Conservation of Momentum vs Lorentz Transformation

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(Dated: February 23, 2018)

An isolated physical system of gravitational force between two identical particles is chosen to manifest the physics law, conservation of momentum, in a random inertial reference frame under Lorentz Transformation. In this random reference frame, the center of mass moves at a constant velocity. By applying Lorentz transformation to the velocities of both particles, total momentum in this random inertial reference frame can be calculated and is expected to remain constant as gravitational force accelerate both particles toward each other. The calculation shows that conservation of momentum fails to hold under Lorentz Transformation.

I. INTRODUCTION

Gravitational force between two identical particles provides an excellent physics system for the demonstration of the physics law, conservation of momentum. The gravitational interaction will be examined in two inertial reference frames, the center of mass (COM) frame and a random inertial reference frame. Conservation of momentum is expected to hold in both reference frames.

Lorentz Transformation[1] transforms the velocities of both particles from COM frame to a random inertial reference frame. The total momentum will be calculated in this random inertial reference frame to verify if conservation of momentum still holds while both particles accelerate toward each other.

The concept of relativistic mass becomes less popular in modern physics. Relativistic force and relativistic momentum do not share the same relativistic mass. The momentum of a particle is represented by either $\gamma(V) * m(0) * V$ or $m(V) * V$. Both representations are equivalent to each other mathematically. In this paper, $\gamma(V) * m * V$ is chosen to emphasize Lorentz Factor, $\gamma(V)$, in Lorentz Transformation.

$$\frac{dm}{dV} = \frac{dm(0)}{dV} = 0 \tag{1}$$

II. PROOF

Consider one-dimensional motion.

A. Gravitational Force

Two identical particles move toward each other under the influence of gravity. In the COM frame (Center Of Mass), both particles move at identical speed but opposite direction. The gravitational force between them is attractive. Both particles accelerate toward each other. The presence of a single force demands that total momentum of both particles should remain constant.

B. Before Collision

Let a reference frame F_1 be stationary relatively to this COM frame. Let the velocity of one particle be V at a random time in F_1.

TABLE I. Velocity and Momentum in COM Frame

Object	Frame	Value
The velocity of particle 1, O_1, in	F_1	is V
The velocity of particle 2, O_2, in	F_1	is -V
The momentum of O_1 in	F_1	is $\gamma(V) * m * V$
The momentum of O_2 in	F_1	is $\gamma(-V) * m * (-V)$

C. Lorentz Transformation

Let another reference frame F_2 move at the velocity -U relatively to F_1. The velocity of F_1 relative to F_2 is U. According to Lorentz Transformation, the velocity of O_1 in F_2 has to be $\frac{V+U}{1+\frac{V*U}{C^2}}$. The velocity of O_2 in F_2 has to be $\frac{-V+U}{1+\frac{-V*U}{C^2}}$.

TABLE II. Velocity Transformation

Object	Frame	Velocity
The velocity of O_1 in	F_1	is V
The velocity of O_2 in	F_1	is -V
The velocity of F_1 relative to	F_2	is U
The velocity of O_1 in	F_2	is $\frac{V+U}{1+\frac{V*U}{C^2}}$
The velocity of O_2 in	F_2	is $\frac{-V+U}{1+\frac{-V*U}{C^2}}$

D. Conservation of Momentum

Let v_1 be the velocity of O_1 in F_2. Let v_2 be the velocity of O_2 in F_2. According to Lorentz Transformation,

$$v_1 = \frac{V+U}{1+\frac{V*U}{C^2}} = \frac{U+V}{1+\frac{U*V}{C^2}} \tag{2}$$

$$V = \frac{v_1 - U}{1 - \frac{v_1 * U}{C^2}} \tag{3}$$

$$v_2 = \frac{-V + U}{1 + \frac{-V * U}{C^2}} = \frac{U - V}{1 - \frac{U * V}{C^2}} \tag{4}$$

$$-V = \frac{v_2 - U}{1 - \frac{v_2 * U}{C^2}} \tag{5}$$

Total momentum P in F_2 is

$$P = \gamma(v_1) * m * v_1 + \gamma(v_2) * m * v_2 \tag{6}$$

Conservation of Momentum demands

$$\frac{dP}{dV} = 0 \tag{7}$$

Total momentum is expected to remain constant as particles move at any velocity V.

From equation (6),

$$\frac{dP}{dV} = \gamma(v_1)^3 * m * \frac{dv_1}{dV} + \gamma(v_2)^3 * m * \frac{dv_2}{dV} \tag{8}$$

From equation (2),(3)

$$\frac{dv_1}{dV} = \frac{1}{1 + \frac{U * V}{C^2}}\left(1 - \frac{U + V}{1 + \frac{U * V}{C^2}} * \frac{U}{C^2}\right) \tag{9}$$

$$= \frac{v_1}{U + V}\left(1 - v_1 * \frac{U}{C^2}\right) \tag{10}$$

$$= \frac{v_1}{U + V}\frac{v_1 - U}{V} \tag{11}$$

From equation (4),(5)

$$\frac{dv_2}{dV} = \frac{1}{1 - \frac{U * V}{C^2}}\left(-1 - \frac{U - V}{1 - \frac{U * V}{C^2}} * \frac{-U}{C^2}\right) \tag{12}$$

$$= \frac{v_2}{U - V}\left(-1 + v_2 * \frac{U}{C^2}\right) \tag{13}$$

$$= \frac{v_2}{U - V}\frac{v_2 - U}{V} \tag{14}$$

Equation (7) fails to hold for 2*V=U=$\frac{C}{2}$ when both particles are approaching each other.

$$v_1 = \frac{U + V}{1 + \frac{U * V}{C^2}} = \frac{\frac{3}{4} * C}{1 + \frac{1}{8}} = \frac{2}{3} * C \tag{15}$$

$$v_2 = \frac{U - V}{1 - \frac{U * V}{C^2}} = \frac{\frac{1}{4} * C}{1 - \frac{1}{8}} = \frac{2}{7} * C \tag{16}$$

$$\gamma(v_1)^3 * m * \frac{dv_1}{dV} = \frac{27}{5\sqrt{5}} * m * \frac{16}{27} = \frac{16}{5\sqrt{5}} * m \tag{17}$$

$$\gamma(v_2)^3 * m * \frac{dv_2}{dV} = \frac{7^3}{45\sqrt{45}} * m * \frac{-48}{49} \tag{18}$$

$$= -\frac{7 * 16}{45\sqrt{5}} * m \tag{19}$$

From equation (8),(17),(19)

$$\frac{dP}{dV} = \frac{16}{5\sqrt{5}} * m - \frac{7 * 16}{45\sqrt{5}} * m = \frac{32}{45\sqrt{5}} * m > 0 \tag{20}$$

Equation (7) fails to hold for V=U=$\frac{C}{2}$ when both particles double their speeds in F_1.

$$v_1 = \frac{U + V}{1 + \frac{U * V}{C^2}} = \frac{C}{1 + \frac{1}{4}} = \frac{4}{5} * C \tag{21}$$

$$v_2 = \frac{U - V}{1 - \frac{U * V}{C^2}} = 0 \tag{22}$$

$$\gamma(v_1)^3 * m * \frac{dv_1}{dV} = \frac{125}{27} * m * \frac{12}{25} = \frac{20}{9} * m \tag{23}$$

$$\gamma(v_2)^3 * m * \frac{dv_2}{dV} = 0 \tag{24}$$

From equation (8),(23),(24)

$$\frac{dP}{dV} = \frac{20}{9} * m + 0 > 0 \tag{25}$$

Under Lorentz Transformation, total momentum in F_2 increases as both particles accelerate toward each other.

III. CONCLUSION

Lorentz Transformation violates conservation of momentum.

Conservation of momentum fails to hold if Lorentz Transformation is applied to an isolated system of two identical particles. The failure of this physics law is due to the addition of velocity from Lorentz Transformation. The correct formula for velocity addition has been derived by Eric Su in 2018[2][10].

Lorentz Transformation was proposed on the assumption that the speed of light is independent of inertial reference frame.

As the result of this incorrect assumption[3], Lorentz Transformation violates Translation Symmetry[4] in physics. Translation Symmetry requires conservation of simultaneity[5], conservation of distance[6], and conservation of time[7]. All three conservation properties are broken by Lorentz Transformation.

Therefore, Lorentz Transformation is an invalid transformation in physics. Consequently, any theory based on Lorentz Transformation is incorrect in physics. For example, Special Relativity[8][9]

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