Lifetimes of the Muon, Hyperons and Tau Lepton

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Abstract: Here, applying the atom-like structure of baryons described within the Scale-Symmetric Theory (SST), we calculated lifetimes of the muon, hyperons and tau lepton. SST gives an opportunity to show the origin of the time distances between the lifetimes of the hyperons. Theoretical results are very close to experimental ones.

1. Introduction

According to the Scale-Symmetric Theory [1], there are two different phenomena responsible for decay of particles. The first type of decay is a result of emission of a radiation mass \( M_{\text{radiation},i} = \alpha_i M_i \), where \( M_i \) is a mass of some Einstein-spacetime condensate which, sometime, can interact with the simplest spin-1 charged lepton pair composed of the electron and electron-antineutrino. Then lifetime is inversely proportional to radiation mass: \( \tau_i \sim 1/(\alpha_i M_i) \). Such type of decay is characteristic for the Higgs boson, \( W \) or \( Z \) bosons [2]. The second type of decay is a result of a change (a result of a transition) in internally exchanged mass, say from \( m_i \) to \( m_j \), or a change of interaction, say from interaction defined by coupling constant \( \alpha_i \) to interaction defined by \( \alpha_j \) – then there are obligatory following relations: \( \tau_{i \rightarrow j} \sim (m_j / m_i)^4 \) [1], [3] and \( \tau_{i \rightarrow j} \sim \alpha_j / \alpha_i \) [1] (i.e. \( \tau \sim 1 / m^4 \) (where \( m \) can be \( m = \alpha M \)) or \( \tau \sim 1 / \alpha \)). The first relation we obtain because the masses \( m_i \) and \( m_j \) are the condensates (they are composed of the Einstein-spacetime components) which behave as ionized gas in the stars [1], [3]. The second relation follows from the definition of the coupling constants and the uncertainty principle [1]. Such type of decay is characteristic for particles which have a rich internal structure such as neutron [3], pions [1], muon, tau lepton or hyperons. Let us emphasize that there may also be two-stage decays, but then we get lifetimes a bit lower than the experimental ones [1] – it follows from the fact that probability of such decays is much lower.

The successive phase transitions of the inflation field, described within SST, lead to the atom-like structure of baryons [1]. Here, the symbols of particles denote their masses also. There is the core of baryons with a mass of \( H^{+,-} = 727.4401 \) MeV. It consists of the electric-charge/torus \( X^{+,-} = 318.2955 \) MeV and the central condensate \( Y = 424.1245 \) MeV both composed of the Einstein-spacetime (Es) components – the Es components are the spin-1 neutrino-antineutrino pairs. The large loops \( m_{LL} = 67.54441 \) MeV with a radius of \( 2A/3 \) (where \( A = 0.6974425 \) fm is the equatorial radius of the electric-charge/torus) are
produced inside the electric-charge/torus – the neutral pions are built of two such loops. In the \(d = 1\) state (it is the \(S\) state i.e. the azimuthal quantum number is \(l = 0\)) there is a relativistic pion – radius of the orbit is \((A + B) = 1.199282\) fm. In the hyperons, relativistic pions are on the orbit with a radius of \((A + 2B) = 1.701122\) fm. Masses, spins, magnetic moments and strangeness of hyperons are calculated in paper [4]. Within SST, we calculated mass of proton \(p = 938.2725\) MeV and mass of neutron \(n = 939.5648\) MeV.

The calculated within SST values of the coupling constants for the weak interactions are as follows [1]:

- for the nuclear weak interactions is \(\alpha_{w(proton)} = 0.0187228615\),
- for the weak electron-muon interactions is \(\alpha_{w(electron-muon)} = 0.9511082 \times 10^{-6}\).

The ratio of these coupling constants is \(\chi_{w} = \frac{\alpha_{w(proton)}}{\alpha_{w(electron-muon)}} = 19,685.3\).

2. Lifetime of the muon

Muons are created as the spin-0 quadrupoles from the \(Y\) condensates. It causes that they conserve electric charge and the half-integral spin of the core of baryons. They become free in distance \(2\pi A\) i.e. in distance equal to circumference of a photon loop created on the equator of the core of baryons. A relativistic muon reaches such places after \(T_o = 2\pi A/c = 1.4617314 \times 10^{-23}\) s. But the weak interactions of the muon increase its lifetime. There is the transition from the nuclear weak interaction (involved mass is \(m_{w,1} = \alpha_{w(proton)} m_{muon}\)) to weak interaction of electron (involved mass is \(m_{w,2} = \alpha_{w(electron-muon)} m_{muon}\)). Such transition increases lifetime of the muon which is

\[
\tau_{muon} = T_o \left( \frac{m_{w,1}}{m_{w,2}} \right)^4 = T_o \chi_{w}^4 = 2.195006 \times 10^{-6} s .
\]  

3. Lifetimes of the hyperons

The relativistic pions responsible for properties of hyperons are in the \((A + 2B)\) state. On the other hand, pions are created in the \(2A/3\) state and there is a relativistic pion in the \((A + B)\) state which is responsible for properties of nucleons [1]. Decay of hyperons can be a result of emission of a pion from the listed three states but there can be also some initial transitions of pions between the \(2A/3\), \((A + B)\) and \((A + 2B)\) states.

For hyperons we have 3 characteristic loops with radii equal to \(R_i = 2A/3\), \((A + B)\), \((A + 2B)\). Ranges of them, \(L_i\), are

\[
L_i = 2 \pi R_i .
\]  

A relativistic pion, which appears in decay of a hyperon, is free after time \(T_i\)

\[
T_i = L_i / c .
\]  

But the weak interactions of such pions increase these times. According to SST, in hyperons, there is transition from the nuclear weak interactions (involved mass is \(Y\)) to the weak interactions of electron via muon (involved mass is \(m_{cond(electron)} = m_{bare(electron)}/2 = 0.5104070 \div 2 \text{ MeV} = 0.2552035 \text{ MeV}\) [1] – it is the mass of the Einstein-spacetime condensate in the centre of the electron). Lifetimes of hyperons we can calculate from following formula
Applying formula (4) we obtain three different lifetimes for hyperons for single decays:
\[ \tau_{\text{hyperon,}2A/3} = 0.7434 \times 10^{-10} \text{ s}, \]
\[ \tau_{\text{hyperon,}(A+B)} = 1.9174 \times 10^{-10} \text{ s}, \]
\[ \tau_{\text{hyperon,}(A+2B)} = 2.7197 \times 10^{-10} \text{ s}. \]

We can compare them with experimental data [5] which are collected in Table 1.

<table>
<thead>
<tr>
<th>Hyperon</th>
<th>Lifetime [5]</th>
<th>Lifetime from SST</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Lambda )</td>
<td>((2.632 \pm 0.020) \times 10^{-10}) s</td>
<td>(2.7197 \times 10^{-10}) s</td>
</tr>
<tr>
<td>( \Sigma^+ )</td>
<td>((0.8018 \pm 0.0026) \times 10^{-10}) s</td>
<td>(0.7434 \times 10^{-10}) s</td>
</tr>
<tr>
<td>( \Sigma^0 )</td>
<td>((7.4 \pm 0.7) \times 10^{-20}) s</td>
<td>(5.5426 \times 10^{-20}) s</td>
</tr>
<tr>
<td>( \Sigma^- )</td>
<td>((1.479 \pm 0.011) \times 10^{-10}) s</td>
<td>(1.9174 \times 10^{-10}) s</td>
</tr>
<tr>
<td>( \Xi^0 )</td>
<td>((2.90 \pm 0.09) \times 10^{-10}) s</td>
<td>(2.7197 \times 10^{-10}) s</td>
</tr>
<tr>
<td>( \Xi^- )</td>
<td>((1.639 \pm 0.015) \times 10^{-10}) s</td>
<td>(1.9174 \times 10^{-10}) s</td>
</tr>
<tr>
<td>( \Omega^- )</td>
<td>((0.821 \pm 0.011) \times 10^{-10}) s</td>
<td>(0.7434 \times 10^{-10}) s</td>
</tr>
</tbody>
</table>

Lifetime of a hyperon can be a mean of lifetimes calculated for different transitions of pions. There can be involved as well the \((A + 4B)\) state. But emphasize that the experimental lifetimes are very close to the theoretical results for the three single decays. It suggests that the single decays dominate.

Why lifetime of hyperon \( \Sigma^0 \) is very short? We suppose that it is due to electromagnetic interaction. But why it does not concern the other neutral hyperons? SST shows that inside baryons, due to the transitions of pions from the \((A + 2B)\) state to the \((A + 4B)\) state, there appear the virtual bosons with a mass of \( E_{\text{virt}} = \pm 19.367 \text{ MeV} \) ([1]: see formula (155)). When we add the absolute value of \( E_{\text{virt}} \) to the mass distance between hyperon \( \Sigma^0 \) and neutron \( n \) then we obtain mass which is a little higher than mass of a quadrupole of four large-loops/photon-loops i.e. there is very high probability for transition from \( Y \) to \( m_{LL} \). When decay is from the \((A + 2B)\) state, we obtain

\[ \tau_{\text{hyperon,}\Sigma^0} = T_{A+2B} (Y / m_{LL})^4 = 5.5426 \times 10^{-20} \text{ s}. \] (5)

But there can be an admixture of the \((A + 4B)\) state which increases the lifetime. For 50% of \((A + 2B)\) states and 50% of \((A + 4B)\) states we obtain \(7.1777 \times 10^{-20}\) s.

To test presented here model, let us consider the decays of hyperons \( \Lambda, \Sigma^+, \) and \( \Omega^- \). According to SST, neutron is a mixture of charged core and charged relativistic pion in the \((A + B)\) state (about 62.6%) and of neutral core and relativistic neutral pion (about 37.4%) [1]. On the other hand, proton is a mixture of charged core and relativistic neutral pion (about 50.8%) and of neutral core and relativistic charged pion (about 49.2%). Emphasize that the listed abundances are in SST not free parameters – they are derived from the initial conditions [1].

In hyperon \( \Lambda \) (it consists of neutron and there is one pion in the \((A + B)\) state and one in \((A + 2B)\) state [1], [4]), there is the initial transition of the pion in the \((A + B)\) state to the \((A + B)\) state.
2B) state (there is an inverse transition as well) and such pion appears in the decay. It leads to conclusion that there should be 62.6% decays to \( p\pi^- \) (experiments give 63.9 ± 0.5 % [5]) and 37.4% decays to \( n\pi^0 \) (experiments give 35.8 ± 0.5 % [5]).

In hyperon \( \Sigma^+ \) (it consists of proton [1], [4]), there is the initial transition of the pion in the \((A + B)\) state to the \((A + 2B)\) state (there is an inverse transition as well) and such pion appears in the decay. It leads to conclusion that there should be 50.8% decays to \( p\pi^0 \) (experiments give 51.57 ± 0.30 % [5]) and 49.2% decays to \( n\pi^+ \) (experiments give 48.31 ± 0.30 % [5]).

The hyperon \( \Omega^- \) consists of neutron, one pion in the \((A + B)\) state and three pions in \((A + 2B)\) state [1], [4]). On the other hand, kaon \( K \) is a binary system of pions. It means that this hyperon can decay to hyperon \( \Lambda \) and negatively charged kaon \( K^- \) – it is an analog to the decay of hyperon \( \Lambda \) to \( p\pi^- \) so we should observe 62.6% such decays (experiments give 67.8 ± 0.7 % [5]). It means that there should be 37.4% decays to \( \Xi \pi \). But the decay to \( \Xi'\pi^- \) is an analog to the decay of hyperon \( \Lambda \) to \( p\pi^- \) so we should observe 62.6% such decays i.e. 37.4% \( \times 0.626 = 23.4\% \) (experiments give 23.6 ± 0.7 % [5]). On the other hand, there should be 37.4% \( \times 0.374 = 14.0\% \) decays to \( \Xi\pi^0 \) (experiments give 8.6 ± 0.4 % [5]).

Is there a selection rule which ties hyperons with states from which they decay?

The selection rule is as follows.

From Table 1 results that when we consider only spin-1/2 hyperons decaying due to the weak interactions (i.e. we neglect \( \Sigma^0 \) and \( \Omega^- \)) then charged positively hyperon \( \Sigma^+ \) decays, first of all, from \( 2A/3 \) state, charged negatively ones (i.e. \( \Sigma^- \) and \( \Xi^- \)) decay from \((A + B)\) state, and neutral ones (i.e. \( \Lambda \) and \( \Xi^0 \)) decay from \((A + 2B)\) state. On the other hand, in neutron and proton, the positive charge \( X^+ \) with a mean radius \( 2A/3 \) dominates, in neutron, the negatively charged pion in the \((A + B)\) state dominates. It suggests that different charge states of hyperons prefer different orbits in baryons from which they decay.

4. Lifetime of the tau lepton

According to SST, the tau lepton is an analog to electron and the radiation mass of the tau is equal to the relativistic mass of the charged pion in the \((A + B)\) state. The decay should be an analog to the beta decay of neutron so we can assume that there is the transition from \( Y \) to a condensate with a mass equal to the mass distance between the neutron and proton. It means that we can calculate lifetime of the tau lepton from following formula

\[
\tau_{\text{tau}} = T_{A+B} \left[ Y / (n-p) \right]^4 = 2.9069 \times 10^{-13} \text{ s}.
\]

This lifetime we calculated taking into account the experimental masses of neutron and proton. For the SST masses of nucleons we obtain \( 2.9161 \times 10^{-13} \text{ s} \). Experimental result: \( (2.903 \pm 0.005) \times 10^{-13} \text{ s} \), is very close to obtained here results.

5. Summary

Here, applying the atom-like structure of baryons described within the Scale-Symmetric Theory (SST), we calculated lifetimes of the muon, hyperons and tau lepton.
Presented here model is mathematically very simple, coherent and concerns all fundamental particles. No other model leads to such high compliance with experimental data. Emphasize that number of parameters applied in SST is at least three times lower than in the Standard Model [6], [1] and within SST we solved all basic problems which are unsolved within the other theories.

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