Two Photon Composite Electron Model: QFT aspects

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Abstract

In a previous paper “A Physical Electron-Positron Model”[1] an electron model was developed in a geometrical algebra (GA) construct developed by Doran et.al. [2] The model shows the mathematical structure, and the physical description required for the existence of a composite electron but not delineating the physical processes. This paper will develop the model from the perspective of classical and QM mechanics and make the connection to the QFT and Lorentz structure that underlies the physical basis, and illustrates how the interaction of photons can create charge. The path integral formulations of QFT fit well with the model and it is absent the infinities indicative of the standard model.

The concept of charge has heretofore not had any theoretical explanation, accept for some unknown substance sprinkling in with the mass. The model therefore offers the QFT community an idea on how to convert the concept of Charge and Pair Production from magic to mechanics.

Introduction

In geometric Algebra (GA) the Dirac Matrices become the spacetime unit coordinate vectors. This indirectly changes the view of QM by defining some of the aspects QM as actually features of Lorentz covariant spacetime. Parity, time reversal, charge, positive & negative mass, become part of the spacetime structure, simplifying the mapping of the Dirac relativistic quantum representation into the eight dimensional, subalgebra of the GA spacetime representation. This allows a GA functional description of a photon. [1], which in turn allows a four-dimensional composite electron.

The authors previous paper [1], proposed a model of an electron formulated as the composition of two photons using the AG rotor structures for QM formulated by Doran et.al. [2]. (Fig. 1). This paper will develop the physical structure of the composite particle primarily in classical physics and QM that can be adapted into a QFT path integral calculation. This representation is not unlike the Bohr atomic model in developing a classical visual physical model but relying on the Schrodinger equations to develop the probability amplitudes.

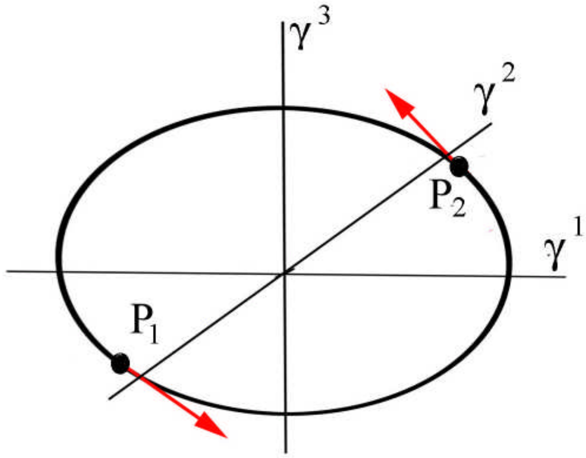

Fig.1 General configuration chowing the orbiting of photons in a GA cooridnate syatem

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Primary Physical Mechanisms

The interaction of the two orbiting photons as described in the earlier paper is not the result of the electric binding force, but by the motion under the influence of the mutual gradient in the index of refraction generated by the vacuum polarization. The binding mechanism is the circular gradient in c induced by the nonlinear E&M effects of the vacuum polarization. Vacuum polarization is generally considered as a scattering mechanism, but a radial gradient in the index of refraction can be a binding mechanism that holds orbiting photons together. This paper will explore a composite electron model based on that concept.

The vacuum polarization between two interaction photons is a well researched process both from a theoretical and experimental. The first theoretical development by Sauter, Serber, Euler and others,[3],[4],[5], and later by more sophisticated methods of QFT by Schwinger and others[8],[9]. The study of the vacuum polarization on the index of refraction is quite extensive in the lower levels of E when birefringence on photons in static fields effects are predominant, [10],[11],[12],[13],[14],[15],[16],[17],[18], [19],[20],[21]. Others have studied and proposed experiments investigated the effects of photon-photon scattering in the higher energy levels,

,[22],[23], and there have been several proposals for studies on the effects on the index of refraction by intense laser beams,[24],[25],[26],[27],[28],[29].

At the higher E levels, that are more appropriate to this work, the processes of Delbruck scattering and pair production dominates, these processes gave been studied extensively, originally proposed by Max Delbruck and first observed, by Robert Wilson [30], and has been the subject of intensive research in both theoretical, and experimental since the 1950's. [31],[32], [33],[34],[35],[36], [37]. Appropriate to this paper, but not at the same energy levels is the research done by J. Kim et.al. [38], on light bending in a Coulombic field.

The theoretical and experimental ideas proposed in this development are not outside the plausible bounds of the current developments.

Outline of Presentation

As will be presented the mechanisms holding the photons in a constant radial direction as they revolve around the center of momentum are: The Thomas precession which counteracts the helicity, and: the vacuum polarization that maximizes the energy density along the radial vector. The maintenance of the electrical vectors of the polarized photons in a constant radial direction provides the net charge for the composite particle.

The mechanism that provides a barrier, and prevents escape of the photons is a discontinuity in the index of refraction created by the non linear vacuum polarization cross-section. This barrier provides a total internal reflection that only allows only the escape of the radially polarized virtual photons which transfer the effect of the electric charge to other particles.

Interacting Index of Refraction: Binding Mechanism

It is proposed that vacuum polarization initiated by the interaction of two photons of sufficient energy, provides radial index of refraction sufficient to hold the photons in circular paths.

Most important to this is the derivation by Schwinger of the leading nonlinear corrections to the vacuum polarization that allows calculations of the local index of refraction below the critical electron–positron limit [3].

$$E_{CR} = \frac{m_e^2 c^3}{Q\hbar} = \frac{c\hbar}{2Q\lambda\!\!\!^{-}{}_e^2} \tag{1}$$

If a pair of interacting photons has insufficient energy to create a photon pair, there is still polarization of the vacuum, and a change to the local index of refraction because of the “probability “of creating electron-positron pairs.

At the low-energy end with non parallel fields generally defined by the Heisenberg-Euler Lagrangian are the studies of birefringence changes in the index of refraction induced at low levels $\left(E << E_{cr}\right)$ .These have been conducted by a large number of researchers [10-21], and the results are generically similar to:

$$\eta^{\parallel,\perp} = 1 + \frac{\alpha\left(11 \mp 3\right)}{45\pi}\frac{E_2^2}{E_{cr}^2} \tag{2}$$

The $\parallel,\perp$ suffix indicates parallel and perpendicular field polarizations.

For two photons moving around the centre of momentum each experiences the electromagnetic field of the other. The relation for that interaction at $E << E_{CR}$ from Kim et.al, “Light bending in radiation background” [38], and Light bending in a Coulombic field the index of refraction can be expressed as:

$$\eta^{-1} = \frac{c}{c_0} = 1 - \frac{(14\perp,8\|)\alpha^2\hbar^3}{45m^4c^5}\left(u \times E_2\right)^2 \qquad (3)$$

At the higher end of the energy levels above the Kim et.al, work closer to the Schwinger limit $\left(E \sim E_{cr}\right)$, the index of refraction is better understood and by the processes related to Delbruck scattering, and pair production developed by Schwinger and others.

The reflection coefficient expressed in the relative index of refraction and the high end scattering experiments, lead to the conclusion that the index of refraction has infinity at the Schwinger Limit, thus it is postulated that as $E \rightarrow E_{cr}$ index of refraction η^{-1} becomes:

$$\eta^{-1} \rightarrow \left(1 - \frac{E^2}{E_{cr}^2}\right) \qquad (4)$$

Required Index of Refraction for Photons containment

The required index of refraction to maintain photons in a circular path can be determined from classical physics by variational methods applied to Fermat's principle. It is straight forward and well done by J. Evans, et.al. [39]. For stable orbits Fermat's principle requites the index of refraction to be proportional to1/r, thus in terms of the Compton radius for an electron, and from Eq.(4),

$$\eta^{-1} = \frac{c}{c_0} = \frac{kr}{\lambda\!\!\!^{-}_e} = \left(1 - \frac{E^2}{E_{cr}^2}\right) \qquad (5)$$

A linear function is be defined by two points. The first point is at $r = 0$. From Eq.(5), $E^2 = E_{cr}^2$. The sum of the energy density of the two photons must equal to the Schwinger Limit:

$$E_{1,2} = (1/2)E_{cr} = \frac{m^2c^3}{2Q\hbar} = \frac{c\hbar}{2Q\lambda\!\!\!^{-}{}_e^2} \qquad (6)$$

Although the polarized photons have an electric field, the binding mechanism of photons is not the electric field, but the gradient in the index of refraction induced by the vacuum polarization of the two interaction photons.

We will consider however, the electric field necessary to hold polarized photons in inertial orbit to be, at least approximately, equal to the electric field that induces the gradient in the index of refraction that provides the same result. The value of the linear proportionality constant k In Eq.(5), can then be calculated.

The value of k must be less than one; else when $r = \lambda\!\!\!^{-}{}_e$ else the particle would not be bound. **Appendix I** calculates the value of k using this approach and finds it should be on the order of ¾. As long as the value is less than one, the exact value is not critical to the model.

$$k = \frac{3}{4} \qquad (7)$$

The vacuum polarization at the location of each of the photons including its own electric density as a function of r is then:

$$E^2 = E_{cr}^2\left(1 - \frac{3}{4}\frac{r}{\lambda\!\!\!^{-}{}_e}\right) \qquad (8)$$

Period, Frequency, & Gradient

For a photon P, having propagation velocity that is proportional to the radius of the orbit, the orbital period of revolution is constant for all radii. Using Eq.(5):

$$T = \frac{2\pi r}{c} \times \frac{c \lambda\!\!\!^{-}_{P}}{c_0 k r} \quad \rightarrow \quad T = \frac{2\pi \lambda\!\!\!^{-}_{P}}{c_0} \tag{9}$$

Thus, the frequency for all radii is the same as the free particle frequency.

$$\omega = \varepsilon_0 / \hbar \tag{10}$$

From Eq.(5), the gradient in c That is responsible for maintaining the photons in orbit as a function of r is:

$$\frac{dc}{dr} = \frac{c_0 k}{\lambda\!\!\!^{-}_{0}} \tag{11}$$

Vector Orientation, Stability, and Net Charge

It has been asserted in the earlier paper that the rotating photons can have electromagnetic vectors maintaining a constant radial direction along the radial vector. Two physical mechanisms are responsible for this. One is the Thomas precession which counteracts the photon helictical rotation, and two is the maximizing of the vacuum polarization energy density along the radial rotation axis.

Thomas Precession

As a pair of photons rotates around the center of momentum in a variable index of refraction, the Thomas precession reduces the helictical rotation frequency of the photon by exactly the axial frequency of the rotation. As the circumference is reduced to the wavelength is the helictical frequency is stopped. The rotation frequency is then equal to the original free particle frequency of the photon and the photon electromagnetic vectors are polarized along the orbital radius.

This is easily shown from Lorentz geometric principles, the Thomas reduction to the frequency of an orbiting photon is:

$$\omega_T = \frac{1}{c^2}\left(\frac{\gamma^2}{\gamma+1}\right) a \times v \tag{12}$$

a is the circular acceleration dr/dt in the moving frame thus:

$$\Delta t' = \gamma\ \Delta t \tag{13}$$

and for a photon moving in a variable index of refraction the precession is:

$$\vec{\omega}_T = \frac{1}{c^2}\left(\frac{\gamma^2}{\gamma+1}\right) a \times v \rightarrow \frac{1}{c^2}\left(\frac{\gamma^2}{\gamma+1}\right)\frac{dv}{\gamma dt} \times v = \frac{1}{c^2}\left(\frac{\gamma^2}{\gamma+1}\right)\frac{dv}{dt} \times v$$

For the photon the circular acceleration is:

$$\frac{dv}{dt} = \frac{c^2}{r} \tag{14}$$

And for the photon orbiting at the Compton radius:

$$r = \frac{c}{\omega_P} \tag{15}$$

Thus as the radius is reduced to the Compton radius the Thomas precession frequency reduces the helictical frequency to zero, whereas the axial frequency in the orbit plane $\vec{\omega}_R$ becomes equal to the free photon frequency.

$$\vec{\omega}_T = \omega_P \uparrow \tag{16}$$

The Thomas thus establishes a radial polarization, but does not provide a preferential direction for that polarization. The vector orientation mechanism is provided by the vacuum polarization energy density.

E Field Maximizing and Vector Orientation

From the center of momentum frame of two identical orbiting photons, the photons are going in opposite directions and are in effect in colliding. The momentums are opposite, the helictical rotations are opposite, and thus by CPT in the center of momentum frame, the electric field contribution to the vacuum polarization are reversed. For the purpose of vacuum polarization the sum of the opposite electric fields are additive to the field strength.

$$\mathrm{E} = \mathrm{E}_1 - \left(-\mathrm{E}_2\right) = \mathrm{E}_1 + \mathrm{E}_2 \tag{17}$$

The energy density ε, for the colliding photons is maximal for a head-on collision and expressed in 3vector notation is[11]:

$$\varepsilon = \mathrm{E}^2 + \mathrm{B}^2 - 2\mathrm{S}\bullet\boldsymbol{k} - \left(\mathrm{E}\bullet\boldsymbol{k}\right) - \left(\mathrm{B}\bullet\boldsymbol{k}\right) \tag{18}$$

For the case of two equal & opposite photons, all but the first square terms of the energy density vanish, and in addition the birefringent terms of the Lagrangian first loop also vanish.

$$\begin{aligned} L &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}\left(\mathrm{E}^2 - \mathrm{B}^2\right) = 0 \\ L &= -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} = \frac{1}{2}\left(\mathrm{E}\bullet\mathrm{B}\right) = 0 \end{aligned} \tag{19}$$

From Eq.(17), and Eq.(18), the maximal energy density for the two photons occurs when the electric and magnetic vectors are parallel, thus the electric and magnetic vectors add without any birefringent terms:

$$\varepsilon = \left(\mathrm{E}_1 + \mathrm{E}_2\right)^2 + \left(\mathrm{B}_1 + \mathrm{B}_2\right)^2, \tag{20}$$

The maximum energy density at the location of a single photon is when the square of the sums of the second photon maximizes at that point. That is the contribution to the electric density at P_1 by P_2 is when this sum maximizes. (Fig, 2)

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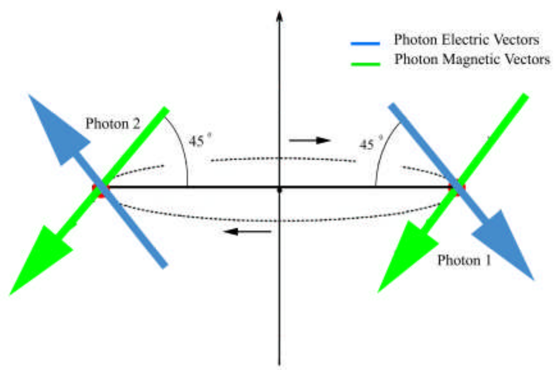
$$\varepsilon_1 = \left(E_1 + E_2 \sin\phi\right)^2 + \left(B_1 + B_2 \cos\phi\right)^2 \tag{21}$$

This occurs when the radial vectors and the electromagnetic vectors from P_2 are at a 45 degree angle.

$$\frac{d\varepsilon_1}{d\phi} = E^2\left(\sin\phi - \cos\phi\right) \rightarrow 0 \ @ \ \phi = 45^0 \tag{22}$$

The result is the same from the perspective at P_2



The combined effect of the Thomas precession and the vector orientation for the maximal field strength gives the stability to the composite electron.

Charge & Magnetic Moment

As these vectors presses around the orbit there is a time average net spherical electric vector, (blue), in the 4π radial directions. The time average of the magnetic vectors, (Green) is dipolar 2π. These time integrals give the net charge to the composite particle and a magnetic dipole moment.

The model defined in electric vectors should only be considered as a classical visual description, whereas the actual electric field effects are generated by probability distribution density of the polarized virtual photons that escape the boundary.

Binding Barrier: Index of Refraction Discontinuity

For the photon interaction to provide the index of refraction specified for stability Eq.(8), there has to be a certain relation for the contribution to the index of refraction at one photon to the other. Since the value of the energy density at one of the photons is fixed the contribution of the other is a function of the distance between the photons.

The value of the vacuum polarization coefficient at one of the photons determines the index of refraction, and the gradient determines the curvature, or the radius of the orbit. If we set $\boldsymbol{f}(\mathrm{r})\mathrm{P}_2$ to be the function of r for the contribution of P_2 to P_1, then the total value of E at P_1 in Eq. (8),is:

$$\frac{\mathrm{E}^2}{\mathrm{E}_{cr}^2} = \frac{\left(\mathrm{E}_1 + \boldsymbol{f}(\mathrm{r})\mathrm{E}_2\right)^2}{\mathrm{E}_{cr}^2} = \left(1 - \frac{3}{4}\frac{\mathrm{r}}{\lambda_e}\right) \qquad \mathrm{E}_1 = \mathrm{E}_2 = \mathrm{E}_{CR} / 2 \tag{23}$$

The function $\boldsymbol{f}(\mathrm{r})$ is the function that determines the value of the contribution of Photon P_2 Ti the value of the electric vector at P_1 r is the radius of the photon orbit.

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Solving, the interaction function is:

$$\mathrm{f}(\mathrm{r}) = 2\sqrt{\left(1 - \frac{3}{4}\frac{\mathrm{r}}{\lambda_e}\right)} - 1 \tag{24}$$

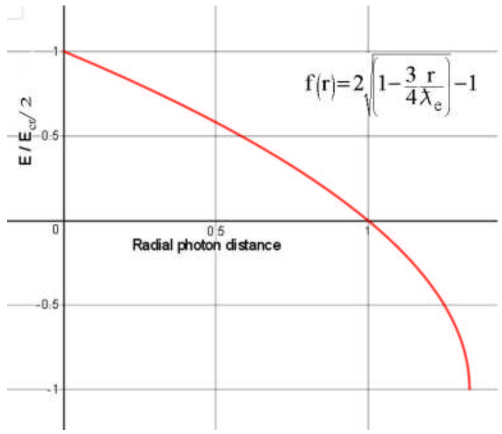

Fig.2 Graph of Eq.(24),

Note that when the orbit radius is equal to the Compton radius the contribution of P_2 to P_1 goes to zero thus beyond this radius there is no contribute to the energy density at P_1.

Since the energy density cannot be negative there is an abrupt discontinuity in the gradient of E^2 at P_1 implying that the crossection for the intraction is sharp, and falls to zero at this distance.

$$\frac{d}{dr}\left(\frac{E^2}{E_{cr}^2}\right) \to 0 \text{ ,and } \frac{dc}{dr} \to 0 \tag{25}$$

The value of the reciprocal index of refraction η^{-1} then goes abruptly from ¾ to 1:

$$\eta^{-1} = \frac{3}{4} \to 1 \tag{26}$$

This discontinuity in the index of refraction provides the barrier preventing escape.

Cross-Section

The implication of Eq., is that the vacuum polarization cross section has a sharp edge with similarity to a solid particle. Graphically this cross-section interaction has a linear energy density contribution until the second photon edges separate Fig. 3.

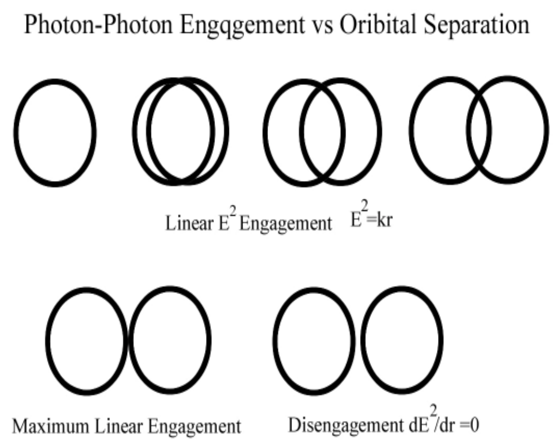

Fig. 3 Cross-section engagement

Photon Velocity

A graph (Fig. 4) of the photon velocity vs. radius illustrates the linear c-r range and the transition to the higher velocity in free space when the particles cross section disengages. The discontinuity in the index of refraction going from high to low creates a total internal reflection for the orbiting photons and thus, a barrier for escaping photons. Action paths outside the Compton radius for the polarized photons must be virtual. It is the interaction of the polarized virtual photons with other particles that constitutes the effects of electrons charge.

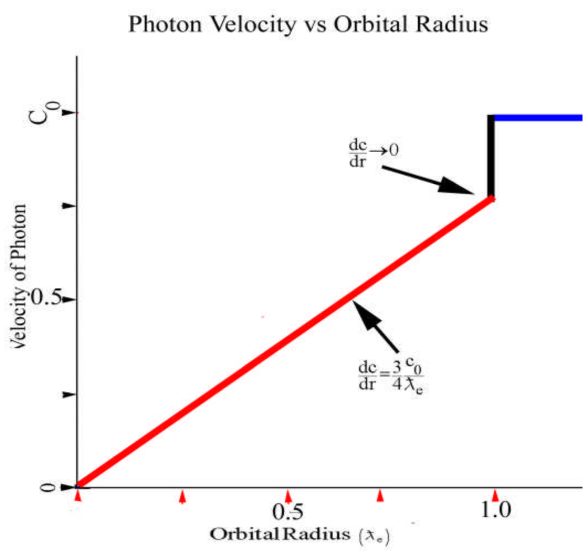

Fig. 4 A graph of the photon velocity vs. radius, and photon barrier

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Orbits

Fig. 5 illustrates the photon orbits of the proposed model. The Compton radius is the location of the boundary that separates the real photons that constitutes the mass, from the virtual photons that generate the electric field effects.

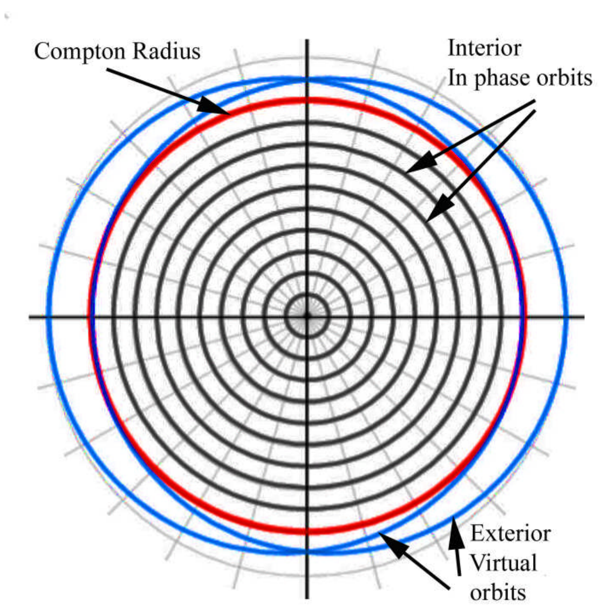

Fig. 5 This is a sketch illustrating the range of orbits between 0 and the electron Compton radius that has an index of refraction inversely

proportional to r. The period is constant and the spinnor function wavelength is twice the conferential distance. Each point of the radius has a circular orbit that is stable and obeys Fermat's principle. All the circular orbit actions are in phase and contribute equally to the total action. The discontinuity in the index of refraction confines real photons to radii less than the Compton radius, thus only virtual photon paths exist outside this radius.

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Composite Particle Spin Angular Momentum

The angular momentum for the orbiting photons is properly calculated by Path integral methods integrating the action over all possible paths. By knowing however that the sum of the spin angular momentum of the two spin-one vector boson is $1/2\,\hbar$ the most probable or classical path can be calculated

The classical angular momentum around the center of mass perpendicular to the orbit is:

$$S = \frac{p}{c_0} cr \tag{27}$$

Putting in the value of the velocity as a function of r from Eq.(5), and equating, gives the angular momentum for the two photons to be:

$$S = \frac{\hbar}{c_0 \lambda\!\!\!^{-}_e}\left(\frac{3}{4}\frac{c_0 r}{\lambda\!\!\!^{-}_e}\right) r \quad , \tag{28}$$

Assume the value of the angular momentum for an electron along the z axis is ½. , into Eq.(28), and yields a most probable value of r to be:

$$r = \sqrt{\frac{2}{3}}\lambda\!\!\!^{-}{}_{e} \tag{29}$$

Proper Path Integrals calculation of the spin for the composite electron should yield the same result thus allowing a test of the methodology.

For path integration of the electrons action, it is noted that circular orbits inside the Compton radius have the same period, Eq.(9), and thus the imaginary action for all circular orbits are in phase and contribute to the action integral. Non-circular orbits are out of phase and tend to cancel.

Conclusion

There are a number of assumptions in the foregoing, but a model of an electron has been presented, based on the composite of two equal, half Schwinger energy, photons, that address a lot of unknown aspects of particle properties. All of the mechanics are well understood and provide a plausible connection to QFT and path integral methodology. The purpose has been to provide a general concept to open a window allowing QFT to be applied to more fundamental mechanisms of particle physics, absent infinities that plague standard methods.

The concept of charge has been an ongoing dilemma for physics, both in its creation and its connection to mass. This paper presents a concept within the plausibility of current theoretical developments to change the view of charge from some unknown substance distributed with mass, to the electrical vectors associated with vacuum polarization.

Although there has been a traditional reluctance to move mechanics into the internal aspects of particle models, a better defined electromagnetic photon model function, and the developments in vacuum polarization a makes this a plausible area of inquiry. The assumptions made in this presentation are both to the structure of the electron, and to the concepts of vacuum polarization, thus if there is merit to the model then both arenas will be advanced.

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Appendix I

Estimating the Index of Refraction Constant k

The centrifugal force necessary to hold one of the photons in orbit is:

$$F_c = \frac{p_1 v^2}{cr} \rightarrow \frac{p_1 c}{r} \tag{30}$$

Or:

$$QE = \frac{p_1 c}{r} \tag{31}$$

then:

$$E = \frac{p_1 c}{Qr} \tag{32}$$

Noting that the orbital radius is just the electron Compton radius and the Compton radius is twice that of the electron, Eq.(32), becomes:

$$E = \frac{p_1 c}{Q\lambda\!\!\!^{-}_e} \rightarrow \frac{c\hbar}{Q\lambda\!\!\!^{-}_e \lambda\!\!\!^{-}_1} \rightarrow \frac{Q}{2\alpha \lambda\!\!\!^{-}_e{}^2} \tag{33}$$

This is the value of E at of the Compton radius of the electron. (Noting that $m_e = 2m_1$)

The value of k can now be determined from Eq.(5). The linear constant at the orbit $r = \lambda\!\!\!^{-}_e$ is:

$$k = \frac{\lambda\!\!\!^{-}_e}{\lambda\!\!\!^{-}_e}\left(1 - \frac{E^2}{E_{cr}^2}\right) \tag{34}$$

Putting the value of E from Eq.(33), and the Schwinger Limit, the value of linear constant, k, is 3/4:

$$k = 1 - \left(\frac{Q}{2\alpha \lambda\!\!\!^{-}_e{}^2}\right)^2 \left(\frac{\alpha \lambda\!\!\!^{-}_e{}^4}{c\hbar}\right) = 3/4 \tag{35}$$

Note that the value of this is not necessarily exact, but some difference will not disrupt the model. The exact value can only be determined by methods of QFT.