

Equivalence, Locality, Torsion, and Λ CDM

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Abstract

A model of a quantum cosmology motivated from fundamental physical principles is proposed, in which the intrinsic spin of both matter and the vacuum explains the existence of a cosmological scale factor a with a constant H_0 , the existence and magnitude of the cosmological constant Λ , the thermal history of the Universe, and the nature of the Dark Matter.

1 Introduction

By the end of the 1920's, shortly after Einstein's publication of General Relativity, the expansion of the universe had been observed by Hubble and explained by Lemaître [1, 2]. By the turn of the century, we had accumulated evidence for a dark energy and a dark matter component in this model, summarised in the Λ CDM cosmology [3]. Most recently, laboratory-based detections of gravitational waves have corroborated General Relativity in the strong-field regime [4] in addition to its many other experimental successes over the past century. This is not only a strong corroboration of Einstein's theory, but of the conceptual foundations from which it was built (Locality, the geometric nature of spacetime, the Equivalence Principle, etc.).

Parallel to this, physicists have witnessed the development of quantum field theory, born out of early attempts to reconcile quantum mechanics with (special) relativity. By contrast with path integration, renormalisation and other mathematically dubious features of QFT, gauge ghosts and the BRST symmetry became part of the Standard Model around 1975 with solid formal foundations. The overwhelming experimental corroboration of the Standard Model is again to the credit of its conceptual foundations (Gauge invariance, etc.).

The last century is also full of theoretical efforts

that have, perhaps unexpectedly, failed to yield tangible fruit. Around the end of the 1920's Einstein and Cartan had investigated the gravitational effects of spacetime torsion; though this theory would reappear later in the work of Sciama and Kibble [5, 6], it never gained mainstream popularity because the effects of the torsion due to matter are too small to be measured (even in the most compact objects known) and the torsion due to matter vanishes in empty space [7]. In parallel to these gravitational studies, attempts to construct a mathematically well-defined QFT have resulted in Haag's theorem, which essentially proves that mathematically consistent local quantum field theories with global vacua do not exist (cf. e.g., Ref. [8] and therein).

In this paper, we present a prototype for a cosmological model based on an extension of the notion of Gauge Invariance and the Equivalence Principle in Einstein-Cartan gravity. We explicitly (and uniquely) recover the Λ CDM cosmology. The structure of this paper is as follows. In Sec. 2, we motivate our hypothesis that torsion in Riemann-Cartan spacetime, much like curvature in Riemann spacetime, is an observable feature of our universe inadequately recognised in our local inertial frames due to the Equivalence Principle. In Sec. 3, we look at the observable consequences of this hypothesis in cosmology and derive the redshift-distance relation of the FLRW cosmology. In Sec. 4 we investigate the origin

of this torsion, and in Sec. 5 its relation to renormalisation theory. In Sec. 6 we discuss fine-tuning issues and the dark sector of Λ CDM. In Sec. 7 we show how the Copernician principle applies to this model, and in Sec. 8 we speculate about quantum gravity.

The primary focus of this work is on empirical (cosmological) and meta-theoretical observations. The secondary focus is on conceptual rigour, rather than mathematical formalisation, regarding the physical principles involved in constructing the proposed model.

2 Gauge Theory and Locality

In classical field theory, by analogy with the definition of the Hamiltonian in classical mechanics, one obtains a *canonical* energy density tensor from the Lagrangian density $\mathcal{L}(\psi, \nabla\psi)$ of some field ψ by the Legendre transformation (cf. e.g. Eqn. (7.3.34) in Ref. [9]),

$$T_{(\text{can})}^{\mu\nu} = \frac{\delta\mathcal{L}}{\delta\nabla_\nu\psi} \nabla^\mu\psi - \mathcal{L}g^{\mu\nu}. \quad (1)$$

A similar expression exists for an \mathcal{L} including the Lagrangian densities of ordinary matter and any non-gravitational gauge fields¹; this stress-energy tensor is the *total energy* of the fields in our theory, as can be defined using only ψ and its derivatives (local measurements). This is why gravity is said to be ‘universal’, and why Einstein’s theory of gravitation conceptually follows from applying the equivalence principle to the mass in the celebrated formula $E = mc^2$.

2.1 Belinfante Symmetrisation and Gravity

The canonical tensor Eqn. (1) works just fine for matter fields, but it notoriously fails to respect gauge-invariance when used to determine the energy in gauge fields (both coupled to matter and in isolation). Yet in order to understand the energy as a measurable, ‘real’, physical quantity, it must be gauge-

¹i.e. \mathcal{L} does NOT contain the Lagrangian density $R^\mu{}_\mu$ for the ‘gravitational field’ $\Gamma_{\mu\nu}^\rho$.

invariant. In order to reconcile $T_{(\text{can})}^{\mu\nu}$ with the experimental observation that $T^{\mu\nu}$ is measurably gauge-invariant, it is customary to add to the canonical tensor a superpotential term [10],

$$T^{\mu\nu} = T_{(\text{can})}^{\mu\nu} + \nabla_\rho\chi^{\mu\rho\nu}, \quad (2)$$

where a clever choice of χ ensures gauge invariance and an assumed antisymmetry $\chi^{\mu\rho\nu} = -\chi^{\rho\mu\nu}$ maintains the divergencelessness. Amongst many possible choices for χ , the Belinfante-Rosenfeld (BR) procedure [10, 9], designed to symmetrise the indices of the stress-energy tensor using the antisymmetry of the intrinsic spin density tensor Σ of the fields ($\Sigma^{abc} = -\Sigma^{bac}$), offers a superpotential

$$\chi^{\mu\rho\nu} = -\frac{1}{2}(-\Sigma^{\mu\nu\rho} + \Sigma^{\nu\rho\mu} - \Sigma^{\rho\mu\nu}). \quad (3)$$

This superpotential also happens to ensure gauge invariance in certain cases (e.g. in Yang-Mills theories [10]).

The application of the BR symmetrisation procedure to $T_{(\text{can})}^{\mu\nu}$ is further motivated in the General Relativistic context by the symmetry of the Ricci tensor $R^{\mu\nu} = R^{\nu\mu}$ in the Einstein Field Equations. This is especially true for e.g. Dirac fields, which have non-symmetric contributions to the canonical stress-energy despite being (by construction) gauge-invariant. But although this choice of superpotential is motivated by both observation and theoretical consistency, from a meta-theoretical perspective it is on both accounts no more than an ad-hoc and post-hoc fix for $T_{(\text{can})}^{\mu\nu}$. Furthermore, the fact that this post-hoc fix is the only situation where spin densities Σ appear to be relevant in all of modern physics seems somewhat arbitrary, if not suspicious.

On the other hand, in the Einstein-Cartan Theory of gravity (ECT) it has been well-established that the geometry is sourced jointly by the canonical (unmodified) stress-energy tensor and the spin tensor [11]. In this theory, the spacetime is not Riemannian but Riemann-Cartan, and the Ricci tensor need not be symmetric; however one may choose to hide all of the torsion in the stress energy tensor, obtaining the Einstein-Cartan field equations on a Riemannian

manifold [11, 7]:

$$\frac{c^4}{8\pi G} G^{\mu\nu} = T_{(\text{can})}^{\mu\nu} - \frac{1}{2} \nabla_\rho (-\Sigma^{\mu\nu\rho} + \Sigma^{\nu\rho\mu} - \Sigma^{\rho\mu\nu}) + \frac{8\pi G}{c^4} (\Sigma^2 \text{ terms})^{\mu\nu} \quad (4)$$

This is interesting in two respects: Firstly, we see that ECT yields essentially the same predictions as BR-symmetrised GR (up to Σ^2 terms that we shall discuss later). Secondly, the Belinfante–Rosenfeld symmetrization of the energy-momentum tensor thus appears as a natural consequence of “removing” the torsion from ECT and using Riemannian spacetimes. Neither of these observations are new, but both deserve some more elaboration.

2.2 Equivalence

If we assume Einstein-Cartan Theory is the correct theory of gravity, and that GR is only an approximation of ECT, then the BR symmetrisation is no longer an ad-hoc fix but a feature our gravitational theory, as per Eqn. (4). To achieve this, we must sacrifice the gauge-invariance of the stress-energy of gauge fields, which is no longer fundamental and necessary, but contingent on our use of Riemannian (as opposed to Riemann-Cartan) spacetimes in the analysis of our experiments. The energy and momentum of gauge fields are fundamentally given by the gauge-dependent canonical tensor of Eqn. (1), but *appear* to be gauge-invariant (i.e. described by Eqn. (2)) when we *insist* the manifold be torsionfree (as dictated by Eqn. (4)).

This supposed relation between torsion and the gauge-invariance of stress-energy becomes less strange when one recalls that analogously, the conservation of stress-energy-momentum is contingent on the Riemannian spacetime’s Killing symmetries. Just as the symmetries of Minkowski space guarantee that conservation laws for $T^{\mu\nu}$ hold in locally inertial frames, the torsionlessness of Minkowski space guarantees that any observer sees $T^{\mu\nu}$ as a gauge-invariant quantity (when performing only local measurements). And just as we mistake spacetime’s curvature for a gravitational force when we insist (based

on local measurements) that $T^{\mu\nu}$ be conserved, we mistake spacetime’s torsion for a superpotential in the gauge field’s $T^{\mu\nu}$ when we insist (based on local measurements) the latter be gauge-invariant.

We’re already very familiar with the equivalence, for local observers, between Euclidean and Riemannian geometries in the theories of Newton and Einstein. The absence of the BR superpotential in Eqn. (1) and its presence in Eqn. (4) suggests a refinement of the Equivalence principle, connecting Riemannian and Riemann-Cartan geometries. Not only are the gravitational ($T^{\mu\nu}$) and inertial ($T_{(\text{can})}^{\mu\nu}$) masses equal, but also we posit a sufficient reason for their equality: because observers misrecognise the gravitational effects of intrinsic spin as a superpotential in the latter. This refinement does not modify the way equivalence works in GR. Strictly speaking it is not even a modification of gravity: it remains consistent with the ‘classical’ tests of GR (perihelion precession, gravitational lensing and redshifts) and of the equivalence principle (Eötvös balance experiments), because it adds specificity to the Equivalence Principle rather than modifying it.

We emphasise that this shift in perspective relies only on local quantities. No spacetime integrations are required in the canonical $T_{(\text{can})}^{\mu\nu}$, nor in the BR symmetrisation that follows from an observer’s choice of a nontorsional frame of reference. This contrasts to the Hilbert energy tensor $T_{(\text{Hil})}^{\mu\nu} = \delta S / \delta g_{\mu\nu}$, built from an action $S = \int d^4x \mathcal{L}$ which encodes information about the entire spacetime and is therefore conceptually unsuitable for use in a theory predicting the outcomes of local measurements. On a more formal note, the vierbein stress-energy $T_{(\text{vier})}^{\mu\nu} = e_a^\nu \partial \mathcal{L} / \partial e_\mu^a$ directly built from local reference frames e_a^μ , also coincides with $T_{(\text{can})}^{\mu\nu}$ [6]. The fact that the stress-energy is local in the sense of vierbeins is a prerequisite for us to measure it from within the confines of our Minkowski frames [8], again preferring $T_{(\text{can})}^{\mu\nu}$ over $T_{(\text{Hil})}^{\mu\nu}$ as the ‘fundamental’ stress-energy.

The importance of locality is explicit when reformulating our refinement of the Equivalence Principle more carefully, as follows: The local inertial frame of a freefalling observer is local not only in the conventional meaning that they are free to work in a patch of

\mathbb{M}^4 that is ‘smaller than gravity gradients’, but also (we posit) in the stronger sense that the Minkowski patch they work in is torsionfree, and all stress-energy measurements they make are understood in relation to their local (symmetrised, gauge-invariant) $T^{\mu\nu}$.

2.3 Gauge Ghosts

The main implication of this discussion of observers’ local frames is that the torsion field, like the gravitational field, cannot be localised into the frame of a single observer. Therefore (again like the gravitational field) it has no stress-energy tensor itself, despite having locally measurable physical consequences, namely the gauge-invariance and symmetry of stress-energies.² However, there is one important difference between curvature and torsion: a local observer can easily escape the inertialness of their freefall (that is, in fact, the typical state of our species on Earth) and thereby verify that gravity is an inertial force; but no local observer can escape the local nature of their own frame of reference. Instead, such observers must turn to the unexplained features and the formal structures of their theories, hoping these might reflect some nonlocal aspects of the torsion.

These observers can notice that the fundamental gauge symmetries in the Standard Model always appear as local symmetries. Furthermore they can notice that ‘unphysical’³ ghost fields appear in loop calculations, the nonlocal quantum corrections to their classical results. Much like the superpotential χ , these ghost fields are introduced ad hoc and post hoc, when we insist *petitio principii* that measurements are gauge-invariant. For these quantum phenomena, locality is again understood in the sense of frame fields e_a^μ , a defining element of the BRST quantisation from which these ghost fields arise [8, 12].

There is a formal basis for identifying gauge ghosts with the nonlocal effects of torsion. Faddeev-Popov ghosts are the Maurer-Cartan form α on the gauge group [13, 14, 15]. With a flat connection, as in the

local Minkowski frame, the torsion is given by $d\alpha$. The Maurer-Cartan equation $d\alpha = -\frac{1}{2}[\alpha, \alpha]$ then shows that ghosts are nothing more than an algebraic re-expression of torsion. Torsion disappears in a local Minkowski frame, and so do the ghosts disappear from exterior branches of a Feynman diagram (as a consequence of their nonzero ghost quantum numbers). However, without the insight that gauge-invariance is an artefact of our use of Riemannian geometry, an observer would misrecognise torsion as ghost fields in the nonlocal contributions from loops.

* * *

The appearance of the Belinfante superpotential (Eqn. (3)) in the Einstein-Cartan field equation (Eqn. (4)), prompted the hypothesis that the gauge-invariance of the stress-energy tensor might be an artefact of our use of Riemannian geometries to describe the background spacetime. This entails an elaboration of the Equivalence Principle that weaves together locality, torsion, the symmetry of $T^{\mu\nu}$, and (in the case of gauge fields) its gauge-invariance. The structure of the Standard Model of particle physics (a local gauge theory, with ghosts to ensure gauge-invariant observables, on a Riemannian spacetime) is then a very strong hint that our spacetime is actually Riemann-Cartan. Another Standard Model hint that gauge-invariance and gravity are related is that $SU(2)_L \times U(1)_Y$ gauge invariance forbids massive gauge bosons and fermions ($T^{00} \equiv 0$). An equivalence between quantum theoretic and gravitational phenomena in this model will be a recurring theme of this study, though the possible reasons for this equivalence are only worth investigating (cf. Sec. 8) once we’ve studied the viability of the model as an explanation of actual phenomena.

3 Cosmological Expansion

Let’s now turn from particle physics to the cosmological implications of this new perspective on the equivalence principle.

Suppose, as is observationally the case in the early universe, that $T^{\mu\nu}$ is to a good approximation constant throughout the observable universe; not just

²Not to be confused with the *effective* stress-energies introduced in Sec. 6, which are nonlocal in the same sense as, e.g., the Landau-Lifshitz pseudotensor.

³From Ref. [12], §15.7: “[T]he physical Hilbert space is free of ghosts and antighosts.” [emphasis original]

in the Riemannian spacetime, but in the equivalent spacetime with torsion. In ECT the torsion is algebraically related to the spin of this homogenous matter, so it is plausible that the torsion is also constant. At this point we simply assume homogeneity of $T^{\mu\nu}$, and we assume (against the common wisdom) that the macroscopic torsion / quantum spin density is homogeneous and does not vanish. These assumptions will be justified later (Sec. 4) once their implications are clearly formulated.

3.1 Hubble Expansion

We know (by construction) that the canonical stress-energy is conserved with respect to the covariant derivative on the spacetime with torsion, but it need not be so for the symmetrised stress-energy with respect to the Riemannian spacetime we impose. The difference between the connections in these covariant derivatives is the contorsion tensor $K_{\mu\nu\rho}$ [6, 16]. This quantity is uniquely determinable (in ECT) at any point in spacetime from the spin density tensor [7, 6],

$$K_{\mu\nu\rho} = \frac{8\pi G}{c^4} (\Sigma_{\nu\rho\mu} + \Sigma_{\nu\mu\rho} - \Sigma_{\rho\mu\nu}), \quad (5)$$

and (given the antisymmetry of Σ in its first two indices) is manifestly antisymmetric in its last indices.

We can use Eqn. (2) and this difference $K_{\mu\nu\rho}$ in connections, to extract the ‘‘Riemannian’’ stress-energy gradient from the one in the torsionful spacetime. Recalling the assumption that T and Σ are constant in the spacetime with torsion, and furthermore neglecting terms $K^2\chi$ (which are a factor of $\sim G^2/c^8$ smaller than the stress-energies and spin densities) we can use Eqn. (2) and Eqn. (5) to reduce

$$\begin{aligned} \overset{\text{(tor)}}{\nabla}_\psi T_{\text{(can)}}^{\mu\nu} &= \nabla_\psi (T^{\mu\nu} - \overset{\text{(tor)}}{\nabla}_\rho \chi^{\mu\rho\nu}) \\ &- \overset{\text{(tor)}}{\nabla}_\psi (K_{\rho\sigma}{}^\mu \chi^{\sigma\rho\nu} + K_{\rho\sigma}{}^\rho \chi^{\mu\sigma\nu} + K_{\rho\sigma}{}^\nu \chi^{\mu\rho\sigma}) \\ &- K_{\psi\sigma}{}^\mu (T^{\sigma\nu} - \overset{\text{(tor)}}{\nabla}_\rho \chi^{\sigma\rho\nu}) \\ &- K_{\psi\sigma}{}^\nu (T^{\mu\sigma} - \overset{\text{(tor)}}{\nabla}_\rho \chi^{\mu\rho\sigma}) \\ &+ (K^2\chi \text{ terms}), \quad (6) \end{aligned}$$

into

$$\nabla_\psi T^{\mu\nu} = 2K_{\psi\sigma}{}^{(\mu} T^{\nu)\sigma}. \quad (7)$$

This nonconservation of stress-energy is not locally measurable, since K vanishes in our local frame as a consequence of the equivalence principle. But, when we integrate over cosmologically large distances it cumulates (linearly, since K and T are assumed constant) into an observable quantity:

$$T^{\mu\nu}(x + \Delta x) - T^{\mu\nu}(x) = \int_x^{x+\Delta x} \nabla_\psi T^{\mu\nu} dx^\psi, \quad (8)$$

$$\Delta T^{\mu\nu} = 2K_{\psi\sigma}{}^{(\mu} T^{\nu)\sigma} \Delta x^\psi. \quad (9)$$

This integral can be more rigourously defined by considering the geodesic $x^\psi(\lambda)$ connecting the points x and $x + \Delta x$, and treating it as the line integral

$$\int_x^{x+\Delta x} [\dots] dx^\psi \equiv \int_{\lambda_0}^{\lambda_1} [\dots (dx^\psi/d\lambda)] d\lambda, \quad (10)$$

but heuristically $\int dx^\psi$ merely relates the (nonlocal) finite-difference Eqn. (9) to the (local) differential Eqn. (7).

If we insert a perfect fluid equation of state into the above, then for spacetime separations Δx in the fluid’s rest frame ($T^{0i} = 0$) we find

$$\Delta T^{00} = 2K_{\psi 0}{}^0 T^{00} \Delta x^\psi = 0 \quad (11)$$

$$\Delta T^{ii} = 2K_{\psi i}{}^i T^{ii} \Delta x^\psi = 0 \quad (12)$$

by antisymmetry of K . This highlights that perfect fluids (including, on cosmological scales, experimental physicists) can’t observe this energy shift from within their local rest frame – even when waiting for cosmologically long proper times – except for shifts in off-diagonal elements, such as

$$\Delta T^{0i} = K_{\psi 0}{}^i (\rho - p) \Delta x^\psi. \quad (13)$$

In this context, distinct points in a perfect (pressureless) matter fluid appear to have an extra momentum difference ΔT^{0i} proportional to their separation, which outside of their rest frame would appear superposed onto their peculiar momenta. In the nonrelativistic regime we have $\rho \propto m$ and a momentum mv ,

so with ρ and K constant Eqn. (13) appears to yield a recessional velocity,

$$v_H \propto \Delta x. \quad (14)$$

The antisymmetry of K ensures that $\Delta T^{0i} = -\Delta T^{i0}$, so this recessional velocity is isotropically outwards from any point x . We will return to this quantity in Sec. 4. Note however that in the ‘rest frame’ of a radiation fluid, we have spacetime separations of $\Delta x = 0$, so we need to be a bit more clever to actually derive Hubble’s Law.

Now suppose instead that an observer located at x is looking at the stress-energy of a light ray that has propagated across the universe along the null geodesic $\{x + \Delta x \rightarrow x; t - \Delta t \rightarrow t\}$. Unlike for a perfect fluid in its rest frame, the Poynting vector T^{0i} of this radiation is nonzero, and changes in the radiation’s energy density are nonzero. The difference Eqn. (9) in the electromagnetic gauge field between two lightlike-separated locations in spacetime, when understood in relation to the local stress energy tensor of the observer, would then appear to be a *photon redshift*, $z = \Delta T/T$. While this ‘tensor division’ is formally suspicious, recall that what is experimentally measured is not the redshift but the photon frequency $\omega_{\text{obs}} = E/h$, which satisfies Eqn. (9) with no such division under the form $\omega_{\text{emit}} - \omega_{\text{obs}} = z \times \omega_{\text{obs}}$.

By observing many such light rays, our observer finds that this redshift increases proportionally with distance [17, 2]. Renaming the proportionality constant to $H_0/c \sim 2K$ yields Hubble’s Law,

$$z = H_0/c \Delta x, \quad (15)$$

with H_0 constant as long as the spin density in Eqn. (5) remains constant on cosmological scales. In this context the spacelike Δx acquires the interpretation of a comoving distance.

3.2 Scale Factor

If an observer, faced with this apparent recessional velocity, insists that the spacetime is torsion-free and obeys Einstein’s equations [1], they’d need to impose the Bianchi Identity $\nabla_\mu G^{\mu\nu} = 0$ and therefore also $\nabla_\mu T^{\mu\nu} = 0$, at odds with the nonconserva-

tion Eqn. (7) we’ve derived from the torsionful spacetime. We argue that this observer resolves this tension as follows: In the same way that observers can use Equivalence to hide the spacetime curvature into an inertial ‘gravitational force’, or hide some of the spacetime torsion in the stress-energy in Eqn. (2), or into ghosts in the Maurer-Cartan equation, in this cosmological setting the excess contorsion is absorbed into $G^{\mu\nu}$ and becomes the scale factor of the FLRW spacetime.

We can make this argument not only in terms of the apparent redshifts of stress-energies, as above, but also directly in terms of geometry and connections. The connection on the torsionfree FLRW spacetime is the Levi-Civita connection, $\Gamma_{(\text{FRW})}$. The connection on the full spacetime with torsion is however $\Gamma = \Gamma_{(\text{FRW})} - K$. By Equivalence, the observer will find $\Gamma_{(\text{FRW})} = K$ when they extrapolate a General Relativistic homogenous cosmology beyond their local inertial frame (in which $\Gamma_{\mathbb{M}^4} = 0$). Notice that the Levi-Civita connection on the FLRW metric, evaluated locally, is indeed the Hubble constant; therefore, the contorsion and the Hubble expansion are Equivalent.

In the Riemannian geometries of General Relativity, this Levi-Civita connection uniquely determines the metric [18]. Here, the connection \dot{a}/a determines the metric to be FLRW, $ds^2 = a^2(d\eta^2 + d\vec{x}^2)$, and likewise it uniquely determines the curvature tensor, the Friedmann equations... In short, the contorsion has become a cosmological scale factor a . The extrapolation of this scale factor, far from our contemporary local frame $a = 1$ and deep into the distant past $a \rightarrow 0$, makes it appear (to the observer unaware of the torsion) that the universe started with a Big Bang.

3.3 Cosmological Constant

The integration Eqn. (9) above is of course only the linear, leading order effect in the full Taylor expansion of $T(x + \Delta x)$. Corrections to Hubble’s Law may be added by refining the discussion above to include small perturbations. Working with Eqn. (6) and perturbation theory, we write $\Sigma = \bar{\Sigma} + \epsilon\Sigma'$ with $\bar{\Sigma}$ a constant, $\epsilon \ll 1$, and Σ' a stochastic perturbation, so

that $\chi = \bar{\chi} + \epsilon\chi'$ and similarly for K . The nonconservation of energy Eqn. (7) then becomes

$$\nabla_{\psi} T^{\mu\nu} = +2\bar{K}_{\psi\sigma}^{(\mu} T^{\nu)\sigma} + \epsilon \nabla_{\psi} \overset{(\text{tor})}{\nabla}_{\rho} (\chi')^{\mu\rho\nu} + (\epsilon^2, \epsilon G/c^4, \text{ and } G^2/c^8 \text{ terms}). \quad (16)$$

In what follows we denote the torsionful superpotential divergence $\epsilon(\nabla\chi')^{\mu\nu} \equiv \Upsilon^{\mu\nu}$. The integral Eqn. (9) over cosmological scales in our almost homogeneous cosmology is then promoted to

$$\Delta T^{\mu\nu} = 2\bar{K}_{\psi\sigma}^{(\mu} T^{\nu)\sigma} \Delta x^{\psi} + [\Upsilon^{\mu\nu}(x + \Delta x) - \Upsilon^{\mu\nu}(x)], \quad (17)$$

which has the linear-with-distance redshift from earlier, plus (using the fundamental theorem of calculus) a perturbing difference $\Delta\Upsilon$. In our local Minkowski frames we are allowed to choose a vanishing torsion, but we cannot choose its divergence $\Upsilon(x) = 0$ since this is our Belinfante correction. We can, however, Taylor expand the quantity $\Upsilon(x + \Delta x)$, such that

$$(\Delta\Upsilon)^{\mu\nu} = A_{\rho}^{\mu\nu} \Delta x^{\rho} + \frac{1}{2} B_{\rho\sigma}^{\mu\nu} \Delta x^{\rho} \Delta x^{\sigma} + \dots \quad (18)$$

where any antisymmetry in the lower indices of $B_{\rho\sigma}^{\mu\nu}(x) = [\nabla_{\rho}\nabla_{\sigma}\Upsilon^{\mu\nu}](x)$ – a quantity related to the Ricci Identity for the Belinfante superpotential – is made irrelevant by the symmetry of $(\Delta x^2)^{\rho\sigma}$.

Now, consider the observable consequences of this perturbation, order by order in Δx , and their reinterpretation in light of the Equivalence principle. An observer looking at cosmological radiation and insisting on a Riemannian geometry, would interpret this difference in superpotentials as new term $z_{\Upsilon} \sim \Delta\Upsilon/T$ of the redshift-distance relation $\Delta T/T$ (recall that ω_{obs} rather than z is actually observed). The zeroth-order term turns out to be zero identically (consistently with today being $z = 0$) since we are looking at a superpotential difference, and the Δx term is a correction (of size ϵ) to the Hubble Constant. More importantly, this observer would identify the new term at Δx^2 as a *deceleration parameter*. Indeed, the Taylor-expansion of the FLRW scale factor $a(t)$ has historically been parameterised (cf. e.g. Ref. [18], p.773,

781) such that

$$z \approx H_0 \Delta x + \frac{1}{2}(1 + q_0) H_0^2 \Delta x^2 + \dots \quad (19)$$

Very heuristically, we can then identify

$$B_{\rho\sigma}^{\mu\nu} \sim [(1 + q_0) H_0^2]_{\rho\sigma\alpha\beta}^{\mu\nu} T^{\alpha\beta} \quad (20)$$

where the association $H_0 \leftrightarrow \bar{K}_{\bullet\bullet}$ above gives the right number of indices to the term in brackets.

This deceleration parameter is permitted to be anisotropic and inhomogeneous, but it is extremely likely to be statistically isotropic, since the Belinfante term Υ that fixes the gauge-invariance of our statistically isotropic matter is likely also statistically isotropic.

It should come as no surprise that, having found the reasons for the expansion of the universe, we thereby gain insight into the reasons for the latter's acceleration. In fact the term $(B\Delta x^2)^{\mu\nu}$ in Eqn. (18), with B in units of stress-energy-density, is directly reminiscent of the 'dark energy' $\Lambda g^{\mu\nu}$ in Einstein's Field Equations. In this model, the cosmological acceleration is equivalent to the difference in Belinfante corrections over cosmological scales.

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By insisting on a torsion-free cosmological model with a nearly homogenous universe an observer insists, in essence, on a photon redshift consistent with a scale factor subject to a minuscule cosmological constant. The Equivalence Principle in Einstein-Cartan gravity can therefore explain the origin of our FLRW cosmology.

Somewhat orthogonal to the issues of fine-tuning and observations we discuss throughout this paper, there are a number of conceptual paradoxes in Λ CDM that follow from our use of an FLRW spacetime, and disappear when we realise that the spacetime is not actually FLRW. For instance, how can a photon's cosmological redshift can be a 'gravitational redshift' without there being a stress-energetic source for this gravity in the FLRW spacetime? Or, how should we reconcile strong cosmic censorship with the existence of a naked singularity at $z \rightarrow \infty$ in the FLRW metric? These paradoxes (directly related to Einstein's

conception of a Machian and deterministic universe, which underlie Λ CDM) are easily resolved by denying their common premise that the FLRW expansion is physical.

In addition to this added conceptual clarity, we expect that this model can quantitatively reproduce all of the successes of Λ CDM on large scales (cf. Sec. 6). However, this model does not (yet) explain why these constants (H_0 , Λ , etc.) have the specific values they do, since this depends on the details of the torsion distribution, which will be specified in the next section.

4 The Cosmological Vacuum

We have a model for the cosmological expansion and its acceleration, in which these arise from a torsion field and (leveraging the Equivalence principle) our use of Riemann rather than Riemann-Cartan spacetimes; however we still need to explain where this torsion comes from and why it should be nearly, but not exactly, homogeneous.

4.1 Origin of Cosmological Torsion

We are able, in our Einstein-Cartan theoretical cosmology, to relate the torsion to the comoving density of particles with intrinsic spin in the universe. As a consequence, there could in principle be discontinuities in H_0 (in addition to the Friedmannian evolution we impose upon it), as the result of sudden changes in the particle composition of the universe at each of its major phase transitions. In order that H_0 remain constant despite hadronization, BBN, recombination, reionization, etc., the particles carrying the overwhelming majority of this spin density must be decoupled from the Standard Model forces at work in these phase changes.

A ‘spin budget’ of all the known matter and radiation should easily quantify the ‘missing spin’ in our cosmology, in addition to the ‘missing mass’ we already observe. However we should not overeagerly conflate the two. Using Eqn. (5), the best-fit value $H_0 = 67.3$ from Planck [3], and the identification $2K \leftrightarrow H_0$, we can construct a constant of the order

of the macroscopic spin momentum density,

$$\bar{\Sigma} \sim \frac{c^2(H_0/2)}{8\pi G} \approx 5.55 \times 10^{41} \text{ } \hbar/\text{m}^3 \quad (21)$$

which for a particle with an intrinsic spin of no more than a few \hbar would be far too abundant (and therefore too light) to be the *cold* dark matter we observe.

The spin-carrying particles, despite their abundance, are therefore absent from the ‘stress-energy budget’ $\Omega = 1$ of the universe (which, of course, counts them even if they haven’t been discovered yet in particle detectors). Another way to see this, which is also fundamentally more correct, is to note that we’ve treated $T^{\mu\nu}$ classically, i.e. in the limit $\hbar \rightarrow 0$, so that nothing in the stress-energy budget can carry intrinsic spin. This points to particles we’ve willfully neglected in our classical treatment of empty spacetime as the source of the cosmological torsion. These particles are either the spin-2 graviton (or more properly whichever quantum is appropriate in a Riemann-Cartan spacetime), or the various constituents of the quantum vacuum. We favor the latter hypothesis, not only since the quantum vacuum is experimentally known to exist while quantum gravity remains speculative, but also since gauge ghosts (which we’ve tied to torsion) are directly related to the renormalisation of quantum gauge theories and therefore to the quantum vacuum.

4.2 Side effects of vacuum torsion

The connection between locality and the quantum vacuum is reinforced by Haag’s Theorem, which (formally) prohibits relativistic local QFTs with a global vacuum [8]. In light of the successes of QFT, there is a tendency to disregard this strong result and assume such a global vacuum anyway, despite the fact that Unruh Radiation (a smoking gun for the existence of a global vacuum) has never been observed [ref!!!]. We argue that the local successes of local QFTs remain valid, but that this global vacuum torsion has observable effects when considering the nonlocal effects of our local QFTs. For instance, a global vacuum will by definition have a global (constant) spin density $\bar{\Sigma}$: We have already seen that this can explain a number of cosmological observations, and we propose

that the vacuum condensate may also be useful in understanding the transition from nonlocal-torsion-as-ghosts to nonlocal-torsion-as-cosmic-expansion. Our model is therefore not a quantum cosmology in the traditional sense of a quantum mechanical cosmology, but instead it is a model in which any phenomenon may be interpreted equivalently as quantum mechanical or as cosmological.

Every physics student has wondered why the expansion of the universe doesn't tear atoms apart [18] – but perhaps it does. The macroscopic spin density of the vacuum becomes the dominant source of torsion at the scales at which we expect nuclear confinement to occur: In Table 1 we show the cosmological spin density per nuclear volume for various chemical elements, calculated by approximating the spherical atomic radius as $r = R_0 \times A^{-1/3}$ so that $V \propto A$. Tantalisingly, nuclear isotopes with a specific spin density of $\gtrsim 1$ have an atomic mass $A \gtrsim 220$; these isotopes are famously unstable. This suggested identity between nuclear fission and Hubble expansion is also supported by the resounding success of Big Bang Nucleosynthesis, which relates this same expansion to nuclear fusion [19] and will be discussed shortly.

If this coincidence is physically significant, perhaps the torsion's transition from “ghosts on \mathbb{M}^4 ” to “FLRW expansion” can be conceived (from the perspective of the \mathbb{M}^4 theory) as a breakdown of the applicability of the theory for large-scale phenomena. In other words, cosmic expansion is an infrared regulator that effectively creates a mass gap in Yang-Mills theories on \mathbb{M}^4 ; if string theory has taught us anything in the last fifty years, it is that theories attempting to explain QCD confinement actually look a lot like theories of gravity with nonlocal phenomenology! In corroboration of this conjecture, notice that the photon is decoupled from its ghosts (i.e. from the background torsion / expansion), and that vacuum QED has no mass gap.

Anticipating our discussion in the next section, notice that what determines stability of an isotope, in this picture, is whether its spin density is greater or smaller than that of the (Haag-violating) global vacuum we've chosen for our quantum theory on \mathbb{M}^4 . If we were to change our choice of the vacuum, for

Table 1: Cosmological spin density per nuclear volume for various chemical elements.

	$\bar{\Sigma}(H_0)/\text{Vol}$
Halo radius R_0 (A=1)	4.5×10^{-3}
Nickel (A=62)	0.28
Radon (A=222)	1.0

instance by renormalising the theory, then we also change which QCD states we consider to be bound states. Conversely, notice that QED is trivial under renormalisation.

4.3 Torsion's Explanatory Power

This hypothesis about the quantum global vacuum sourcing the cosmological torsion field – the result of an empirically driven process of elimination (Sec. 4.1), with empirically testable consequences – naturally explains some of the open puzzles in our cosmological picture. Firstly, the extrapolation of our local vacuum to a global vacuum (despite Haag's Theorem) explains the origin of a constant torsion density, and therefore a constant Hubble parameter. Secondly, this conjecture explains how the expansion of the *nearly empty* spacetime we live in can occur, despite the torsion being algebraically (Eqn. (5)) related to the universe's *contents* at any given point in Einstein-Cartan theory: empty spacetime is full of vacuum! Finally, it explains why the vacuum's energy density we calculate in quantum theory and the ‘dark energy’ density are so different: the former does not couple to classical gravity [ref Padmanabhan], and the latter is actually a vacuum torsion / deceleration parameter (due to torsion inhomogeneities) rather than a vacuum energy density.

* * *

The cosmological expansion we've been measuring with ever-increasing accuracy over the last century – and not the cosmological constant – provides the first direct experimental evidence of the gravitational influence of the quantum vacuum.

5 Hubble's Law for the Vacuum

5.1 Homogeneity of the Vacuum

Consider how inhomogeneities in the superpotential χ relate to those of the stress-energy T . If $\bar{\Sigma}$ is the vacuum spin density, perhaps \bar{T} is the vacuum stress-energy and T' (the perturbative correction to \bar{T}) is the 'classical' stress energy – in this interpretation it comes as no surprise that \bar{T} should be constant everywhere, even in the emptiness of space. Note that we do not need an inflationary fine-tuning to get the initial conditions for \bar{T} : the vacuum is by definition empty everywhere.

It then also comes as no surprise that the perturbations on top of our global vacuum are tiny: the vacuum energy density is a *huge* number relative to classical energies! Let's start once more from Eqn. (6), this time writing not only

$$\Sigma = \bar{\Sigma} + \eta\Sigma' = \bar{\Sigma} + \eta(\bar{\Sigma}' + \epsilon\Sigma'') \quad (22)$$

but also $T = \bar{T} + \eta(\bar{T}' + \epsilon T'')$. These contributions refer to the vacuum contribution, and the homogeneous and inhomogeneous contributions of the matter, respectively. We find, at order η^0 in perturbations, exactly Eqn. (7) with $T \rightarrow \bar{T}$ and $K \rightarrow \bar{K}$. We will return to the interpretation of this exact Hubble scaling of the vacuum energy shortly. At order η^1 , we find the following more 'careful' version of Eqn. (6):

$$\begin{aligned} \nabla_\psi T'^{\mu\nu} &= 2\bar{K}_{\psi\sigma}^{(\mu} T'^{\nu)\sigma} + \nabla_\psi \Upsilon^{\mu\nu} \\ &+ \nabla_\psi^{(\text{tor})} T'_{(\text{can})}{}^{\mu\nu} + \nabla_\psi (3\bar{K}\Sigma' + 3K'\bar{\Sigma}) \\ &+ 2K'_{\psi\sigma}^{(\mu} \bar{T}^{\nu)\sigma} - \bar{K}_{\psi\sigma}{}^\mu \Upsilon^{\sigma\nu} - \bar{K}_{\psi\sigma}{}^\nu \Upsilon^{\mu\sigma} \\ &+ (K^2\chi \text{ terms}) \quad (23) \end{aligned}$$

The first two terms will of course turn out to be Hubble's Law and the deceleration parameter as seen by the gauge-invariant stress-energies T' , reproducing Eqn. (16) as before. The remaining terms are new and curious – but notice that we're no longer using the fact that the universe is very homogeneous now that we've identified \bar{T} as a vacuum energy: we need to split the perturbation T' into a homogeneous

part (that vanishes under ∇) and a truly inhomogeneous part (which is what we thought we were studying above): $T' = \bar{T}' + \epsilon T''$ and correspondingly $\Sigma' = \bar{\Sigma}' + \epsilon\Sigma''$. For exactly homogeneous matter and torsion densities, the complicated expression above reduces to

$$\nabla_\psi \bar{T}'^{\mu\nu} = 2\bar{K}_{\psi\sigma}^{(\mu} \bar{T}'^{\nu)\sigma} + 2\bar{K}'_{\psi\sigma}^{(\mu} \bar{T}^{\nu)\sigma} \quad (24)$$

where we set all barred quantities under $\nabla^{(\text{tor})}$ to zero, e.g. in

$$\Upsilon^{\mu\nu} = \nabla_\rho^{(\text{tor})} (\chi'^{\mu\rho\nu}) = \epsilon \nabla_\rho^{(\text{tor})} (\chi''^{\mu\rho\nu}). \quad (25)$$

The vacuum energy \bar{T} on its own is a large number; but the coherent macroscopic torsion due to the matter's spin is well known to be $\bar{K}' = 0$ (or more accurately the average $\langle \bar{K}' \rangle \rightarrow 0$ for a macroscopically large number of particles [16]), so that $\bar{K}'\bar{T} \rightarrow 0$: this term vanishes and we recover Hubble's law (with the same $H_0/c = 2\bar{K}$ as for the vacuum energy density) for exactly homogeneous matter. The well-known result in Einstein-Cartan theory that the torsion due to matter vanishes on cosmological scales is therefore a crucial ingredient of this model.

Now, we see that there are in fact two Hubble's laws at different orders of perturbation theory: one valid for the vacuum energy density up to perturbations η which are the universe's matter contents, and one valid for the universe's homogeneous contents up to perturbations ϵ which are matter inhomogeneities. While reproducing Hubble's Law for the large-scale structures of the universe, this model does not predict that the gauge-non-invariance of small-scale inhomogeneous perturbations would admit a reinterpretations as an FLRW scale factor. The extra terms in Eqn. (23), which conveniently disappeared for the homogeneous vacuum \bar{T}' and the homogeneous matter density \bar{T}' , cannot be ignored when considering the $\nabla T''$ of the inhomogeneous perturbations themselves; therefore the effect of torsion on e.g. the dynamics of the Solar system or the Earth-moon system are not just the effects of an FLRW scale factor with $\dot{a}/a = H_0$ (which would immediately refute this model experimentally!), but need to be computed explicitly using these higher order perturbations.

5.2 Why the Big Bang looks Hot

The observation of a cosmic microwave background with a blackbody spectrum and anisotropies of order $\Delta\Theta/\Theta_0 = 10^{-5}$ (under a ± 3.5 mK dipole, related to our peculiar velocity with respect to the global vacuum we define in terms of the Hubble flow) provides a spectacular confirmation of the simple FLRW cosmology that this model reproduces. This model would clearly be incomplete, then, without explaining the behaviour of the CMB in terms of Einstein-Cartan theory.⁴

If the universe’s expansion is just the side-effect of a local choice of a Riemannian frame, how can we explain that the temperature of the CMB seems to scale as $\Theta/(2.725\text{ K}) = 1 + z$? More precisely: why does the typical gauge-invariant energy of particle interactions outside of our local Riemannian frame appear to follow a linear-with-redshift law?

We postulate that the redshift of the vacuum energy Eqn. (9) is the reason the early universe appears to have a temperature $\Theta/\Theta_0 = (1 + z)$: Even though the vacuum energy $\bar{T}^{\mu\nu}$ is constant in the Riemann-Cartan spacetime (by assumption), in the Riemannian FLRW we’re not “seeing” the same vacuum energy in our local frame as in the early universe because the virtual particles in the Riemann-Cartan vacuum of the early universe would also “appear” redshifted. The early universe appears not only redshifted, but also appears to be hotter!

But as always, because equivalence operates locally, observers confined to small patches of the universe don’t necessarily realise this. They believe in a global Riemannian vacuum, and they *impose* that the vacuum at points in the early universe is the same as theirs: they’re unwittingly letting the renormalisation group flow as they probe the cosmic distance ladder. The distant universe appears to be at a different temperature, because they’re (unwittingly) changing the energy scale of the theory they use to study it!

For this explanation to be possible, there must ex-

⁴The photon field being a gauge field, its stress-energy tensor is only gauge-invariant insofar as our description of the universe is Riemannian. An explanation of the origin of the CMB is postponed until an explanation of its apparent behaviour can be established.

ist a spatially local prescription for renormalisation, such that the notion of applying different renormalisations at different spacetime points (here different redshifts) is sensible. It turns out that the Epstein-Glaser (“rigorous BPHZ”) prescription for perturbative renormalisation not only guarantees locality by construction [20], but is also the contemporary golden standard for mathematically rigorous renormalisation prescriptions.

5.3 Cosmological Phase Transitions

As a corollary, cosmological phase transitions may be characterised in the Riemannian spacetime by S-matrices connecting different particle bases for ‘in’ and ‘out’ states occurring at different locations in spacetime (different redshifts), depending on the (in)stability of such states at any given renormalisation scale.

We traditionally say Big Bang Nucleosynthesis *occurs* because the dilution of the energy density as the (Riemannian) universe expands allows quantum fields to *form* bound states with increasingly small unbinding energies; background photons would immediately unbind any states with unbinding energies smaller than some energy scale. However, one might recast this as a running of the renormalisation scale: nucleosynthesis *appears to occur* because the unwitting decrease of the renormalisation scale with redshift allows quantum fields to *be described by* long-lived bound states with increasingly large binding energies; for instance deuterium exists only as an unstable (virtual) state when the renormalisation scale exceeds 2.23 MeV.

Notice that nucleosynthesis-as-renormalisation does not rely on thermodynamic equilibrium of nuclei with the photon bath – the energy content of gauge fields is not a gauge-invariant quantity outside of the Riemannian spacetime – and in fact is not a (thermo)dynamic process at all. Therefore, we avoid the “Low-entropy problem” of the Big Bang cosmology (cf. e.g. Ref. [21] and therein). However, it’s important to emphasize that this description of cosmological phase transitions is *equivalent* to the description as evolution in time in the Riemannian-geometric Λ CDM model.

5.4 Structure Formation

The above picture of the universe’s thermal history also allows us to treat structure formation in terms of the same renormalisation theory. In fact, this treatment already exists and needs only be recontextualised.

The Press-Schechter model of self-similar collapse [refs] does not follow the gravitational collapse of perturbations over time, but rather as a function of a resolution parameter “which acts like a pseudo-time variable” [22]. In this model, the density perturbation fields of any two (lightlike) separated parts of our universe are statistically identical, the distinction between them being only the resolution scale R with which they are observed. The fluctuations that have gravitationally collapsed, at some scale R , are those with amplitudes higher than some threshold value δ . Therefore this resolution parameter is usually identified with the variance σ^2 of the density field at some scale R ,

$$\sigma^2(R) = \frac{1}{(2\pi)^3} \int_0^\infty P(k)W^2(k, R)d^3k, \quad (26)$$

where $P(k)$ is the perturbation field’s power spectrum, where W^2 is the R -scaled smoothing function, and where observationally we have [3]

$$\sigma_8 = \sigma(R = 8h^{-1} \text{ Mpc}) = 0.816 \pm 0.009. \quad (27)$$

Note that the amplitude of perturbations evolves with redshift as $\sigma \propto (1+z)^{-1}$: this defines a one-to-one mapping between the renormalisation scale R and the redshift z . The relation between σ and R above depends on Λ CDM parameters (σ_8, n, \dots) and the choice of the window function. Any mapping $R(z)$ must therefore also depend on these parameters. It is also common to identify the resolution with the mass $M(R) \propto R^3$ of the collapsing perturbation, and again Λ CDM parameters are involved in this conversion.

The abundances of cosmic structures of a given size σ or M (and even the abundances of their mergers and substructures!) are predicted in Press-Schechter theory by looking at statistical features of random fields of perturbations at different resolutions. For instance, the fraction of the universe in gravitationally

bound structures $\Omega_\delta(R)$ is identified with the fraction of the amplitudes (smoothed with scale R) above the threshold value δ [22]. The self-similarity / scale invariance of the abundance distributions may then be interpreted as triviality under changes of this scale (‘renormalisations’); this interpretation is supported by the fact that we don’t need any nongravitational physics to explain, to a relatively good approximation, the self-similar structure formation in Λ CDM. Suggestively, this self-similarity breaks down only on small scales, where non-gravitational physics start to matter.

More speculatively, the stellar evolution that occurs during structure formation can also be reinterpreted as ‘stellar renormalisation’: Stars follow the Main Sequence of an HR diagram because their spectral type and luminosity depend only on their mass $M \propto R^3$. Populations of stars of decreasing metallicities at increasing redshifts roughly fits with the idea that heavy metals are not stable bound states until the renormalisation scale is sufficiently low. Pushing this speculation further, note that there is another interesting phase transition at redshifts at which the first stars and quasars form. In the gauge-invariant, Riemannian description, we would say the first stars and quasars emit gauge bosons (ultraviolet light) that ionise the neutral intergalactic medium. In the torsionful description, a description in terms of ionising photons is still possible, although it is no longer gauge-invariant.

6 The Dark Universe

We’ve argued that torsion is intricately tied to the quantum vacuum and its renormalisation theory, and that it can explain the scale factor and deceleration parameter of the FLRW spacetime. In this section, we combine all of these elements to see how the other, unexplained observational features of Λ CDM cosmology emerge as nonlocal effects in this model.

6.1 Torsion as a Perfect Fluid

Consider the classical limit of Einstein-Cartan theory. Particularly, consider the extraneous spin-density-

square terms of Eqn. (4),

$$(\Sigma^2)^{\mu\nu} = \left(\eta_m^\mu \eta_n^\nu - \frac{1}{2} \eta^{\mu\nu} \eta_{mn} \right) \times \\ \left(-4\Sigma^{ma}{}_{[b}\Sigma^{nb}{}_{a]} - 2\Sigma^{mab}\Sigma^n{}_{ab} + \Sigma^{abm}\Sigma_{ab}{}^n \right), \quad (28)$$

where an overall factor of $8\pi G/c^4$ has been suppressed for concision [11, 7]. Classically, the shell theorem tells us that homogeneous torsion terms of Eqn. (28) act as a dynamically irrelevant constant energy shift to the gravitational Poisson equation. We have, up to and including order η^2 in perturbations,

$$(\partial_x^2 + \partial_y^2 + \partial_z^2) \frac{\phi}{4\pi G} = \rho(x, y, z) \\ + \frac{8\pi G}{c^4} (\bar{\Sigma}^2 + \eta \bar{\Sigma} \bar{\Sigma}' + (\eta \bar{\Sigma}')^2) \\ + \mathcal{O}(\epsilon), \quad (29)$$

where we assume the observer has already recognised the classical, gauge-invariant matter density $\rho = \rho_{(\text{can})} + \partial\chi$. The constant torsion of the quantum vacuum therefore has no dynamical effect on gravitational dynamics in the classical limit of Eqn. (4); as found experimentally, the choice of a vacuum and a renormalisation scale do not affect classical dynamics. However a dynamical effect does occur at perturbative order $\epsilon G/c^4$.⁵ We can rewrite the leading inhomogeneous contribution to the Poisson Equation, heuristically and suppressing factors of c for concision, as

$$8\pi G(2\epsilon \bar{\Sigma} \Sigma'') \sim \epsilon(2\bar{K})\Sigma'' \sim \epsilon H_0 \Sigma''. \quad (30)$$

This is a modification of Newtonian gravity where macroscopically inhomogeneous spin density perturbations $\epsilon \Sigma''$ behave like a gravitational source that is only relevant whenever the classical matter density is small: $\rho c^4 \lesssim \epsilon H_0 \Sigma''$ in Eqn. (29). We also emphasize that this modification of the Poisson equation (contrary to theories such as MOND) is not a modification of the gravitational *force law*: the effect of the perturbing torsion field $\epsilon \Sigma''$ on the stress-energy contents of the universe, is Equivalent to that of a classical perfect fluid ρ_ϵ .

⁵This order of perturbations was neglected in Eqn. (16).

More formally, and now in the fully relativistic theory, the perturbation term Eqn. (30) appearing in Eqn. (28) is

$$(2\epsilon \bar{\Sigma} \Sigma'')^{\mu\nu} = 2\epsilon \times \left(\eta_m^\mu \eta_n^\nu - \frac{1}{2} \eta^{\mu\nu} \eta_{mn} \right) \times \\ \left(-4\bar{\Sigma}^{(m|a}{}_{[b}\Sigma''^{n)b}{}_{a]} \right. \\ \left. - 2\bar{\Sigma}^{(m|ab}\Sigma''^{n)}{}_{ab} \right. \\ \left. + \bar{\Sigma}^{ab(m}\Sigma''^{n)} \right) \quad (31)$$

where the explicit factor of two comes from the symmetrisation $m \leftrightarrow n$ of this $(\bar{\Sigma} + \epsilon \Sigma'')^2$ cross-term.

The divergencelessness of Eqn. (28), required by the divergencelessness of the Einstein Tensor and the symmetrised stress energy in Eqn. (4), yields

$$\nabla_\mu (2\epsilon \bar{\Sigma} \Sigma'')^{\mu\nu} = 0 \quad (32)$$

when we require order-by-order divergencelessness in perturbation theory. So, identifying this symmetric and divergenceless perturbation as an effective $T_{(\epsilon)}^{\mu\nu}$ in the Einstein Field Equations is plausible even beyond the classical limit. This is very relevant since this apparent stress-energy tensor $T_{(\epsilon)}^{\mu\nu}$ naturally produces gravitational redshifts and gravitational lensing in the Riemannian spacetime, while *ad hoc* modifications of the gravitational force do not necessarily have such a phenomenology.

The other contributions from $(\Sigma\Sigma)^{\mu\nu}$ also share in this divergencelessness, and so also behave as effective stress-energy tensors. We show in the rest of this section that these effective stress-energies contribute to the “dark” universe of contemporary Λ CDM cosmology.

6.2 Critical Density (EdS)

Our universe appears fine-tuned to have a critical density, and this is strange; stranger still is that our universe, which will be asymptotically deSitter rather than Einstein-deSitter in the infinite future, is actually fine-tuned to the critical density $\rho_c = 3H_0^2/8\pi G$ as defined by the asymptotic stability of an Einstein-deSitter universe $\Lambda = 0$! Our model of a torsionful

cosmology, besides providing a motivation for the accelerating expansion of Λ CDM, can also explain this curiously incongruous feature of the expansion.

Let's first assume that the contents of the universe are homogeneous, i.e. that $\epsilon(\dots)''$ terms vanish (this simplification will be lifted in Sec. 6.3). As described in Sec. 3, the conformal factor of the FLRW is a homogeneous contorsion K misrecognised as a connection: we actually have a Minkowskian $G^{\mu\nu} = 0$ in Eqn. (4) for the Riemannian part of the spacetime with torsion, i.e. we have

$$0 = T^{\mu\nu} + \frac{8\pi G}{c^4}(\Sigma\Sigma)^{\mu\nu} \quad (33)$$

where $T^{\mu\nu}$ is defined in Eqn. (2). Suppose an observer (mis)identifies this torsion term as $-G^{\mu\nu}$ when they insist that their torsion-free universe satisfies the Einstein field equations. We can then convert the spin densities to contorsions as $8\pi G\bar{\Sigma}^2 \sim K^2/(8\pi G)$ and interpret this contorsion as the Hubble Constant $K \sim H_0$ (by Equivalence) to find that this term is in fact the critical density

$$\rho_c^{\text{EdS}} \leftrightarrow \frac{8\pi G}{c^4}(\bar{\Sigma})^2 \propto \frac{H_0^2}{8\pi G}. \quad (34)$$

We then find (upon dividing Eqn. (33) by an overall factor of ρ_c^{EdS}) that a homogeneous universe has a critical density by definition, rather than as a result of fine-tuning the amount of matter it contains. Recall that in this model the deceleration parameter q_0 is due to torsion inhomogeneities $\nabla^2\Upsilon$, so the cosmological constant is not accounted for (yet) in this homogeneous treatment.

The derivation above shows there is *exactly zero* spatial curvature in the FLRW metric, if the metric of the Riemann-Cartan spacetime is adopted as \mathbb{M}^4 . At face value, this would appear to solve the ‘‘flatness problem’’ that $\Omega_k \propto (1+z)^2$ is required to be incredibly fine-tuned in the early universe in order to observe $\Omega_k = 0.000 \pm 0.005$ today [3]. However, if the metric of the Riemann-Cartan spacetime were de Sitter instead of Minkowski, then the cosmological constant of the spacetime with torsion would look like a spatial curvature term in the torsionfree Friedmann equations. For instance, an AdS ($\ell < 0$) metric in the

Riemann-Cartan spacetime has a positive spatial curvature contribution to the Einstein-Cartan equation,

$$G^{\mu\nu} = -\ell g_{\text{AdS}}^{\mu\nu} \leftrightarrow \frac{-\ell}{a^2(t)} g_{\text{FLRW}}^{\mu\nu}, \quad (35)$$

However unlike for Λ in the FLRW spacetime, the philosophically appealing value $\ell = 0$ (that solves the flatness problem) remains compatible with observations.

Another fine-tuning problem of Λ CDM that is conventionally resolved by inflation but avoided completely in this model is the rotation problem [23]. The local inertial frame of an observer in General Relativity can always be chosen to be irrotational, therefore the \mathbb{M}^4 on which the torsion is reinterpreted as expansion is similarly irrotational, leading to an FLRW metric (rather than some other metric for a rotating universe).

6.3 Critical density (Λ CDM)

Let's now repeat this exercise in the presence of inhomogeneous torsion perturbations $\epsilon\Sigma''$. The Einstein-Cartan equation Eqn. (4) becomes a balance between the vacuum torsion / critical density defined in Sec. 6.2, and the matter-induced torsion stress-energies defined in Sec. 6.1:

$$\left(\frac{H_0^2}{8\pi G}\right)^{\mu\nu} = T^{\mu\nu} + T_{(\eta)}^{\mu\nu} + T_{(\eta^2)}^{\mu\nu} + T_{(\epsilon)}^{\mu\nu} + T_{(\eta\epsilon)}^{\mu\nu} + T_{(\epsilon^2)}^{\mu\nu}. \quad (36)$$

We can again divide out the vacuum-induced critical density $8\pi G(\bar{\Sigma})^2/c^4$, only now we find an energy budget

$$1 = \Omega_{T^{\mu\nu}} + \Omega_{(\eta)} + \Omega_{(\eta^2)} + \Omega_{(\epsilon)} + \Omega_{(\eta\epsilon)} + \Omega_{(\epsilon^2)}. \quad (37)$$

We emphasise that although this rearrangement seems trivial, it is not simply a matter of definition (as it would be if we started from the Friedman equations), since in this model the critical density in Eqn. (36) is derived from first principles. Indeed, the critical density is not just an arbitrarily defined combination of constants, but the torsion density of the (Haag-violating) global vacuum, with vanishing corrections from the homogeneous matter.

If we can identify some or all of the ϵ -perturbation terms in with the cosmological constant density Ω_Λ , then this explains how our universe can be fine-tuned to an EdS critical density despite having a dS asymptotic future. This is plausible, since we've already identified the deceleration parameter q_0 with torsion inhomogeneities in Sec. 3.

Our association of Λ to these effective stress-energy tensors is further motivated by the Friedmannian cosmology to which our model is Equivalent. The well-known relation between cosmological stress-energies and the deceleration parameter is [18]

$$q_0 = \frac{1}{2} + \frac{3}{2} \sum_i w_i \Omega_i = \frac{1}{2} - \frac{3}{2} \Omega_\Lambda + \mathcal{O}(\Omega_{\text{rad}}). \quad (38)$$

Adding one to each side, and then multiplying this equation by the constant $H_0^2 \rho_c^{\text{EdS}}$, we obtain

$$(1 + q_0) H_0^2 \rho_c^{\text{EdS}} = \frac{3H_0^2}{2} (\rho_c^{\text{EdS}} - \rho_\Lambda) \quad (39)$$

where the usual definition $\Omega_\Lambda = \rho_\Lambda / \rho_c^{\text{EdS}}$ was used. Covariantisation of the left-hand-side yields a term reminiscent of the coefficient $B_{\sigma\rho}^{\mu\nu}$ we saw in Sec. 3.3, suggesting that the 'dark energy' ρ_Λ is indeed identical to the accelerated expansion term in the redshift-distance relation.

Another detail of interest regarding these Σ^2 terms is that they are not subject to the redshift of the FLRW spacetime. This has two ramifications: firstly, Ω_Λ is independent of redshift and therefore may indeed be associated to an equation of state $w = -1$ exactly (Ref. [3] finds $w = -1.006 \pm 0.045$ and no evidence for redshift evolution). Secondly, the critical density today is also independent of redshift, which is absolutely essential to its interpretation as a threshold value applicable today in Einstein-deSitter cosmologies. Indeed, this entire discussion occurs at $z = 0$. A more complete picture, which is given below and accounts for renormalisation at higher redshifts, is required to recover modern cosmology's picture of the early universe.

In short, all Einstein-Cartan cosmologies have an Einstein-deSitter critical density, even in the presence of a deceleration parameter. There appears to be a suspiciously fine-tuned balance between gravitational

attraction and expansion, because we've heedlessly imposed torsionlessness on our homogeneous cosmology.

6.4 Dark Energy Renormalised

One very interesting feature of a model in which the cosmological constant is due to inhomogeneities, is that the cosmological constant will not actually be a constant: its effective stress-energy tensor will take the form of a random field over spacetime. Following our discussion in Sec. 6.1, this inhomogeneous torsion will have an effective stress-energy, sourcing a gravitational potential and producing gravitational lensing. With $\Omega_\Lambda = 0.7$ taking up most of the energy density today, it would seem unnatural *not* to observe these effects. The empirical response to such a bold claim is to look for unexplained patterns in gravitational potentials and lensing. These may have already been observed; in the Λ CDM paradigm, these effects are assigned to dark matter! We've already identified the (inhomogeneous) ϵ -perturbation terms in Eqn. (36) with dark energy; we now interpret the remaining (homogeneous) η -perturbation terms as dark matter, yielding the complete Λ CDM relation

$$\Omega_{T^{\mu\nu}} + \Omega_{\text{dm}} + \Omega_\Lambda = 1 \quad (40)$$

with $\Omega_k = 0$ as discussed above.

To understand how the torsion also behaves like dark matter, it is crucial to visualise structure formation as Press-Schechter theory / Renormalisation (cf. Sec. 4). The cosmological constant is sourced by inhomogeneities in the torsion field; however the inhomogeneities of the distant-and-early universe are coarse-grained (homogenised!) when we unintentionally renormalise the vacuum of our observations, as described above. The coarse-graining therefore changes the apparent ratio of homogeneous and inhomogeneous perturbations as a function of redshift: the earlier we look, the less inhomogeneities we find. Therefore, $\Omega_\Lambda / \Omega_{\text{dm}}$ is a decreasing function of z . It should be easy to show specifically that $\Omega_\Lambda / \Omega_{\text{dm}} \sim (1+z)^{-3}$, since this is the matter-like scaling these terms would have in an Einstein-deSitter universe (cf. Sec. 6.2).

This picture obviously solves the Horizon problem, since the surprising homogeneity of the early universe is explained as an observational artefact of unintentional coarse-graining. It also solves the “coincidence problem”: Unintentional renormalisations of observations of the local universe are small, and so the torsion anisotropies in the local universe are preserved. The unintentional renormalisations of observations of the distant-and-early universe ($z \gg 0.55$) are significant enough to wash away the effect of torsion anisotropies. The current era is not special because the components of the dark universe are anthropically fine-tuned in the Big Bang, but on the contrary because any torsion distribution with inhomogeneities of relative magnitude $1/8 \lesssim \Omega_\Lambda/\Omega_{\text{dm}} \lesssim 8$ in the local universe would yield a coincidence redshift of $-0.5 \lesssim z \lesssim 1$. This $\mathcal{O}(1)$ ratio corresponds to an un-fine-tuned torsion distribution, which is neither surprisingly homogeneous nor surprisingly inhomogeneous. Finally, if this torsional effective stress-energy contributes a non-negligible amount to the observed $\Omega_{(\text{dm})}$, then it may also explain the “Milgrom coincidence” that the physical scales at which the dynamical effects of dark matter become noticeable are within an order of magnitude of H_0 (cf. our discussion of the Poisson Equation Eqn. (29)).

One odd feature of this model is related to gravitational lensing. In General Relativity it is not stress-energy, but stress-energy overdensities which act as lenses. Accordingly, the effective stress-energies $T_{(\eta)}^{\mu\nu}$ and $T_{(\eta^2)}^{\mu\nu}$, being homogeneous, cannot contribute to the gravitational lensing of radiation. The observed lensing is therefore entirely due to the inhomogeneous ϵ -terms; and indeed whenever we treat dark matter as accreted into structures and substructures with density profiles, we treat it as inhomogeneous. This phenomenology of the η and ϵ perturbations leads to the counterintuitive claim that “dark matter does not cause lensing, but dark energy does” when this behaviour is described using terminology adopted from the Λ CDM paradigm.

Modified gravity proposals for the Dark Matter typically fail to produce any damping for Baryonic Acoustic Oscillations. In this model, the decoupling phase transition that freezes out these oscillations

and the structure formation that produces them are both aspects of the same (unintentional) renormalisation, so it is easy to argue (at least, conceptually) that the pattern of BAO should be the same as in Λ CDM. Similarly, if Press-Schechter renormalisation does indeed yield the dark matter’s $(1+z)^3$ scaling for the early universe, then perhaps renormalisation that includes the phase transitions observed in baryons could reproduce the Tully-Fisher relation more generally. Of course, until this qualitative account can be improved to quantitatively explain these observations (and the Bullet cluster, rotation curves, etc.) it remains plausible that some of the dark matter we see may be due to a dark particle unrelated to torsion. Either way, the torsion of Einstein-Cartan theory is non-dynamical, so the “torsion dark matter density” is *locally* sourced by spin density inhomogeneities even though the effective $T_{(\text{dark})}^{\mu\nu} \rightarrow 0$ in any local experiment (since \mathbb{M}^4 is torsion-free).

Since this dark phenomenology vanishes in local frames and acts as a dynamically irrelevant energy shift in the classical limit, as an interpolating behaviour between the classical and cosmological regimes it must appear to gradually disappear on smaller and smaller cosmological scales. This feature may be relevant to resolving the Λ CDM small-scale crises.

Unlike superficially similar holographic proposals (cf. e.g. Ref. [24]), we are discussing effects that may be only observable nonlocally but which are fundamentally due to local physics. That said, in Section 8, we will return to quantum mechanical considerations and show that torsion does, in fact, behave holographically.

7 Copernician Principle

Our place in the universe is not special; the isotropy of the FLRW expansion, and the abstract featurelessness of the observer who might extrapolate such a Riemannian cosmology from observations in their local frame, essentially guarantees that our Einstein-Cartan cosmology satisfies the Copernician principle.

However, there is a more important Copernician Principle for a model that explicitly considers gauge-

invariance as a by-product of the Belinfante procedure. The gauge-invariant and Riemannian way we look at the Riemann-Cartan universe is not special.

7.1 Mixing Torsion and Curvature

In analogy to the Lorentz transformations that mix lengths and durations, there exist transformations mixing Lorentzian Curvature and Torsion on Riemann-Cartan spacetimes [25]. These transformations are distinct from the observers' mischaracterisation of torsion as a Belinfante superpotential or an FLRW scale factor in light of the Equivalence Principle; a coordinate transformation cannot change the nature of the spacetime from Riemann-Cartan to Riemann.

Much like lengths and durations are relative to different observers as a result of the Lorentz symmetry, causality and gauge-invariance must be understood as relative to different observers on a Riemann-Cartan spacetime as a consequence of these transformations. The many different choices of Riemannian spacetime correspond to many different choices of vacuum, and so to different choices of torsion field, Belinfante superpotential, and symmetrised stress-energy tensor Eqn. (2). More provocatively: experimentally tangible reality is relative. The many different choices of Riemannian spacetime, or more properly Lorentzian spacetime, also correspond to many different choices of lightcone structure, or causality.⁶ More provocatively: the history of the universe is relative.

The local temperature of the CMB, the local abundances of nucleosynthesised elements, etc., are consistent with our causal FLRW history – but because of this extended relativity an identically located observer with a different curvature/torsion split than us could see the same universe very differently. Although the gauge structure of the Standard Model is presumably the same in each of these alternate histories, their notion of which stress-energies are gauge-invariant would differ from our own. This distinct observer could measure distinct gauge-invariant values of the CMB stress-energy (showing that our Θ_0

⁶An enlightening discussion of the observer-dependence of causality can be found in Ref. [26].

is not special), or may not observe a CMB at all. However, by communicating not only their choice of frame but also their choice of Riemannian spacetime, we could check that all of our observations are consistent: no observer has a privileged position.

7.2 Objectivity in this Cosmology

Our new perspective on cosmology placed a lot of weight on the observers' extrapolative assumptions that their spacetime is torsion-free (\implies accelerated expansion) and has a global vacuum (\implies thermal history); these assumptions are of course related if the quantum vacuum we extrapolate is, as we have argued in Sec. 4, the source of the cosmological torsion.

But these conjectures still lack a physical justification as to why such naïve extrapolations should appear to agree with cosmological data in the first place. Observers' assumptions unequivocally color their interpretations of our observations, but Nature does not care what we assume, and does not yield to the limitations of a human model when presenting us with new data.

Yet, by insisting that the mechanism(s) underlying the agreement of our extrapolations with observations should be *physical*, we effectively demand that it be *gauge-invariant and causal*. The agreement of our extrapolative assumptions with cosmological data, and indeed the regularity of physical law across the universe more generally, is then fundamentally a consequence of our interpretation of our experiences in an Einstein-Cartan spacetime through the lens of gauge-invariance and causality. This *physicalist* lens colors our interpretations of our experiences to produce the structure of the universe, both at the smallest and at the largest scales (as described by Gauge-theoretic QFT and Lorentzian General Relativity).

This reflection on the conditions under which physical truths are conjectured, is not to be misconstrued as an indictment of the physical sciences in particular, or of the scientific method more generally.⁷ Quite to the contrary, this clarifies how physical theories can

⁷This condition on discourse is a 'historical a priori' of the physical sciences, in the terminology of Ref. [27].

be experimentally corroborated by observers embedded in a world which is fundamentally neither gauge-invariant nor causal.

8 Holography

In this section, we ponder the significance of the recurring theme that the cosmological expansion and ideas from the renormalisation in quantum gauge theories are related via the torsion of a Riemann-Cartan spacetime. We argue (qualitatively) that Riemann-Cartan spacetimes have “holographic” properties, from which it follows this recurring theme is utterly unsurprising.

8.1 What is Quantisation?

In Riemann-Cartan spacetime there are many causalities and many gauge-invariances, with many associated histories and experimentally tangible realities. This ultimately Copernician universe, which allows for a multitude of classical observer-dependent histories, is qualitatively not dissimilar to the quantum mechanical picture of reality as a multiverse with many partially consistent histories [28]. This brings us to quantisation, the notion that one can construct quantum field theories perturbatively around classical field theories. This is a little curious if the quantum theory is supposed to be fundamental and the classical theory merely the ‘ $\hbar \rightarrow 0$ ’ limit within the quantum theory. Various QFT formalisms, of varying degrees of sophistication, all start from a classical action, a classical Lagrangian/Hamiltonian, etc., to compute their predictions; and they all rely on a classical background spacetime (be it curved or not), with quantum corrections (“gravitons”) in effective field theories [29, 30]. This may simply be a historical accident, but if the non-renormalisability of gravity points to the fact that gravity cannot be quantised, then for consistency of the Einstein(-Cartan) Field Equations the matter ought not be quantised either. Perhaps the matter really does behave classically, and perhaps quantum mechanical violations of naïve probability theory (wavefunction interference, entanglement, etc.) simply point to a new kind of un-

certainty: an uncertainty tied to the relativity of the observer’s history and experimentally tangible reality. If this is the case, then the purpose of quantising a classical theory becomes clear: it is simply how we deal with this uncertainty.

We propose that perhaps Feynman’s sum over histories is, quite literally, a sum over the relative histories of the Riemann-Cartan universe. The purpose of this sum is to account for the observer’s uncertainty about which consistent history they are participating in. The inclusion of ‘causality-violating histories’ in the sum is a reflection of different classical observers’ understandings of causality; in the same way, the gauge ghosts at the heart of this sum over histories are a reflection of their different understandings of gauge-invariance. And of course, we should highlight that this sum is local.⁸ The rigorous renormalisation of Ref. [20] is also local. So, the nonlocal quantum corrections we compute with path integrals are indeed corrections to the local observables made by a local observer.

This picture of observers’ quantum uncertainties about their classical histories, is clearly tied to the background field method and the use of effective actions, in which vacuum contributions are *explicitly thrown away* while perturbing around the classical solution [12, 31]. It is interesting to note that effective action calculations are explicitly gauge-invariant and causal, and formally similar to standard generating-function computations in probability theory. In fact, background field quantisation is identical to BRST quantisation [32], so the gauge ghosts (tied to torsion via Maurer-Cartan equation in BRST quantisation, cf. Sec. 2) are intimately tied to the choice of a classical background – or more precisely to uncertainties the observer may have about what their background spacetime actually is. One telling feature of the background field method, in the context of this cosmological model, is that gauge ghost fields do not need to be renormalised [31], and so are unaffected by the redshift’s RG flow.

This model of an Einstein-Cartan “multiverse” can also explain the physical content of Haag’s Theorem.

⁸From Ref. [12], §20.7: “Dyson observed that the number of diagrams of n th order typically grows as $n!$, which suggested that the perturbation series has zero radius of convergence”.

If we write down a local QFT to deal with non-local corrections to our classical field theory using the background field method, its domain of applicability is limited to the region of spacetime where the background of alternate histories is not appreciably different from the classical background.⁹ The vacuum in alternate classical histories is allowed to be very different outside that limited region of spacetime, and so we cannot sensibly extrapolate our local vacuum (shared by all classical histories contributing to the effective action) into a global vacuum (since no such vacuum is shared by these histories). This prohibition of local relativistic QFTs with global vacua is Haag’s Theorem.

8.2 Quantisation affects everything

We’ve seen that the background field method allows us to compute quantum corrections to our classical history; the gravitational, gauge and matter fields of Einstein-Cartan theory are then intrinsically classical, but *appear quantised* to a local observer who insist on using a torsionless description (and therefore can’t determine which history they are participating in). The observer’s uncertainty naturally also extends to the gravitational fields of ‘their’ history, and not surprisingly the background field method is the language conventionally used to think about gravitational quantum corrections in QFT [29, 30]. In our interpretation of the path integral with an effective action, gravitons act as corrections accounting for slightly different gravitational field in alternate (classical) universes; the appearance of graviton ghosts (which account for the difference in causality and gauge-invariance of the gravitational fields in other universes) can extend the spacetime region where the QFT is applicable. However, this background-field quantisation of gravity is *conceptually* no different from the quantisation of electromagnetism: neither gravity nor electromagnetism are fundamentally “quantised” in any individual (classical) history of the Einstein-Cartan universe: the quantum behaviour comes from the observer’s uncertainty with

⁹If the backgrounds were appreciably different, the perturbative approach of the background field method would simply break down.

respect to their history.

In a similar vein, the effective stress-energy tensors of the torsion discussed in Sec. 6, which give rise to effective matter-like contributions, should also appear to be quantised to massive observers, with a classical background (the homogeneous $T_{(\eta)}^{\mu\nu}$ terms) and discrete excitations of the field (the inhomogeneous $T_{(\epsilon)}^{\mu\nu}$ terms). Since the torsion field generated by the baryonic matter does not carry any of the Standard Model gauge symmetries, these excitations are uncharged.¹⁰ This model therefore *disguises a modification of gravity as a dark matter particle*.

However, since torsion vanishes in local inertial frames, these particles should only be observed (besides their gravitational effects) in non-local loop corrections. We’ve already argued that classical torsion can take the form of gauge ghosts in Sec. 2.3, but the matter fields don’t require additional fields to maintain gauge-invariance. This does not necessarily mean these fields do not exist: in QED, the gauge ghosts are present but decoupled, only contributing to vacuum bubbles since the $U(1)$ structure constants vanish. If the $T_{(\epsilon)}^{\mu\nu}$ -particles are similarly decoupled (which is plausible, since the matter fields themselves were already gauge-invariant before they were Belinfante-Rosenfeld symmetrised!), then it would not be surprising that these new particles are not visible in contact interactions either. This quantum cosmological model therefore not only predicts a dark matter particle, but can explain naturally and from first principles why this particle is dark in the first place.

This does not mean these $T_{(\epsilon)}^{\mu\nu}$ -particles are completely unobservable. Given that the ghosts associated to the massive Z and W^\pm bosons interact with the Higgs, it is plausible that $T_{(\epsilon)}^{\mu\nu}$ quanta (sourced by massive matter particles) could also interact with the Higgs. If this is not the case, our only hope of seeing these particles in a collider (besides in graviton tadpoles) would be if some or all of them interacted with the (non-abelian) Standard Model gauge ghosts. The “loops-only” phenomenology of these interactions is easy to enforce if the $T_{(\epsilon)}^{\mu\nu}$ particles carry

¹⁰The ghosts are of course charged, as usual, since they are generated by the torsion of the gauge fields!

their own ghost quantum numbers; and integrating out the ghost loops would even reduce to the “spin-spin contact interaction” phenomenology conventionally associated to classical torsion [16].

8.3 Quantum Uncertainty Principle

It is tempting to reinterpret the uncertainty relations $\Delta E \Delta t \sim \hbar$ and $\Delta p \Delta x \sim \hbar$ as scalarisations of a single relation between the uncertainty about which notion of gauge-invariance the observer should use for $T^{\mu\nu}$ (i.e. uncertainty about the spacetime torsion) and the uncertainty about which causal structure they should use for $g_{\mu\nu}$ (i.e. uncertainty about spacetime curvature). To understand the uncertainty principle, we must therefore understand why observers might be uncertain which deterministic history they are participating in. The answer is locality: in their local frame, observers do not have access to the information outside their lightcone. Treating the interior and exterior of the observer’s lightcone as two parts \mathcal{A} and \mathcal{B} of the physical Hilbert space, observers need to trace out \mathcal{B} to compute the expectations of observables. This is of course reminiscent of the paradigm of a ‘holographic’ duality between classical gravitational theories and quantum mechanical ones: The observer’s lightcone would be the holographic null surface, giving rise to an entanglement entropy.

Recall that locality in the relativistic sense we’ve been considering also entails ephemerality: a “long-lived” observer could collect information from a slightly larger region of spacetime. The region \mathcal{B} is a pair of Rindler wedges, with origins spacelike-separated by a distance equal to the rest-frame lifetime of the observer (cf. Fig. 1). The holographic properties of Rindler wedges are by now well-established; what this picture adds is the importance of massive observers, and therefore (once again) the importance of the Belinfante procedure in guaranteeing that such observers be gauge-invariant. The requirements of causality and gauge-invariance are tied together in the Riemann-Cartan spacetime, and so our Riemannian spacetime and the gauge-invariant matter in it are coemergent.

Furthermore, longer-lived observers trace over a smaller regions \mathcal{B} , and so they have more opportuni-

ties to interact with their environment and decohere their history. The classical limit of quantum theory is therefore explicitly recovered when work with large enough spacetime separations that $\mathcal{B} \approx \emptyset$. Besides the consistent histories formalism [28], mathematical support for this picture can be found in the construction of local quantum expectation values, also known as position-postselected weak values [ref original papers, [33]]. Ref. [34], studying the uncertainties associated to these expectation values, shows that the *local variance* of a quantum system’s momentum is tied to its classical limit. An observer who is not sure which history they live in isn’t sure which is the correct Riemannian spacetime to use to define their local rest frame, and so a local momentum uncertainty arises for every system they observe (interact with) in this frame – the quantum uncertainty. Longer-lived observers would be less unsure of their history, giving rise to a smaller local variance: they see a more classical world.

This relation between gravity and renormalisation is already under active investigation in the case of conformal field theories, which behave trivially under renormalisation and are known to be holographically dual to anti-deSitter spacetimes. We speculate that our knowledge of holographic dualities has historically been restricted to CFTs because we’ve been restricting ourselves to torsion-free spacetimes; and further predict that strongly-coupled QFTs with non-trivial renormalisation properties (CMT, QCD) cannot be understood holographically without Einstein-Cartan theory.

9 Conclusions

We proposed an extension of the Equivalence Principle in Einstein-Cartan gravity, with which torsion can unify and parsimoniously explain a variety of physical phenomena at a variety of scales:

1. gauge-invariance via the automatic inclusion of the Belinfante-Rosenfeld superpotential in Eqn. (4),
2. ‘unphysical’ Faddeev-Popov ghosts via the Maurer-Cartan equations,
3. Hubble Expansion, and the FLRW redshift-distance relation and scale factor, via nonlocal stress-energy nonconservation,
4. the apparent temperature evolution and phase transitions of the universe via an unintentional Renormalisation of the early universe,
5. the curious Einstein-deSitter critical density of the asymptotically deSitter universe via the homogeneous torsion
6. the Deceleration Parameter / Cosmological Constant via the inhomogeneous torsion,
7. the Dark Matter via coarse-graining (renormalisation) of the inhomogeneous torsion, and
8. the quantisation of classical fields via the Copernican principle applied to Riemann-Cartan spacetimes.

This model also resolves (avoids) a number of fine-tuning problems and paradoxes of the Λ CDM cosmology, while uniquely predicting that we should observe the Λ CDM cosmology.

It should be clear that the hypotheses made in this study are highly speculative and still require a level of experimental corroboration that no single study can provide. That said, if the insight behind some or all of these hypotheses is correct, then the reinterpretation of various quantum and cosmological phenomena as torsion in a mathematically rigorous model will require the adoption of a new paradigm, in which the torsion of Einstein-Cartan theory is as ubiquitous and important in our daily lives as gravity itself.

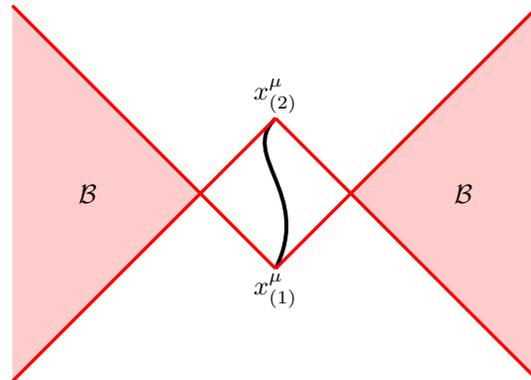


Figure 1: Minkowski diagram of the spacetime region \mathcal{B} inaccessible to a massive observer travelling from its creation at some point $x_{(1)}^\mu$ to its annihilation at $x_{(2)}^\mu$.

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A Derivation of Eqn. (16)

Starting from a homogeneous matter distribution $\overset{\text{(tor)}}{\nabla} T_{(\text{can})} = 0$, Eqn. (6) can be rewritten as:

$$\begin{aligned} \nabla_{\psi} T^{\mu\nu} = & \nabla_{\psi} \overset{\text{(tor)}}{\nabla}_{\rho} (\chi)^{\mu\rho\nu} + \overset{\text{(tor)}}{\nabla}_{\psi} (K_{\rho\sigma}{}^{\mu} \chi^{\sigma\rho\nu} + K_{\rho\sigma}{}^{\rho} \chi^{\mu\sigma\rho} + K_{\rho\sigma}{}^{\nu} \chi^{\mu\rho\sigma}) \\ & + 2\bar{K}_{\psi\sigma}{}^{(\mu} T^{\nu)\sigma} + 2\epsilon K'_{\psi\sigma}{}^{(\mu} T^{\nu)\sigma} - K_{\psi\sigma}{}^{\mu} \overset{\text{(tor)}}{\nabla}_{\rho} (\chi)^{\sigma\rho\nu} - K_{\psi\sigma}{}^{\nu} \overset{\text{(tor)}}{\nabla}_{\rho} (\chi)^{\mu\rho\sigma} \\ & + (K^2\chi \text{ terms}). \end{aligned} \quad (41)$$

Using $\overset{\text{(tor)}}{\nabla} (\chi) = \overset{\text{(tor)}}{\nabla} (\bar{\chi} + \epsilon\chi') = \epsilon \overset{\text{(tor)}}{\nabla} (\chi')$, we can already isolate the two interesting terms:

$$\begin{aligned} \nabla_{\psi} T^{\mu\nu} = & +2\bar{K}_{\psi\sigma}{}^{(\mu} T^{\nu)\sigma} + \epsilon \nabla_{\psi} \overset{\text{(tor)}}{\nabla}_{\rho} (\chi')^{\mu\rho\nu} \\ & + \overset{\text{(tor)}}{\nabla}_{\psi} (K_{\rho\sigma}{}^{\mu} \chi^{\sigma\rho\nu} + K_{\rho\sigma}{}^{\rho} \chi^{\mu\sigma\rho} + K_{\rho\sigma}{}^{\nu} \chi^{\mu\rho\sigma}) + 2\epsilon K'_{\psi\sigma}{}^{(\mu} T^{\nu)\sigma} - \epsilon K_{\psi\sigma}{}^{\mu} \overset{\text{(tor)}}{\nabla}_{\rho} (\chi')^{\sigma\rho\nu} - \epsilon K_{\psi\sigma}{}^{\nu} \overset{\text{(tor)}}{\nabla}_{\rho} (\chi')^{\mu\rho\sigma} \\ & + (K^2\chi \text{ terms}) \end{aligned} \quad (42)$$

After using Leibniz's Law to expand the three $\overset{\text{(tor)}}{\nabla} (K\chi)$ terms, we again use the relation above and the similar relation $\overset{\text{(tor)}}{\nabla} (K) = \overset{\text{(tor)}}{\nabla} (\bar{K} + \epsilon K') = \epsilon \overset{\text{(tor)}}{\nabla} (K')$. We can neglect perturbative cross-terms of order ϵ^2 :

$$\begin{aligned} \nabla_{\psi} T^{\mu\nu} = & +2\bar{K}_{\psi\sigma}{}^{(\mu} T^{\nu)\sigma} + \epsilon \nabla_{\psi} \overset{\text{(tor)}}{\nabla}_{\rho} (\chi')^{\mu\rho\nu} \\ & + \epsilon \overset{\text{(tor)}}{\nabla}_{\psi} (K'_{\rho\sigma}{}^{\mu} \bar{\chi}^{\sigma\rho\nu} + \overset{\text{(tor)}}{\nabla}_{\psi} (K'_{\rho\sigma}{}^{\rho} \bar{\chi}^{\mu\sigma\rho} + \epsilon \overset{\text{(tor)}}{\nabla}_{\psi} (K'_{\rho\sigma}{}^{\nu} \bar{\chi}^{\mu\rho\sigma} \\ & + \epsilon \bar{K}_{\rho\sigma}{}^{\mu} \overset{\text{(tor)}}{\nabla}_{\psi} (\chi')^{\sigma\rho\nu} + \epsilon \bar{K}_{\rho\sigma}{}^{\rho} \overset{\text{(tor)}}{\nabla}_{\psi} (\chi')^{\mu\sigma\rho} + \epsilon \bar{K}_{\rho\sigma}{}^{\nu} \overset{\text{(tor)}}{\nabla}_{\psi} (\chi')^{\mu\rho\sigma} \\ & + 2\epsilon K'_{\psi\sigma}{}^{(\mu} T^{\nu)\sigma} - \epsilon \bar{K}_{\psi\sigma}{}^{\mu} \overset{\text{(tor)}}{\nabla}_{\rho} (\chi')^{\sigma\rho\nu} - \epsilon \bar{K}_{\psi\sigma}{}^{\nu} \overset{\text{(tor)}}{\nabla}_{\rho} (\chi')^{\mu\rho\sigma} \\ & + (\epsilon^2 \text{ terms}) + (K^2\chi \text{ terms}) \end{aligned} \quad (43)$$

Now, recognising that $\epsilon K = \mathcal{O}(\epsilon G/c^4)$ and $\epsilon \nabla K = \mathcal{O}(\epsilon G/c^4)$ are small with respect to the leading terms of magnitudes $\mathcal{O}(G/c^4)$ and $\mathcal{O}(\epsilon)$ respectively, we have the asserted result:

$$\nabla_{\psi} T^{\mu\nu} = +2\bar{K}_{\psi\sigma}{}^{(\mu} T^{\nu)\sigma} + \epsilon \nabla_{\psi} \overset{\text{(tor)}}{\nabla}_{\rho} (\chi')^{\mu\rho\nu} + (\epsilon^2, \epsilon G/c^4, \text{ and } G^2/c^8 \text{ terms}). \quad (44)$$

This *a fortiori* a derivation of Eqn. (7), in the limit $\epsilon \rightarrow 0$ of a homogeneous torsion distribution. ■