Gravitational Interaction between Photons

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Recently, it has been reported an experiment where a very weak laser beam passes through a dense cloud of *ultracold rubidium atoms*. Under these circumstances, it was observed that the photons bound together in pairs or triplets, suggesting an unexpected *attractive* interaction between them. Here, it is shown that mentioned interaction can be related to the *gravitational interaction*.

Key words: Interaction Gravitational, Casimir Force, Interaction between Photons.

1. Introduction

In a paper recently published in *Science* [1], researchers have reported that when they have put a very weak laser beam through a dense cloud of *ultracold rubidium atoms* (as a *quantum* nonlinear medium), the photons bound together in pairs or triplets, suggesting an unexpected *attractive* interaction between them.

Here, it is shown that mentioned interaction is related to the *gravitational interaction*.

2. Theory

I have show in the *Mathematical Foundations of the Relativistic Theory of Quantum Gravity* [2] that, by combination of Gravitation and the *Uncertainty principle* it is possible to derive the expression for the *Casimir force*. The starting point is the expression of correlation between gravitational mass m_g and *rest* inertial mass, m_{i0}, obtained in the mentioned paper, i.e.,

$$\chi = \frac{m_g}{m_{i0}} = \left\{1 - 2\left[\sqrt{1 + \left(\frac{p}{m_{i0}c}\right)^2} - 1\right]\right\} \qquad (1)$$

where p is the variation in the particle's *kinetic momentum*; c is the light speed.

Thus, an uncertainty Δm_{i0} in m_{i0} produces an uncertainty Δp in p and therefore an uncertainty Δm_g in m_g, which according to Eq.(1) , is given by

$$\Delta m_g = \Delta m_{i0} - 2\left[\sqrt{1 + \left(\frac{\Delta p}{\Delta m_{i0}c}\right)^2} - 1\right]\Delta m_{i0} \qquad (2)$$

From the uncertainty principle for position and momentum, we know that the product of the uncertainties of the simultaneously measurable values of the corresponding position and momentum components are at least of the magnitude order of $\hbar$, i.e.,

$$\Delta p \Delta r \sim \hbar$$

Substitution of $\Delta p \sim \hbar/\Delta r$ into (2) yields

$$\Delta m_g = \Delta m_i - 2\left[\sqrt{1 + \left(\frac{\hbar/\Delta m_i c}{\Delta r}\right)^2} - 1\right]\Delta m_i \qquad (3)$$

Therefore if

$$\Delta r << \frac{\hbar}{\Delta m_i c} \qquad (4)$$

Then the expression (3) reduces to:

$$\Delta m_g \cong -\frac{2\hbar}{\Delta r c} \qquad (5)$$

Note that, Δm_g does not depend on m_g .

Consequently, an uncertainty ΔF in the gravitational force $F = -G m_g m'_g / r^2$, will be given by

$$\Delta F = -G\frac{\Delta m_g \Delta m'_g}{(\Delta r)^2} = $$

$$= -\left[\frac{2}{\pi(\Delta r)^2}\right]\frac{hc}{(\Delta r)^2}\left(\frac{G\hbar}{c^3}\right) \qquad (6)$$

The amount $\left(G\hbar/c^3\right)^{1/2} = 1.61\times 10^{-35}m$ is called the *Planck length*, l_{planck} ,(the length scale on which quantum fluctuations of the metric of the space time are expected to be of order unity).

Thus, we can write the expression of ΔF as follows

$$\Delta F = -\left(\frac{2}{\pi}\right)\frac{hc}{(\Delta r)^4} l_{planck}^2 = = -\left(\frac{\pi}{480}\right)\frac{hc}{(\Delta r)^4}\left[\left(\frac{960}{\pi^2}\right) l_{planck}^2\right] = = -\left(\frac{\pi A_0}{480}\right)\frac{hc}{(\Delta r)^4} \tag{7}$$

or

$$F_0 = -\left(\frac{\pi A_0}{480}\right)\frac{hc}{r^4} \tag{8}$$

which is the expression of the *Casimir force* for $A = A_0 = \left(960/\pi^2\right) l_{planck}^2$.

Now, multiplying Eq. (8) (the expression of F_0) by n^2 we obtain

$$F = n^2 F_0 = -\left(\frac{\pi n^2 A_0}{480}\right)\frac{hc}{r^4} = -\left(\frac{\pi A}{480}\right)\frac{hc}{r^4} \tag{9}$$

This is the general expression of the *Casimir force*.

We can then conclude that *the Casimir effect* is just a *gravitational* effect related to the *uncertainty principle*. In this context, the nature of the Casimir *force* is clearly gravitational as shown in the derivation of Eq. (9), which expresses, in turn, the intensity of the gravitational force *in the case of very small scale* (r very small) [1].

Now consider the discovery reported recently in *Science* [1]. When the researchers have put a very weak laser beam through a dense cloud of *ultracold rubidium atoms* [2], the photons bound together in pairs or triplets, suggesting an unexpected *attractive* interaction between them. Now, we will show that the nature of this interaction is *gravitational*.

According mentioned in the paper, the length of the cloud of ultracold rubidium atoms were of approximately $130\mu m$ (along the propagation direction), while the transverse extent of the probe beam waist had about $4.5\mu m$. Therefore, the distances r between the photons of the cloud were very small. As we have already seen, at very small scale, the *gravitational interaction* cannot be trated via usual Newton's equation of gravitation. In this case, Eq. (9) must be used. Thus, assuming $A \approx \lambda^2 = (c/f)^2 \cong 10^{-13} m^2$, and substituting this value into Eq. (9), we obtain:

$$F \approx 10^{-40}/r^4 \tag{10}$$

Using the above equation, and considering the dimensions of the mentioned cloud $(130\mu m \times 4.5\mu m)$, we can calculate the intensity of the *gravitational force* between two photons of the cloud, when the distance r between them were, for example, of the order of $1\mu m$, i.e.,

$$F \approx 10^{-16} N \tag{11}$$

The intensity of this gravitational force is highly significative. Compare for example, with the *Coulombian attractive force* between an *electron and a proton*, separated by *the same distance* ($r \approx 1\mu m$), which is given by

$$F_c = \frac{e^2}{4\pi\varepsilon_0 r^2} \cong \frac{10^{-28}}{r^2} \approx 10^{-16} N \tag{12}$$

The *Coulombian repulsive force* between *two protons* in an atomic *nucleus*, considering that, $r_{proton} = 1.4 \times 10^{-15} m$, and that the distance between them is $r = 4 \times 10^{-15} m$ [4], is given by

$$F_c = \frac{e^2}{4\pi\varepsilon_0 r^2} \cong 14N \tag{13}$$

This enormous repulsive force *must be overcomed* by the intense *attractive nuclear force* (*strong* nuclear force).

Now consider Eq. (9), where we put $A = \pi r_{proton}^2 \cong 6 \times 10^{-30} m^2$ and $r = 4 \times 10^{-15} m$, then the result is

[1] The Casimir force is only significative when the value of r is very small (*microcosm scale*).

[2] The velocities of the photons through the cloud of *ultracold rubidium atoms* are strongly reduced. This is the reason for the laser to pass through the mentioned cloud. Lene Hau et al., [3] showed that light speed through a cloud of *ultracold rubidium atoms* reduces to values much smaller than $100 m.s^{-1}$.

$$F = -\left(\frac{\pi A}{480}\right)\frac{hc}{r^4} \cong 30N \qquad (14)$$

Comparing Eq. (14) with Eq. (13), we can conclude that the *attractive gravitational force* ($30N$) is sufficient to overcome the *repulsive* Coulombian force expressed by Eq. (13).

These results lead us to formulate the following question: What is the true nature of the "strong nuclear force"? Is it *gravitational* as shown above?

This possibility is reinforced by the derivation the *Coupling Constants for the Fundamental Forces* that we will make hereafter, starting from Eq. (9).

It is known that the *weak* force, F_W, which is related to the *strong* force, F_S, by means of the following expression:

$$\frac{F_W}{F_S} = \frac{\alpha_W}{\alpha_S} \qquad (15)$$

where α_W is the *weak force coupling constant*, and α_S is the *strong force coupling constant* [3].

Assuming that $F_S = F$, where F is given by Eq. (9), then Eq.(15) can be rewritten as follows

$$F_W = \left(\frac{\alpha_W}{\alpha_S}\right)\left(\frac{\pi A}{480}\right)\frac{hc}{r^4} \qquad (16)$$

At $r \cong 3\times10^{-18}m$ (*0.1% of the diameter of a proton*), the weak interaction has a strength of a similar magnitude to electromagnetic force, $F_E = e^2/4\pi\varepsilon_0 r^2$ [5]. Thus, making $F_W = F_E$, and substituting the above mentioned value of r, we obtain

$$\frac{\alpha_W}{\alpha_S} = \frac{480r^2e^2}{4\pi^2\varepsilon_0 Ahc} = \frac{480r^2e^2}{4\pi^3\varepsilon_0 r_p^2 hc} \approx 3\times10^{-7} \qquad (17)$$

This is the same value mentioned in the literature for α_W/α_S [6].

Now, considering that $F_W/F_E = \alpha_W/\alpha_E$, where α_E is the *electromagnetic force coupling constant*, then we can write that

$$F_W = \left(\frac{\alpha_W}{\alpha_E}\right)\frac{e^2}{4\pi\varepsilon_0 r^2} \qquad (18)$$

At the maximum range of the weak interaction, $r_{\max}$, we have the minimum value of the weak force, $F_W^{\min}$, which can be expressed by Eq. (16) or Eq. (18) as follows

$$F_W^{\min} = \left(\frac{\alpha_W}{\alpha_S}\right)\left(\frac{\pi A}{480}\right)\frac{hc}{r_{\max}^4} \qquad (19)$$

$$F_W^{\min} = \left(\frac{\alpha_W}{\alpha_E}\right)\frac{e^2}{4\pi\varepsilon_0 r_{\max}^2} \qquad (20)$$

By comparing these equations, we obtain

$$\left(\frac{\alpha_W}{\alpha_S}\right)\left(\frac{\pi A}{480}\right)\frac{hc}{r_{\max}^2} = \left(\frac{\alpha_W}{\alpha_E}\right)\frac{e^2}{4\pi\varepsilon_0} \qquad (21)$$

or

$$\frac{\alpha_S}{\alpha_E} = \frac{4\pi^2 A\varepsilon_0 hc}{480e^2r_{\max}^2} = \frac{4\pi^3 r_p^2\varepsilon_0 hc}{480e^2r_{\max}^2} =$$

$$= \left(\frac{4\pi\varepsilon_0\hbar c}{e^2}\right)\left(\frac{2\pi^3 r_p^2}{480r_{\max}^2}\right) \qquad (22)$$

Experimental data, describing the strong force between nucleons is consistent with a strong force coupling constant of about 1 [6]. Thus, making $\alpha_S = 1$ (*strong force coupling constant*) in Eq. (22), we obtain

$$\alpha_E = \left(\frac{e^2}{4\pi\varepsilon_0\hbar c}\right)\left(\frac{480r_{\max}^2}{2\pi^3 r_p^2}\right) \qquad (23)$$

The maximum range of the weak interaction, $r_{\max}$, is of the order of $10^{-16}m$ [7]. Equation above shows that, for $r_{\max} \cong 5\times10^{-16}m$ the term

[3] Similarly, the weak force is related to the electromagnetic force, F_E, by means of the expression: $F_W/F_E = \alpha_W/\alpha_E$; and the strong force is related to the electromagnetic force, by means of the expression: $F_S/F_E = \alpha_W/\alpha_E$; and the gravitational force, F_G, is related to the electromagnetic force, by means of the expression: $F_G/F_E = \alpha_G/\alpha_E$.

$$\left(\frac{480 r_{max}^2}{2\pi^3 r_p^2}\right) \cong 1 \qquad (24)$$

Consequently, Eq. (23) reduces to

$$\alpha_E = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \cong \frac{1}{137} \qquad (25)$$

this is the expression of the *electromagnetic force coupling constant*.

Multiplying α_W / α_S (given by Eq. (17)) by α_S / α_E (given by Eq. (22)), we get

$$\frac{\alpha_W}{\alpha_E} = \left(\frac{480 r^2 e^2}{4\pi^3 \varepsilon_0 r_p^2 hc}\right)\left(\frac{4\pi\varepsilon_0 \hbar c}{e^2}\right)\left(\frac{2\pi^3 r_p^2}{480 r_{max}^2}\right)$$

whence we obtain

$$\alpha_W = \left(\frac{480 r^2 e^2}{4\pi^3 \varepsilon_0 r_p^2 hc}\right)\left(\frac{4\pi\varepsilon_0 \hbar c}{e^2}\right)\left(\frac{2\pi^3 r_p^2}{480 r_{max}^2}\right)\alpha_E =$$

$$= \left(\frac{480 r^2 e^2}{4\pi^3 \varepsilon_0 r_p^2 hc}\right)\left(\frac{2\pi^3 r_p^2}{480 r_{max}^2}\right) \cong$$

$$\cong \left(\frac{r^2 e^2}{2\varepsilon_0 r_{max}^2 hc}\right) \cong 3 \times 10^{-7} \qquad (26)$$

Now, we will obtain the *gravitational force coupling constant*, α_G, starting of the fact that the *strong* force, F_G, is related to the *electromagnetic* force, F_E, by means of the following expression:

$$\frac{F_G}{F_E} = \frac{\alpha_G}{\alpha_E} \qquad (27)$$

Then, we can write that

$$\alpha_G = \alpha_E\left(\frac{F_G}{F_E}\right) = \alpha_E\left(\frac{G m_p^2}{\frac{e^2}{4\pi\varepsilon_0}}\right) \cong 5.9 \times 10^{-39} \qquad (28)$$

The relative strength of interactions varies with distance [8]. Here, starting from the fact that *the strong nuclear force* and the *weak nuclear force* are *gravitational forces* expressed by Eq. (9), we have showed that, at the range of about 10^{-15} m ($r_{max} \cong 5 \times 10^{-16}\, m$), the *strong force*($\alpha_S = 1$) is approximately 137 *times as strong* as electromagnetic force ($\alpha_E = 1/137$), about *a million times as strong* as the weak force ($\alpha_W \cong 3 \times 10^{-7}$), and about 10^{38} *times as strong as* gravitation($\alpha_G \cong 5.9 \times 10^{-39}$). All these values are in strong accordance with the values widely mentioned in the literature [9, 10], given below

$$\alpha_S = 1$$
$$\alpha_E = 1/137$$
$$\alpha_W \approx 3 \times 10^{-7}$$
$$\alpha_G \cong 5.9 \times 10^{-39}$$

Finally, we complete the *unification* of the Fundamental Forces of the Universe, by deriving from Eq. (9) the equations of the *Coulombian Force* and of the *Newtonian Force*.

Consider two electric charges q_1 and q_2 separated by a distance r. If we define the *area* A in Eq. (9) by means of the following expression

$$A = \sqrt{A_1 A_2} = \sqrt{k_e\left(\frac{q_1}{e}\right)^2 r^2 \times k_e\left(\frac{q_2}{e}\right)^2 r^2} =$$
$$= k_e\left(\frac{q_1 q_2}{e^2}\right) r^2 \qquad (29)$$

where k_e is a constant to be determined, and $e = 1.6 \times 10^{-19} C$, then Eq. (9) can rewritten as follows

$$F = \frac{\pi hc k_e q_1 q_2}{480 e^2 r^2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{4\pi^2 hc k_e \varepsilon_0}{480 e^2}\right)\frac{q_1 q_2}{r^2} \qquad (30)$$

Note that, the term in brackets is equal to 1 for $k_e = 120 e^2 / \pi^2 hc\varepsilon_0 \cong 0.1769$. In this case, Eq. (30) reduces to

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad (31)$$

which is the expression of the *Coulombian Force*.

In a similar way, we can derive the expression of the *Newtonian Force* for two particles with masses m_1 and m_2 respectively, separated by a distance r. First we define the area A in Eq. (9) by means of the following expression

$$A = \sqrt{A_1 A_2} = \sqrt{k_g\left(\frac{m_1}{m_0}\right)^2 r^2 \times k_g\left(\frac{m_2}{m_0}\right)^2 r^2} = $$
$$= k_g\left(\frac{m_1 m_2}{m_0^2}\right) r^2 \qquad (32)$$

where k_g is a constant to be determined, and m_0, is a *minimum* value of mass that will be calculated hereafter. Then substitution of Eq. (32) into Eq. (9) yields

$$F = \frac{\pi\hbar c k_g m_1 m_2}{480 m_0^2 r^2} = G\left(\frac{2\pi^2\hbar c k_g}{480 G m_0^2}\right)\frac{m_1 m_2}{r^2} \qquad (33)$$

The term in brackets is equal to 1 for

$$\frac{k_g}{m_0^2} = \left(\frac{60}{\pi^2}\right)\left(\frac{4G}{\hbar c}\right) \qquad (34)$$

Equation (34) can be rewritten as follows

$$\frac{k_g}{m_0^2} = \frac{\left(\frac{60}{\pi^2}\right)}{\left(\sqrt{\frac{\hbar c}{4G}}\right)^2} \qquad (35)$$

where $\sqrt{\hbar c/4G} = 1.08 \times 10^{-8}\, kg$.

Equation (35) shows that, the term $\left(60/\pi^2\right)$ is a *pure number* such as k_g, and the term $\sqrt{\hbar c/4G}$ is expressed in kg such as m_0, then we can conclude that

$$k_g = \frac{60}{\pi^2} \qquad (36)$$

and

$$m_0 = \sqrt{\frac{\hbar c}{4G}} \qquad (37)$$

This expression it was first derivated by Hawking (1971) [11], and it is known as *Hawking mass limit*. Starting from the principle that the gravitational collapse is a process essentially classic, Hawking have concluded that black-holes could not exist with radius less than the *Planck length* $\left(\sqrt{G\hbar/c^3}\right)$ (limit for which *quantum* fluctuations in the metric of the spacetime are considered of the order of 1). In this way, the *minimum* radius of Schwarzschild, $r_S = 2Gm_0/c^2$, would have this value and, to this radius would correspond to a *minimum* value of mass m_0, given by

$$m_0 = \frac{r_S c^2}{2G} = \frac{c^2\sqrt{G\hbar/c^3}}{2G} = \sqrt{\frac{\hbar c}{4G}} \qquad (38)$$

This would be, obviously, the *smaller* mass value for any *macroscopic* gravitational systems (black-holes, etc).

Now, just substitute Eq. (36) and Eq. (37) into Eq.(33), in order to obtain the expression of the *Newtonian Force*.

$$F = G\frac{m_1 m_2}{r^2} \qquad (39)$$

The derivation of the Equations (31) and (39) via Eq. (9), shows clearly the *unification* of the Fundamental Forces of the Universe, i.e. shows that the *nature* of all the fundamental interactions is *Gravitational*.

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