; how much energy must be taken from the pool increase the frequency to f? With access to a pool of energy and posing $d\overline{x} = 0$, the power-time formulation becomes:

$$TdS = d\overline{E} - Pd\overline{t} \tag{8.38}$$

$$T\frac{dS}{dt} = \frac{d\overline{E}}{d\overline{t}} - P \tag{8.39}$$

We consider the case of speed c. Thus, as we have seen in the section on special relativity, this implies that $dS/d\overline{t} = 0$.

$$d\overline{E} = Pd\overline{t} \tag{8.40}$$

We change to the action-frequency formulation by posing $f = 1/t$ and $-P = \mathcal{S}t^2$

$$-d\overline{E} = \mathcal{S}d\overline{f} \tag{8.41}$$

Then, integrating:

$$-\int d\overline{E} = \mathcal{S}\int d\overline{f} \tag{8.42}$$

$$-\overline{E} + C_1 = \mathcal{S}\overline{f} + C_2 \tag{8.43}$$

Here, $\overline{E}$ is the energy that must be taken from the pool for the system to occupy a micro-state with frequency $\overline{f}$. Reversing the sign of $\overline{E}$ and posing the integration constants to 0, we obtain the energy associated with it: $E = \mathcal{S}f$. Furthermore, recall that we posed $dS/d\overline{t} = 0$ which is associated with speed c.

Indeed, posing $\mathcal{S} = h$, the Planck action, we do recover the energy-to-frequency relation of a photon: $E = hf$.

8.5 Planck units

We have now obtained a definition for three of the fundamental constants.

$$h := \mathcal{S} \qquad c := \frac{P}{F} \qquad G := \frac{L^2 c^3}{\hbar} \tag{8.44}$$

Thus, we can now show that the Lagrange multipliers of the equation of states P and F are indeed the Planck units.

	expression	*quantity*	
$G = \frac{L^2c^3}{\hbar} \implies$	$L = \sqrt{\frac{\hbar G}{c^3}}$	Planck's length	(8.45)
	$t = \frac{L}{c} = \sqrt{\frac{\hbar G}{c^5}}$	Planck's time	(8.46)
	$P = t^{-2}\mathcal{S} = 2\pi\frac{c^5}{G}$	Planck's power*	(8.47)
$\frac{P}{F} = c \implies$	$F = 2\pi\frac{c^4}{G}$	Planck's force*	(8.48)
	$\vdots$	$\vdots$	

*The reader will notice that we have obtained the definitions of P and F with an added multiplication constant 2π; whereas in the literature these quantities are defined without it. The definitions we have here are actually the correct ones. Indeed, in the literature, the Planck time is connected to the Planck angular frequency via $\omega_P = 1/t_P$. In reality, however $\omega = 2\pi/t$. Thus, for our equations to balance out, we cannot ignore the factor 2π and must use the corrected value for the Planck units which are $P = 2\pi c^5/G$ and $F = 2\pi c^4/G$.

8.6 General relativity

In this section, we will show how the term $d\overline{A}$ suggests that general relativity is entropic and emergent. Our goal is to derive the Einstein field equation of general relativity, starting from the $d\overline{A}$ regime. First, we start from the power-time formulation expanded with two Taylor terms:

$$TdS = -Pd\bar{t} + Fd\bar{x} + l_A \frac{F}{4\pi L} d\overline{A} + O(\bar{x}^2)d\bar{x} \tag{8.49}$$

Then, we pose $d\bar{t} = 0$ and $O(\bar{x}^2)d\bar{x} \to 0$. We obtain:

$$TdS = Fd\bar{x} + l_A \frac{F}{4\pi L} d\overline{A} \tag{8.50}$$

We notice that the term $d\bar{x}$ grows linearly and the term $d\overline{A}$ grows quadratically. Thus, as $\bar{x}$ is increased, there will be a point where $d\overline{A} \gg d\bar{x}$. The approximation yields:

$$TdS = l_A \frac{F}{4\pi L} d\overline{A} \tag{8.51}$$

Deriving general relativity from $TdS = l_A \frac{F}{4\pi L} dA$ has indeed been done before in the literature, notably by Ted Jacobson[30], then later (and differently) by Erik Verlinde[31]. Furthermore, key insights were provided by Christoph Schiller[32]. Here, we will provide a sketch of the proof by Ted Jacobson as summarized by Schiller.

[30] Ted Jacobson. Thermodynamics of spacetime: The einstein equation of state. Phys. Rev. Lett., 75:1260–1263, Aug 1995. DOI: 10.1103/PhysRevLett.75.1260. URL https://link.aps.org/doi/10.1103/PhysRevLett.75.1260

[31] Erik P. Verlinde. On the origin of gravity and the laws of newton. Journal of High Energy Physics, 2011(4):29, Apr 2011. ISSN 1029-8479. DOI: 10.1007/JHEP04(2011)029. URL https://doi.org/10.1007/JHEP04(2011)029

[32] Christoph Schiller. General relativity and cosmology derived from principle of maximum power or force. International Journal of Theoretical Physics, 44(9):1629–1647, Sep 2005. ISSN 1572-9575. DOI: 10.1007/s10773-005-4835-2. URL https://doi.org/10.1007/s10773-005-4835-2

First, the entropic force F is constant throughout the system as a result of being a Lagrange multiplier. We have already shown that F is the Planck force. This has allowed us to derive special relativity and the speed of light; therefore, we must continue to use F as the Planck force here.

What then is L? Recall that earlier we used the Unruh temperature to link T to an acceleration and derive $F = ma$. Here and likewise, we will use special relativity to derive a relation between length and acceleration and use it to replace L. As per Schiller's paper, we select L as the maximum length that an accelerated object can have under special relativity[33].

[33] Christoph Schiller. General relativity and cosmology derived from principle of maximum power or force. International Journal of Theoretical Physics, 44(9):1629–1647, Sep 2005. ISSN 1572-9575. DOI: 10.1007/s10773-005-4835-2. URL https://doi.org/10.1007/s10773-005-4835-2; Wolfgang Rindler. Relativity: special, general, and cosmological, 2003; and Ray A D'Inverno. Introducing einstein's relativity. 1992

$$L = \frac{c^2}{2a} \tag{8.52}$$

L is perhaps better understood as the acceleration of circular motion ($r = v^2/a$) at the speed of light ($v = c$). In the present context, L is the length associated with the maximum force, the Planck force. In the context of maximums, the force cannot accelerate the object beyond the speed of light, and therefore is best defined for a circular motion produced by a force perpendicular to the direction of motion. The maximum acceleration changes the direction of the motion, but does not increases the speed beyond the speed of light.

With $F = 2\pi c^4/G$, we obtain:

$$TdS = l_A \frac{c^2}{G} ad\overline{A} \tag{8.53}$$

With this result, Jacobson's proof directly follows. Starting from $dE = TdS$, he first connects TdS to an arbitrary coordinate system and energy flow rates:

$$TdS = \int T_{ab} k^a d\Sigma^b \tag{8.54}$$

Here T_{ab} is an energy-momentum tensor, k is a killing vector field, and $d\Sigma$ the infinitesimal elements of the coordinate system. Jacobson then shows that the area part can be rewritten as follows:

$$ad\overline{A} = c^2 \int R_{ab} k^a d\Sigma^b \tag{8.55}$$

where R_{ab} is the Ricci tensor describing the space-time curvature. This relation is obtained via the Raychaud-Huri equation, giving it a geometric justification. Combining the two with a local law of conservation of energy, he obtains

$$\int T_{ab} k^a d\Sigma^b = l_A \frac{c^2}{G} \int R_{ab} k^a d\Sigma^b \tag{8.56}$$

, which can only be satisfied if

$$T_{ab} = l_A \frac{c^2}{G} \left[R_{ab} - \left(\frac{R}{2} + \Lambda \right) g_{ab} \right] \tag{8.57}$$

Here, the full field equations of general relativity are recovered, including the cosmological constant (as an integration constant). Only the numerical value of l_A remains. The exact formulation of the field equation is obtained by posing the numerical value to $g_A := 1/(8\pi)$.

Remark: Had we not used the corrected Planck force ($F = 2\pi c^4/G$), we would have a 2π term dividing T_{ab} and l_A would have been $1/4$. Thus, the difference would have been absorbable. However, using the corrected Planck force has the consequence that all dimensionless numerical multipliers are attributed to the Taylor coefficient, making the derivation more aesthetically pleasing.

8.7 Dark energy

Connecting dark energy to a volumetric entropy has been suggested and discussed by other authors before[34]. First, we start from the power-time formulation expanded with three Taylor terms:

$$TdS = -Pd\bar{t} + Fd\bar{x} + l_A \frac{F}{4\pi L} d\overline{A} + l_V \frac{3F}{8\pi A} d\overline{V} + O(\bar{x}^3) d\bar{x} \tag{8.58}$$

Them we pose $d\bar{t} = 0$ and $O(\bar{x}^3)d\bar{x} \to 0$. We obtain:

$$TdS = Fd\bar{x} + l_A \frac{F}{4\pi L} d\overline{A} + l_V \frac{3F}{8\pi A} d\overline{V} \tag{8.59}$$

We notice that as $d\bar{x}$ grows linearly, $d\overline{A}$ grows as the square and $d\overline{V}$ as the cube. Thus, there will be a point where $d\overline{V} \gg d\overline{A} \gg d\bar{x}$. The approximation yields:

$$TdS = l_V \frac{3F}{8\pi A} d\overline{V} \tag{8.60}$$

We notice that the factor F/A has the units of pressure. Hence, our goal will be to derive a value of the pressure p associated with volumetric entropy. As suggested by the factor F/A and in line with our earlier derivations, we will select F to be the corrected Planck force ($F = 2\pi c^4/G$) and will take A as the area of a sphere. In this case, the pressure relates to the force as

$$F = -pA \tag{8.61}$$

$$\implies p = -\frac{F}{A} = -\frac{F}{4\pi\bar{x}^2} \tag{8.62}$$

$$p = -\frac{c^4}{2G\bar{x}^2} \qquad \text{entropic pressure} \tag{8.63}$$

The sign of the force is negative because the force points in the direction of increased entropy, which is oriented outward of the enclosing area. Physically and as argued by Easson et al., it makes sense to connect the size of the sphere to the Hubble horizon. Therefore, we take the radius of the sphere to be the Hubble radius $x := c/H$. Finalizing our derivation, we obtain:

$$p = -\frac{c^2 H^2}{2G} \tag{8.64}$$

This is close to the current measured value for the negative pressure associated with dark energy[35]. As we can see, the suggested entropic derivation of dark energy applies to the third term of the Taylor expansion.

[34] Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic accelerating universe. Physics Letters B, 696(3): 273 – 277, 2011. ISSN 0370-2693. DOI: https://doi.org/10.1016/j.physletb.2010.12.025. URL http://www.sciencedirect.com/science/article/pii/S0370269310014048; and Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic inflation. International Journal of Modern Physics A, 27(12):1250066, 2012. DOI: 10.1142/S0217751X12500662. URL http://www.worldscientific.com/doi/abs/10.1142/S0217751X12500662

[35] Damien A. Easson, Paul H. Frampton, and George F. Smoot. Entropic accelerating universe. Physics Letters B, 696(3): 273 – 277, 2011. ISSN 0370-2693. DOI: https://doi.org/10.1016/j.physletb.2010.12.025. URL http://www.sciencedirect.com/science/article/pii/S0370269310014048

9 Discussion - Arrow of time

Adding a time variable to a partition function adds a whole new dynamic to a thermal system. The system now becomes aware of future, past, and present configurations and can translate from time to space and from space to time for an entropic cost (provided that various limits are respected). By studying thermodynamic cycles involving space and time, we investigated what happens to the entropy when a system is translated forward or backward in time and draw conclusions that pertain to the arrow of time. In the model presented, space serves as an entropy sink for time; whose role is to deplete future alternatives to power *change* in the universe.

9.1 Negative power

In the power-time formulation, increasing $\bar{t}$, while keeping the other variables constant, decreases the entropy. Indeed, starting with the power-time formulation and posing $d\bar{x} = 0$, we obtain:

$$TdS = -Pd\bar{t} \tag{9.1}$$

$$\implies T\frac{dS}{d\bar{t}} = -P \tag{9.2}$$

$$\tag{9.3}$$

This result is expected for the following reason: to obtain the relation $d\bar{x} = cd\bar{t}$ with the correct signs, the power P must have a different sign than the force F in the equation of states. Thus, a positive force implies a negative power and vice versa. As we require a positive force to recover $F = ma$ (and not $F = -ma$), the sign of the force is already chosen for us. Therefore, the power must be negative.

We will now discuss this result in more detail.

Question: What is a negative power?

Let's take an example. Consider the case of an electric car; whose engine is powered by a battery. To propel the car, the battery supplies power to the engine. If the driver hits the breaks, such that regenerative breaking kicks in, the flow of power will reverse and the engine will supply power to the battery. Thus, the power is now considered to be negative and occurs when the engine depletes the energy of the system (e.g. the car slows down) to supply power to the battery.

Question: Why does time have a negative power?

Power is associated with time because it powers all *changes* that occur in the universe. To understand why it is negative, it helps to understand negative power in the context of thermodynamics. To do so, let's first recall its more familiar cousin: the negative temperature. If we understand temperature as the random movements of molecules, then a temperature is always equal to or above zero. However, statistical physics admits a generalized definition of temperature as the trade-off between energy and entropy. Most systems cannot admit a negative temperature because their entropy will always increase at higher energies; however, for some systems, e.g. the population inversion in a laser, the entropy saturates at higher energies. Thus, a negative temperature is possible.

In regards to time, the negative power has essentially the same interpretation; increasing time, while keeping the other variables constant, decreases the entropy. A decrease in entropy over time produces a negative entropic power.

9.2 *The second law of thermodynamics as an opposition to negative power*

Question: How does this result reconcile with the second law of thermodynamics, which states that entropy increases with time (or in some ideal cases stays constant)?

The power-time formulation admits other terms: $d\overline{x}$, $d\overline{A}$, and $d\overline{V}$. The term $-Pd\overline{t}$ encourages a reduction in the entropy over time, but the other variables, as their signs are positive, work in the other direction. Thus, the entropy of the system as a whole need not necessarily decrease over time. It is more accurate to say that increasing $\overline{t}$, while keeping the other variables constant, decreases the entropy. We will now study this into more detail.

To offset the decrease in entropy caused by the negative power, we suggest a proportional increase in the quantities $\overline{x}$, $\overline{A}$, and $\overline{V}$.

To simplify the power-time formulation, let us rename $\kappa := \frac{F}{16\pi L}$ and $p := \frac{3g_V F}{4\pi A}$ and pose $O(\overline{x}^3)d\overline{x} \to 0$. We obtain:

$$TdS = -Pd\overline{t} + Fd\overline{x} + \kappa d\overline{A} + pd\overline{V} \tag{9.4}$$

Dividing both sides by $d\overline{t}$, we obtain:

$$T\frac{dS}{d\overline{t}} = -P + F\frac{d\overline{x}}{d\overline{t}} + \kappa\frac{d\overline{A}}{d\overline{t}} + p\frac{d\overline{V}}{d\overline{t}} \tag{9.5}$$

This result puts in opposition the change of entropy caused by a change of $\overline{t}$ to the change in entropy caused by a change of $\overline{x}$, $\overline{A}$ and $\overline{V}$. To investigate this result, let us look at these three cases:

$$F\frac{dx}{dt} + \kappa\frac{dA}{dt} + p\frac{dV}{dt} < P \quad \implies \frac{dS}{dt} < 0 \quad \text{decreasing entropy} \tag{9.6}$$
$$F\frac{dx}{dt} + \kappa\frac{dA}{dt} + p\frac{dV}{dt} = P \quad \implies \frac{dS}{dt} = 0 \quad \text{constant entropy} \tag{9.7}$$
$$F\frac{dx}{dt} + \kappa\frac{dA}{dt} + p\frac{dV}{dt} > P \quad \implies \frac{dS}{dt} > 0 \quad \text{increasing entropy} \tag{9.8}$$

At (9.7), we have an inflexion point and a shift occurs in the direction of the production of entropy over time. It is the point at which the production of entropy caused by the space quantities overtake and exceed the reduction in entropy caused by the time quantity. The second law of thermodynamics states that $dS/dt \geq 0$ and will hold for (9.7) and (9.8), but will be violated for (9.6).

9.3 *Arrow of time*

In this section, we will explain why these results provide us with an understanding of the arrow of time. Indeed, it links the arrow of time to three concepts: 1) a reduction in entropy over time caused by the negative power, 2) an increase in entropy over time caused by the space quantities, and 3) a closed system's inability to reduce its own entropy. We will see how it corresponds to an observer's perception of time.

1. *At the beginning of time* all possible future alternatives are compatible with the present. Thus, the pool of entropy accessible to $\bar{t}$ is maximal. In contrast, the entropy associated with the space quantities is zero. Thus, the occupied micro-states have to be located at the same point in space. This matches our current empirical data regarding the Big Bang for which the entropy of space was very low and the entropy of time, as the future was as of yet undetermined, was very high.

2. *During the evolution* the future becomes past and the possible future alternatives are rarefied. This reduction in entropy caused by a growth in $\bar{t}$ produces a negative entropic power fuelled by the growth of entropy in the space quantities.

3. *At the "end of time"* there is no future alternatives. The full history of the system is now "set in stone". The system can no longer produce an entropic power to fuel changes and the entropy associated with the space quantities is at its maximum.

Question: The conventional wisdom is that the arrow of time is connected to an increase in entropy with time. Are you suggesting something else?

A partition function constructed without the use of a time quantity will follow the second law of thermodynamics. This statistical effect is partially explained by the H-theorem of Boltzmann; however, this changes when time is inserted as a thermodynamic quantity. Such a partition function then becomes aware of past, present, and future configurations. The rarefaction of futures configurations as time is increased is associable to a time which moves forward by closing future alternatives as it creates a past. Thus, an increase in the time quantity, while keeping other quantities constant, must be followed by a decrease in entropy.

To help fixate the idea, let us look at an example:

9.4 *The physics of future alternatives*

Here, we give a simple system which follows the requirements of the equation of states.

Suppose a system with n open binary future alternatives. At $\bar{t} = 0$, there are 2^n possible futures each equally compatible with the present macroscopic state. Thus, the entropy of the system (which includes a description of its possible futures) is equal to $S = k_B n \ln 2$. As time is increased, *events* occurs and future alternatives are closed. Say, at $\bar{t} = 1$, one event occurs: Thus, one future alternative becomes fixed to a specific value and the entropy of the system is reduced to $S = k_B(n-1)\ln 2$.

For instance, we might have:

$\bar{t}$	*event*	*future alternatives*	*entropy*	
0	Big Bang	$\{b_1, b_2, b_3, b_4, ..., b_n\}$	$k_B n \ln 2$	(9.9)
1	$b_3 \to 0$	$\{b_1, b_2, b_3 := 0, b_4, ..., b_n\}$	$k_B(n-1)\ln 2$	(9.10)
2	$b_1 \to 1$	$\{b_1 := 1, b_2, b_3 := 0, b_4, ..., b_n\}$	$k_B(n-2)\ln 2$	(9.11)
$\vdots$	$\vdots$	$\vdots$	$\vdots$	

As events occurs over time, an entropic power is generated. Furthermore, the second law of thermodynamics imposes that the space quantity ($Fd\bar{x}$) must grow proportionally. To maintain $dS/d\bar{t} = 0$, the growth must correspond to $d\bar{x} = cd\bar{t}$; special relativity. Extending this example to the continuous partition function, we also recover general relativity and dark energy as per the earlier derived equations of states. In the continuous case, we would use the natural bit (the nat, in base e) to express future possibilities. A continuous event would consume a non-integer quantity of future possibilities.

Question: But a system cannot decrease its entropy over time without violating the second law of thermodynamics!

A system can decrease its entropy if it is connected to an entropy sink. For example, biological life can reduce its entropy but only at the cost of severely increasing it in its environment. This requires excess energy and, in the case of Earth, the Sun supplies it. Thus, the power-time conjugate can decrease the entropy as long as it is connected to a sink.

Question: So, there should be a sink in the universe available to offset the decrease in entropy caused by increasing $\overline{t}$?

In the case of time, the sink is the universe itself. The laws of physics that we have derived are in fact the limits required to produce an entropy sink of sufficient size to accommodate a forward direction of time for an observer (we will discuss this more rigorously in a moment in the section on limiting relations).

Question: Can we calculate the exact future before it occurs?

An observer cannot pre-calculate his exact future before it occurs without increasing the size of the entropy sink. Here we make a distinction between calculating a probable future versus the exact future. Calculating a probable future does not necessarily imply a reduction of entropy within the system, but calculating the exact future requires consuming the entropy of all possible alternative futures. Therefore, an entropy sink is required to offset the reduction. Calculating an exact future is equivalent to causing it.

Question: Does the second law of thermodynamics need to be corrected for the wider system, which includes future states?

Yes. Time is usually considered to be an independent background to statistical physics and, to our knowledge, statistical physics has not been used with a time quantity before. When we do add time as a thermodynamic quantity to a partition function, a new behaviour emerges. Indeed, an observer cannot move into the future unless all alternative futures are 'closed'. Thus, its time-entropy must decrease when he does. The second law of thermodynamics is a consequence of the system increasing its space-entropy to offset the reduction in future alternatives as time moves along. Thus, this system follows a general entropy conservation law.

9.5 *Limiting relations*

With our new interpretation of space as an entropy sink for time, let us immediately prove three limits from first principle: the speed of light, a limiting stiffness, and a limiting volumetric flow rate applicable to the universe. To prove that these are limits, we will consider the assumption that an observer who evolves forward in time must see a growth in the size of its available entropy sink to offset the reduction in future alternatives. The limit occurs when the sink exactly offsets the reduction in entropy attributable to time (in which case $dS/dt = 0$). First, let us see how the power-time formulation implies a limiting speed.

$$TdS = -Pd\bar{t} + Fd\bar{x} \tag{9.12}$$

$$\frac{T}{F}\frac{dS}{d\bar{t}} = -\frac{P}{F} + \frac{d\bar{x}}{d\bar{t}} \tag{9.13}$$

To see why this implies a limiting speed, first consider that the units of this equation are *length/time* and hence are indeed describing a speed. Second, consider the following three cases:

$$\frac{d\bar{x}}{d\bar{t}} = \frac{P}{F} \implies \frac{dS}{d\bar{t}} = 0 \tag{9.14}$$

$$\frac{d\bar{x}}{d\bar{t}} < \frac{P}{F} \implies \frac{dS}{d\bar{t}} < 0 \tag{9.15}$$

$$\frac{d\bar{x}}{d\bar{t}} > \frac{P}{F} \implies \frac{dS}{d\bar{t}} > 0 \tag{9.16}$$

We notice a reversal in the production of entropy at the inflection point where $dS/d\bar{t} = 0$. Therefore, for an observer at rest to evolve forward in time, it must see its entropy sink grow at the speed of $c := P/F$. Therefore, the entropy sink of an observer moving forward in time must grow at the speed of light.

The following relations each characterize a limiting quantity.

limited quantity	*units*	*limiting relation*	
Power	J/s	$\frac{T}{F}\frac{dS}{d\bar{t}} = -P$	(9.17)
Speed	m/s	$\frac{T}{F}\frac{dS}{d\bar{t}} = \frac{d\bar{x}}{d\bar{t}} - \frac{P}{F}$	(9.18)
Stiffness	m^2/s	$\frac{T}{\kappa}\frac{dS}{d\bar{t}} = \frac{d\overline{A}}{d\bar{t}} - \frac{P}{\kappa}$	(9.19)
Volumetric flow rate	m^3/s	$\frac{T}{p}\frac{dS}{d\bar{t}} = \frac{d\overline{V}}{d\bar{t}} - \frac{P}{p}$	(9.20)

Each relation can easily be obtained from the power-time formulation by posing the other quantities as 0. To show that the quantities are inflection limits, it suffices to notice that they each correspond to a growth of the entropy sink that an observer at rest must see to fuel its forward translation in time.

It is well known that a limiting speed implies special relativity, but what about the other two limits? It is less known, but a maximum stiffness does imply general relativity. In this context, we can interpret space as being very stiff but nonetheless compressible. The maximum volumetric flow rate is associated with dark energy and is responsible for the Hubble horizon - beyond which the flow rate would be exceeded. These are in fact the approaches (in disguise) that we took to derive general relativity and dark energy earlier.

10 *Fluctuating space-time*

What is thermal time and thermal space? Consider the thermodynamic quantities t and x of the power-time formulation. Their average value is given by the standard relations (from 5.10):

	quantity	average	
thermal-time	t	$\bar{t} = \frac{-\partial \ln Z}{\partial P}$	(10.1)
thermal-space	x	$\bar{x} = \frac{-\partial \ln Z}{\partial F}$	(10.2)

Furthermore, as thermal-time and thermal-space are thermodynamic averages, they will undergo fluctuations (from 5.10):

	quantity	fluctuation	
thermal-time	t	$\overline{(\Delta t)^2} = \frac{\partial^2 \ln Z}{\partial P^2}$	(10.3)
thermal-space	x	$\overline{(\Delta x)^2} = \frac{\partial^2 \ln Z}{\partial F^2}$	(10.4)

Using the original argument made by Einstein in 1905, which led to the derivation of Brownian motion, we argue here that fluctuations of the t and x variables produce a universal Brownian motion along the axis themselves. What does a thermal space-time with fluctuations look like? The consequences of such are nothing to be feared; indeed, we will shortly show that Brownian motion over $\bar{x}$ will produce the Schrödinger equation and that Brownian motion over both $\bar{x}$ and $\bar{t}$ will produce the Dirac equation.

Question: Are we suggesting a pilot-wave interpretation where particles undergo Brownian motion until a measurement is made?

Not at all. Rather, we are suggesting that any positional or time information undergoes a "Dirac equation-like diffusion" so as to make positional or time information perishable over time. To illustrate, we can imagine placing a mark at a position in space. After a certain time, Brownian motion will diffuse the position of the marker at any number of possible locations until its actual position is measured again. Instead of being punctual, the marker could be continuous and weighted and the same diffusion-like behavior will be observed. This Brownian motion would universally apply to the axis itself. This is not a claim that a particle is punctual.

10.1 Schrödinger equation

The derivation of the Schrödinger and Dirac equations as a result of universal Brownian motion has already been done by other authors. Therefore, we can import their proofs into our derivation. Here, we will offer a sketch and refer to their respective authors for the more rigorous treatment. The derivation of the Schrödinger equation from Brownian motion was done by Nelson[36] and reviewed by the same author some 46 years later[37]. The field is stochastic mechanics and it connects very nicely to our thermodynamic description of the world.

[36] Edward Nelson. Derivation of the schrodinger equation from newtonian mechanics. Phys. Rev., 150:1079–1085, Oct 1966. DOI: 10.1103/PhysRev.150.1079. URL https://link.aps.org/doi/10.1103/PhysRev.150.1079

[37] Edward Nelson. Review of stochastic mechanics. In Journal of Physics: Conference Series, volume 361, page 012011. IOP Publishing, 2012

Nelson first considers the Langevin equation,

$$d\left[x(t)\right] = v(t)dt \tag{10.5}$$

$$d\left[v(t)\right] = -\frac{\gamma}{m}v(t)dt + \frac{1}{m}W(t)dt \tag{10.6}$$

, which describes a particle in a fluid undergoing a Brownian motion as a result of the random collisions with the water molecules. Here $W(t)$ is a *noise* term responsible for the Brownian motion and $v(t)$ is a viscosity term specific to the properties of the fluid.

Nelson replaces the acceleration $d[v(t)]/dt$ by F/m (from $F = ma$). Then, he is able to show that the Langevin equation in gradient form becomes:

$$\nabla\left(\frac{1}{2}\vec{u}^2 + D\nabla\cdot\vec{u}\right) = \frac{1}{m}\nabla V \tag{10.7}$$

where $D := \hbar/(2m)$ is the diffusion coefficient, where $\vec{F} = -\nabla V$, where $\vec{u} = v\nabla\ln\rho$ and ρ is the probability density of $x(t)$. As this is a sketch, the proof of 10.7 is omitted here but can be reviewed in Nelson's paper. Eliminating the gradients on each side and simplifying the constants, Nelson obtains:

$$\frac{m}{2}\vec{u}^2 + \frac{\hbar}{2}\nabla\cdot\vec{u} = V - E \tag{10.8}$$

where E is the arbitrary integration constant. Nelson then converts this equation to a linear equation via a change of variable applied to the term $\vec{u}^2$. Posing,

$$\vec{u} = \frac{\hbar}{m}\frac{1}{\psi}\nabla\psi \tag{10.9}$$

Nelson obtains

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V - E\right]\psi = 0 \tag{10.10}$$

which is the time-independent Schrödinger's equation. The time-dependent Schrödinger's equation is recovered as per the usual replacement $\psi := e^{R+iS}$. Finally, Nelson obtains:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(x,t)\right]\psi(x,t) \tag{10.11}$$

, which is the time-dependent Schrödinger's equation.

10.2 Dirac equation

We recently used the entropic force $TdS = Fdx$ and the Unruh temperature to recover $F = ma$. Then, we used $0 = -Pdt + Fdx$ to recover special relativity. Finally, we showed that a Brownian motion resulting from the thermal fluctuations on x recovers the Schrödinger equation as a thermo-statistical analogue to $F = ma$. Of course, the natural question to ask is this: will the thermal fluctuations of both t and x be enough to recover the Dirac equation as a thermo-statistical analogue to special relativity? The answer is yes!

Similarly to the stochastic-mechanical derivation of the Schrödinger equation, other authors previously derived the Dirac equation from universal Brownian motion[38]. In the original stochastic-mechanical derivation, the origin of such universal Brownian motion is ambiguous and is, at best, imported as a hypothesis. Thus, the benefit of our construction is to provide a thermal source of such universal Brownian motion. Hence, the derivation of the Dirac equation and the Schrödinger equation by these authors can nicely be imported into our thermodynamic construction.

[38] D Mckeon and G N. Ord. Time reversal in stochastic processes and the dirac equation. Physical review letters, 69:3–4, 08 1992

The derivation of the Dirac equation was noticed by studying random walk effects that were applicable to telegraphic communication. McKeon and Ord propose a random walk model in space and in time which, once applied to the telegraph equations, produces the Dirac equation. We provide a sketch of the proof here and refer to the authors' paper for the rigorous treatment. Starting from the equation for a random walk in space, the authors obtain:

$$P_\pm(x, t+\Delta t) = (1 - a\Delta t)P_\pm(x \mp \Delta x, t) + a\Delta t P_\mp(x \pm \Delta x, t) \tag{10.12}$$

Afterward, the authors extend this equation with a random walk in time and obtain:

$$F_\pm(x,t) = (1 - a_L\Delta t - a_R\Delta t)F_\pm(x \mp \Delta x, t - \Delta t) + \\ a_{L,R}\Delta t B_\pm(x \mp \Delta x, t + \Delta t) + a_{R,L}\Delta t F_\mp(x \pm \Delta x, t - \Delta t) \tag{10.13}$$

where $F_\pm(x,t)$ is the probability distribution to go forward in time and $B_\pm(x,t)$ the probability distribution to go backward in time. They then introduce a causality condition such that $F_\pm(x,t)$ and $B_\pm(x,t)$ only depends on probabilities from the past.

$$F_\pm(x,t) = B_\mp(x \pm \Delta x, t + \Delta t) \tag{10.14}$$

From equation 10.13 and 10.14, they get

$$B_\pm(x,t) = (1 - a_L\Delta t - a_R\Delta t)B_\pm(x \mp \Delta x, t + \Delta t) + \\ a_{L,R}\Delta t B_\mp(x \pm \Delta x, t + \Delta t) + a_{R,L}\Delta t F_\pm(x \mp \Delta x, t - \Delta t) \tag{10.15}$$

In the limit $\Delta x, \Delta t \to 0$ and with $\Delta x = v\Delta t$, they get

$$\pm v\frac{\partial F_\pm}{\partial x} + \frac{\partial F_\pm}{\partial t} = a_{L,R}(-F_\pm + B_\pm) + a_{R,L}(-F_\pm + F_\mp) \tag{10.16}$$

$$\pm v\frac{\partial B_\mp}{\partial x} + \frac{\partial B_\mp}{\partial t} = a_{L,R}(-B_\mp + F_\mp) + a_{R,L}(-B_\mp + B_\pm) \tag{10.17}$$

Posing these changes of variables,

$$A_\pm = (F_\pm - B_\mp)\exp[(a_L + a_R)t] \tag{10.18}$$

$$\lambda := -a_L + a_R \tag{10.19}$$

then 10.17 becomes

$$v\frac{\partial A_\pm}{\partial x} \pm \frac{\partial A_\pm}{\partial t} = \lambda A_\mp \tag{10.20}$$

Finally, they pose $v = c$, $\lambda = mc^2/\hbar$ and $\psi = F(A_+, A_-)$, and they get

$$i\hbar\frac{\partial \psi}{\partial t} = mc^2\sigma_y\psi - ic\hbar\sigma_z\frac{\partial \psi}{\partial x} \tag{10.21}$$

which is the Dirac equation in 1+1 space-time.

11 Quantum Field Theory and the actual world

The connection between classical statistical physics and quantum field theory is well established[39]. In classical statistical physics, we have:

[39] Barry M McCoy. The connection between statistical mechanics and quantum field theory. arXiv preprint hep-th/9403084, 1994

$$\overline{O_j} = \frac{1}{Z}\sum_{q\in\mathcal{Q}} O_j e^{-E(q)/kt} \qquad \text{where } Z = \sum_{q\in\mathcal{Q}} e^{-E(q)/kT} \tag{11.1}$$

and in quantum field theory, we have:

$$\overline{O_j} = \frac{1}{Z_E}\int [dq] O_j e^{-S_E(q)/h} \qquad \text{where } Z_E = \int [dq] e^{-S_E(q)/h} \tag{11.2}$$

This is the Feynman path integral formulation of quantum field theory. The constructions are reciprocal; the thermal fluctuations of the first one are the quantum fluctuations of the second one. The Euclidean-space representation (above) can be connected to the Lorentzian representation via a Wick rotation $t \to it$.

The partition function for analytical facts (without temperature) can be formulated as a quantum field theory quite directly. We start with the power-time formulation:

$$Z = \sum_{q\in Q} e^{Pt(q)-Fx(q)} \tag{11.3}$$

$$Z = \sum_{q\in Q} e^{-\frac{1}{h}[-hPt(q)+hFx(q)]} \tag{11.4}$$

Then, posing $S_E(q) = hPt(q) - hFx(q)$

$$Z = \sum_{q\in Q} e^{-S_E(q)/h} \tag{11.5}$$

Then, posing the smoothness approximation we get

$$Z = \int_Q e^{-S_E(q)/h} dq \tag{11.6}$$

where the average value of each observables O_j is given by:

$$\overline{O_j} = \frac{1}{Z}\int O_j e^{-S_E(q)/h} dq \tag{11.7}$$

Here, the action S_E is an space-time event function dependent of both x and t. Quantum fluctuations around the averages replaces the thermal fluctuations of statistical physics. The usual interpretation of the path integral applies to understanding the actual world; incompatible paths over the micro-states interfere destructively and compatible ones interfere constructively, etc.

12 Conclusion

The starting point of the theory is the self-referential notion that a formal theory requires logical artefacts to be formulated. Thus, a derivation exclusively from $\exists$*(logical-artefacts)* is as safe as safe can be. Using logical artefacts, each synthetic fact is converted to an analytical fact via Miniversal logic. Then, because Miniversal logic is universal and only includes tautologies, we are able to claim that it is autological.

$$\textit{tautological} \wedge \textit{universal} \implies \textit{autological} \tag{12.1}$$

An autological physical theory has the same properties as reality. Thus, studying it using a formal meta-theory (such as Set theory) should be equivalent to using mathematics to understand the universe. This, is indeed what we find. Explicitly, we consider the objective properties of facts (description-length and proof-length) and avoid "poetic" properties (such as how *interesting* a fact might be to us). Doing so, the natural description of these facts is as an ensemble Z of feasible mathematics.

We find that when Z describes an ensemble of facts statistically weighted by their proof-length and description-length, we recover a description for which the familiar laws of physics emerge - including space and time. The world that is actual can be tentatively understood as an emergent average over the set of all possible facts of reality for certain fixed resources associated with a purely description-length and proof-length description. This interpretation is directly mappable to a thermodynamic system from which we understand and derive the laws of physics from.

Understanding the world from purely thermodynamic principles holds several conceptual advantages. The construction provides a possible mean to explain the origins of the laws of physics as per John Wheeler's suggestion of law without law (or as order from disorder) - in this case thermo-statistical disorder. Indeed, the obscure origin of the Dirac and Schrödinger equations is now clearly shown to be a result of thermal fluctuations applicable to x and t. Second, the laws of inertia, general relativity, and dark energy are simply the result of taking the Taylor expansion of an arbitrary space-encoding function. Third, as these laws are derived from the general equation of state of the system, the laws of physics do not need to be invoked as a 'special case'. In the present construction, the laws of physics are a consequence of the mere fact that the world can be expressed as a statistical ensemble involving time and space; hence, the 'axiomatic-load' of the construction is minimal.

The construction allows a possible explanation of the arrow of time. Indeed, moving into the future requires a negative power. A possible cause of negative power is closing future alternatives, which works towards reducing the entropy over time. To preserve the second law of thermodynamics, an entropy sink must be grown as time moves forward to offset said entropy reduction. Thus, the passage of time is heavily connected to the size of the entropy sink. The minimal growth rate requirements of this entropy sink are precisely the limits required to derive special relativity, general relativity, and dark energy. Therefore, we conclude that the entropy sink spawns the observable universe. The second law of thermodynamics, understood as an increase in entropy over time, is only half the truth. The second law is perceived in the entropy sink while the larger system, made to include future possibilities, has a constant entropy. In this system, future possibilities are consumed as time moves forward.

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