Conservation of Momentum vs Lorentz Transformation

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An isolated physical system of gravitational force between two identical particles is chosen to manifest the physics law, conservation of momentum, in a random inertial reference frame under Lorentz Transformation. In this random reference frame, the center of mass moves at a constant velocity. By applying Lorentz transformation to the velocities of both particles, total momentum in this random inertial reference frame can be calculated and is expected to remain constant as gravitational force accelerate both particles toward each other. The calculation shows that conservation of momentum fails to hold under Lorentz Transformation.

I. INTRODUCTION

Gravitational force between two identical particles provides an excellent physics system for the demonstration of the physics law, conservation of momentum. The gravitational interaction will be examined in two inertial reference frames, the center of mass (COM) frame and a random inertial reference frame. Conservation of momentum is expected to hold in both reference frames.

Lorentz Transformation[1] transforms the velocities of both particles from COM frame to a random inertial reference frame. The total momentum will be calculated in this random inertial reference frame to verify if conservation of momentum still holds while both particles accelerate toward each other.

The concept of relativistic mass becomes less popular in modern physics. Relativistic force and relativistic momentum do not share the same relativistic mass. The momentum of a particle is represented by either \( m \) or \( \gamma m \) \( \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \), which is equivalent to each other mathematically. In this paper, \( \gamma \) is chosen to emphasize Lorentz Factor, \( \gamma \), in Lorentz Transformation.

\[
\frac{dv}{dt} = \frac{dv(0)}{dt} = 0
\]  

II. PROOF

Consider one-dimensional motion.

A. Gravitational Force

Two identical particles move toward each other under the influence of gravity. In the COM frame (Center Of Mass), both particles move at identical speed but opposite direction. The gravitational force between them is attractive. Both particles accelerate toward each other. The presence of a single force demands that total momentum of both particles should remain constant.

Let a reference frame \( F_1 \) be stationary relatively to this COM frame. Let the velocity of one particle be \( V \) at a random time in \( F_1 \).

<table>
<thead>
<tr>
<th>Object</th>
<th>Frame</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The velocity of particle 1, ( O_1 ), in ( F_1 )</td>
<td>( F_1 )</td>
<td>is ( V )</td>
</tr>
<tr>
<td>The velocity of particle 2, ( O_2 ), in ( F_1 )</td>
<td>( F_1 )</td>
<td>is ( -V )</td>
</tr>
<tr>
<td>The momentum of ( O_1 ) in ( F_1 )</td>
<td>( F_1 )</td>
<td>is ( \gamma(V) \ast m \ast V )</td>
</tr>
<tr>
<td>The momentum of ( O_2 ) in ( F_1 )</td>
<td>( F_1 )</td>
<td>is ( \gamma(-V) \ast m \ast (-V) )</td>
</tr>
</tbody>
</table>

C. Lorentz Transformation

Let another reference frame \( F_2 \) move at the velocity \( -U \) relatively to \( F_1 \). The velocity of \( F_1 \) relative to \( F_2 \) is \( U \). According to Lorentz Transformation, the velocity of \( O_1 \) in \( F_2 \) has to be \( \frac{V+U}{1+\frac{V+U}{c^2}} \). The velocity of \( O_2 \) in \( F_2 \) has to be \( \frac{-V+U}{1+\frac{V+U}{c^2}} \).

<table>
<thead>
<tr>
<th>Object</th>
<th>Frame</th>
<th>Velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>The velocity of ( O_1 ) in ( F_1 )</td>
<td>( F_2 )</td>
<td>is ( U )</td>
</tr>
<tr>
<td>The velocity of ( O_2 ) in ( F_2 )</td>
<td>( F_2 )</td>
<td>is ( -U )</td>
</tr>
</tbody>
</table>

D. Conservation of Momentum

Let \( v_1 \) be the velocity of \( O_1 \) in \( F_2 \). Let \( v_2 \) be the velocity of \( O_2 \) in \( F_2 \). According to Lorentz Transformation,

\[
v_1 = \frac{V+U}{1+\frac{V+U}{c^2}} = \frac{U+V}{1+\frac{V+U}{c^2}}
\]
Equation (7) fails to hold for \( 2V = U = \frac{C}{2} \) when both particles are approaching each other. From equation (4),(5)

\[
v_2 = \frac{-V + U}{1 + \frac{U}{C^2}} = \frac{U - V}{1 - \frac{U}{C^2}} \quad (4)
\]

\[-V = \frac{v_2 - U}{1 - \frac{V}{C^2}} \quad (5)
\]

Total momentum \( P \) in \( F_2 \) is

\[
P = \gamma(v_1) m v_1 + \gamma(v_2) m v_2 \quad (6)
\]

Conservation of Momentum demands

\[
\frac{dP}{dV} = 0 \quad (7)
\]

From equation (6),

\[
\frac{dP}{dV} = \gamma(v_1)^3 m v_1 + \gamma(v_2)^3 m v_2 + \frac{dv_1}{dv} \quad (8)
\]

From equation (2),(3)

\[
\frac{dv_1}{dv} = \frac{1}{1 + \frac{U}{C^2}} (1 - \frac{U + V}{1 + \frac{U}{C^2}} \frac{U}{C^2}) \quad (9)
\]

\[= \frac{v_1}{C + V} (1 - v_1 \frac{U}{C^2}) \quad (10)
\]

\[= \frac{v_1}{C + V} \frac{v_1 - U}{V} \quad (11)
\]

From equation (4),(5)

\[
\frac{dv_2}{dv} = \frac{1}{1 - \frac{U}{C^2}} (-1 - \frac{U + V}{1 - \frac{U}{C^2}} \frac{U}{C^2}) \quad (12)
\]

\[= \frac{v_2}{V - U} (-1 + v_2 \frac{U}{C^2}) \quad (13)
\]

\[= \frac{v_2}{V - U} \frac{v_2 - U}{V} \quad (14)
\]

Equation (7) fails to hold for \( 2V = U = \frac{C}{2} \) when both particles are approaching each other.

\[
v_1 = \frac{U + V}{1 + \frac{U}{C^2}} = \frac{\frac{3}{4} C}{1 + \frac{1}{8}} = \frac{3}{8} C \quad (15)
\]

\[
v_2 = \frac{U - V}{1 - \frac{U}{C^2}} = \frac{\frac{3}{4} C}{1 - \frac{1}{8}} = \frac{3}{8} C \quad (16)
\]

\[
\gamma(v_1)^3 m v_1 + \frac{dv_1}{dv} = \frac{27}{5 \sqrt{5}} m \cdot \frac{16}{27} = \frac{16}{5 \sqrt{5}} m \quad (17)
\]

\[
\gamma(v_2)^3 m v_2 + \frac{dv_2}{dv} = \frac{73}{45 \sqrt{5}} m \cdot \frac{-48}{45} \quad (18)
\]

\[
\frac{dP}{dV} = \frac{16}{5 \sqrt{5}} m - \frac{716}{45 \sqrt{5}} m = \frac{32}{45 \sqrt{5}} m > 0 \quad (20)
\]

From equation (8),(17),(19)

\[
\frac{dP}{dV} = \frac{16}{5 \sqrt{5}} m - \frac{716}{45 \sqrt{5}} m = \frac{32}{45 \sqrt{5}} m > 0 \quad (20)
\]

Equation (7) fails to hold for \( V = U = \frac{C}{2} \) when both particles double their speeds in \( F_1 \).

\[
v_1 = \frac{U + V}{1 + \frac{U}{C^2}} = \frac{C}{1 + \frac{1}{4}} = \frac{4}{5} C \quad (21)
\]

\[
v_2 = \frac{U - V}{1 - \frac{U}{C^2}} = 0 \quad (22)
\]

\[
\gamma(v_1)^3 m v_1 + \frac{dv_1}{dv} = \frac{125}{27} m \cdot \frac{12}{25} = \frac{20}{9} m \quad (23)
\]

\[
\gamma(v_2)^3 m v_2 + \frac{dv_2}{dv} = 0 \quad (24)
\]

From equation (8),(23),(24)

\[
\frac{dP}{dV} = \frac{20}{9} m + 0 > 0 \quad (25)
\]

Under Lorentz Transformation, total momentum in \( F_2 \) increases as both particles accelerate toward each other.

III. CONCLUSION

Lorentz Transformation violates conservation of momentum.

Conservation of momentum fails to hold if Lorentz Transformation is applied to an isolated system of two identical particles. The failure of this physics law is due to the addition of velocity from Lorentz Transformation. The correct formula for velocity addition has been derived by Eric Su in 2018[2][10].

Lorentz Transformation was proposed on the assumption that the speed of light is independent of inertial reference frame.

As the result of this incorrect assumption[3], Lorentz Transformation violates Translation Symmetry[4] in physics. Translation Symmetry requires conservation of simultaneity[5], conservation of distance[6], and conservation of time[7]. All three conservation properties are broken by Lorentz Transformation.

Therefore, Lorentz Transformation is an invalid transformation in physics. Consequently, any theory based on Lorentz Transformation is incorrect in physics. For example, Special Relativity[8][9].