Kinetic Energy and Conservation of Momentum

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In the history of physics, kinetic energy has been represented by two expressions. One from Issac
Newton, the other from Special Relativity. Both expressions are expected to describe a physical
system that demands conservation of momentum. By examining the expression of momentum in a
projectile motion, the kinetic energy from Issac Newton is found to obey conservation of momentum
while the kinetic energy from Special Relativity is found to violate conservation of momentum.

I. INTRODUCTION

In 17th century, Issac Newton proposed a definition of force, \( F = m \times a \). Based on this definition, both kinetic
energy and momentum can be derived.

In 20th century, Special Relativity\[1\] proposed a new definition of kinetic energy. This results in new defini-
tions of both momentum and force.

However, the physics law, conservation of momentum, remains intact. Any definition of kinetic energy is ex-
pected to generate a force that results in the conservation of momentum.

This paper examines both expressions of momentum in a projectile motion. The x-component of total momen-
tum is calculated for both expressions of kinetic energy.

The concept of relativistic mass becomes less popular in modern physics. Relativistic force and relativis-
tic momentum do not share the same relativistic mass. The momentum of a particle is represented by either
\( \gamma(v) \times m(0) \times v \) or \( m(v) \times v \). Both representations are equivalent to each other mathematically. In this paper,
\( \gamma(v) \times m \times v \) is chosen to emphasize Lorentz Factor, \( \gamma(v) \),
in Lorentz Transformation.

\[
\frac{dm}{dv} = \frac{dm(0)}{dv} = 0
\]  

II. PROOF

Consider two-dimensional motion.

A. Kinetic Energy and Momentum

In Newtonian Mechanics, force \( F \) is defined as multi-
plication of mass and acceleration.

\[
F = m \times a
\]  

In Special Relativity, kinetic energy \( K \) is defined as

\[
K = (\gamma(v) - 1) \times m \times C^2
\]  

These two definitions generate different expressions for momentum. However, the derivation of momentum from
kinetic energy has not been changed. Kinetic Energy \( K \) is defined as integration of force over distance.

\[
K = \int F \, dx
\]  

Momentum \( P \) is defined as integration of force over time.

\[
P = \int F \, dt
\]  

\[
\frac{dP}{dt} = F
\]  

In Newtonian Mechanics,

\[
K = \frac{1}{2} \times m \times v^2
\]  

\[
P = m \times v
\]  

In Special Relativity,

\[
K = (\gamma(v) - 1) \times m \times C^2
\]  

\[
P = \gamma(v) \times m \times v
\]

The difference in expression indicates that only one ex-
pression of momentum can be correct By applying con-
servation of momentum to both expressions of momentum
in a physical system such as projectile motion, the correct
expression can be distinguished.

B. Projectile Motion

A particle moves along x axis under a force along y
axis. The acceleration in x direction is zero. The acceleration
in y direction is \( A \). The single force on this particle

demands that total momentum in x direction \( P_x \) should
remain constant.

\[
\frac{dP_x}{dt} = F_x = 0
\]
C. Conservation of Momentum

Let $a_x$ be the acceleration on $x$ direction. Let $a_y$ be the acceleration on $y$ direction.

\[
    a_x = 0 = \frac{dv_x}{dt}
\]  

(12)

\[
    a_y = A = \frac{dv_y}{dt}
\]  

(13)

In Newtonian Mechanics, change of momentum in $x$ direction is zero.

\[
    \frac{dP_x}{dt} = \frac{d(mv_x)}{dt} = m \cdot a_x = 0
\]  

(14)

In Special Relativity, change of momentum in $x$ direction is not zero.

\[
    \frac{dP_x}{dt} = \frac{d(\gamma(v)mv_x)}{dt} = \gamma(v) m \cdot v_x
\]  

(15)

\[
    = \gamma(v)^3 \cdot \frac{v}{c^2} \cdot \frac{dv_x}{dt} m \cdot v_x
\]  

(16)

\[
    = \gamma(v)^3 \cdot \frac{v}{c^2} \cdot \sqrt{\frac{v^2 + v_x^2}{v^2 + c^2}} m \cdot v_x
\]  

(17)

\[
    = \gamma(v)^3 \cdot \frac{v}{c^2} \cdot \frac{v_x}{v} \cdot A \cdot m \cdot v_x
\]  

(18)

Total momentum in $x$ direction remains constant in Newtonian Mechanics but not in Special Relativity.

III. CONCLUSION

Special Relativity violates conservation of momentum in projectile motion.

Conservation of momentum fails to hold if momentum is defined as $\gamma(v) m \cdot v$. The failure of this physics law is due to the introduction of Lorentz factor, $\gamma(v)$, from Lorentz Transformation.[8][11].

Lorentz Transformation was proposed on the assumption that the speed of light is independent of inertial reference frame.

As the result of this incorrect assumption[3], Lorentz Transformation violates Translation Symmetry[4] and Conservation of Momentum[10] in physics. Translation Symmetry requires conservation of simultaneity[5], conservation of distance[6], and conservation of time[7]. All three conservation properties are broken by Lorentz Transformation.

Therefore, Lorentz Transformation is an invalid transformation in physics. Consequently, any theory based on Lorentz Transformation is incorrect in physics. For example, Special Relativity.