

# **P ≠ NP using the power key, a proof by logical contradiction**

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## **Abstract**

Using a new technique called the power key, it's possible to imply P ≠ NP using a proof by logical contradiction.

## **Part 1**

- Let  $PS(x)$  be the list of all sublists of natural list  $x$ , with each sublist folded over the sum operation, such that, given some natural  $n$ ,  $PS(x)[n]$  is the  $n$ th element of  $PS(x)$ , well ordered as if the  $n$ th element of  $x$  was the  $n$ th power of 2 before each sublist was folded over the sum operation
- **NOTE: To clarify what "folded over the sum operation" means, here is the list [1, 2, 3] folded over the sum operation in pseudocode: "[1, 2, 3].fold(sum) = 1 + 2 + 3 = 6"**
- **NOTE: To clarify,  $PS(x)$  is the list of all sublist sums of  $x$ , well ordered as if each element of  $x$  was a unique power of 2**
- **NOTE: To clarify, "well ordered" means smaller naturals are always before larger naturals. This does not well order  $PS(x)$ , unless each element of  $x$  was well ordered and much larger than the previous element. However, in this proof,  $x$  is always unordered, therefore  $PS(x)$  is always unordered**
- Let a "valid power key" be a natural such that, for some list  $x$ , for all natural  $n$ ,  $PS(x)[n \oplus (\text{the valid power key of } PS(x))]$  is the  $n$ th largest element of  $PS(x)$
- **NOTE:  $\oplus$  is the Boolean exclusive or operation. If you apply  $\oplus$  against some natural  $x$  to every natural from 0 (inclusive) to  $2^n$  (exclusive), those naturals are reordered such that every unique  $x$  gives a unique order**
- **NOTE: Deciding the valid power key that works for all elements of  $PS(x)$  is the same as well ordering  $PS(x)$ . This is because  $PS(x)[n]$  is the  $n$ th element of  $PS(x)$ , unordered, and  $PS(x)[n \oplus (\text{the valid power key of } PS(x))]$  is the  $n$ th element of  $PS(x)$ , well ordered,**

so having the valid power key that works for all elements of  $PS(x)$  means you effectively have a well ordered  $PS(x)$

• **NOTE: If all elements of  $PS(x)$  are unique, there is only 1 valid power key for  $PS(x)$ . Again, 1 valid power key works for all elements of  $PS(x)$**

- Let A be an unordered natural list, given as input
- Let KEY be a natural, given as input
- Let the decision problem be "Given unordered list A as input and natural KEY as input, is KEY not the valid power key of  $PS(A)$ ?"
- A deterministic polynomial time verifier can verify a YES solution to the decision problem if list A, natural KEY, natural x, and natural y are given, such that  $(x < y) \neq (PS(A)[x \oplus KEY] < PS(A)[y \oplus KEY])$
- If a deterministic polynomial time verifier exists for a YES solution to a decision problem such that all deterministic Turing machines deciding it must run in superpolynomial time,  $P \neq NP$ 
  - If the decision problem can't be solved in polynomial time,  $P \neq NP$
  - If the decision problem can be solved in polynomial time, see part 2

## Part 2

- It's implied that algorithm ALGORITHM exists such that ALGORITHM can determine if a power key is invalid or not in polynomial time
- **NOTE: If ALGORITHM is polynomial time for a YES solution to a decision problem, ALGORITHM polynomial time for a NO solution to a decision problem, and vice versa**
- If ALGORITHM exists, deterministic polynomial time verifier V exists such that V can verify if a power key is valid for any set of subsets and also determine if that power key is even (YES) or odd (NO)
- Let M be some deterministic time Turing machine such that M, given only A, decides the power key of A, then determines if it's even (YES) or odd (NO)
  - Any such deterministic Turing machine runs in superpolynomial time. Otherwise, it could sort a set of subsets without looking at every subset, which is a logical contradiction
- It is implied that V can verify M's superpolynomial decision problem in polynomial time, given A and the power key of A, using ALGORITHM, therefore,  $P \neq NP$