Is Mass Real?

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Abstract

The classical theories making up modern physics are: Newtonian mechanics, Maxwell’s equations, statistical mechanics, thermodynamics, special relativity and general relativity. We also mention nonrelativistic quantum mechanics. Excluding Maxwell’s equations the Newtonian concept of mass plays a fundamental role in each of these theories even though no one really knows what mass is. Starting with relativistic mass we will reason our way to the conclusion that mass is an illusion, and tidal effects are local. That is, behind the illusion of mass lay the mass-free foundations of a mass-free paradigm of Relativity that transcends all the theories mentioned and shows just why general relativity does not go far enough. Further, the true significance of Planck’s four-dimensional constant is to be found in this new paradigm of Relativity. The maths employed here have been used in [1] and [2].

Keywords: Mass, The Light, Equivalence Identity, Bekenstein-Hawking formula, Paradigm, Relativity, Planck’s constant, Laws of physics
1. Introduction

In this article we begin with relativistic mass and reason our way to the conclusion that mass never really existed. What we will end up with are the foundations for a mass-free paradigm of Relativity. It will be shown that there is a natural evolution from classical physics to Relativity where the significance of Planck’s four-dimensional constant will be apparent.

2. The Light

We begin with the fact that many practical applications of special relativity are found in the theory of relativistic mechanics. The basis of relativistic mechanics is the following four equations

\[ M = m_0 \sqrt{1 - \left(\frac{v}{c}\right)^2} \]  

which is a function of velocity \( v \) where \( M \) is the relativistic mass, \( m_0 \) the rest mass and \( c \) the speed of light in a vacuum. Multiplying both sides of Eq. (1) by the velocity vector \( \mathbf{v} \) gives us the expression for relativistic momentum

\[ \mathbf{p} = M \mathbf{v} = m_0 \mathbf{v} \sqrt{1 - \left(\frac{v}{c}\right)^2} \]  

and multiplying both sides of Eq. (1) by \( c^2 \) gives the total energy of a particle

\[ E = Mc^2 = m_0 c^2 \sqrt{1 - \left(\frac{v}{c}\right)^2} \]  

Ignoring \( M \) in Eqs. (2) and (3) and combining the two equations gives us

\[ E = \sqrt{(m_0 c^2)^2 + (p c)^2} \]

where we have ignored the negative root, which we shall justify below. Of these four equations only Eqs. (2) and (3) are theoretically necessary, and as the basis of relativistic dynamics they are routinely confirmed in elementary-particle physics.

We now introduce the energy of a photon \( E = hf \) where \( h \) is Planck’s four-dimensional constant and \( f \) is frequency, and equating this with the expression above we get
\[ hf = \sqrt{(m_0c^2)^2 + (pc)^2} \]

Now, if \( m_0 = 0 \) then using \( f = c/\lambda \) we derive de Broglie’s hypothesis

\[ \lambda = h/p \tag{4} \]

where \( \lambda \) is wavelength, and if \( p = 0 \) we similarly derive the Compton wavelength

\[ \lambda_c = h/m_0c \tag{5} \]

This much was known before 1925 with the possible exception of the derivation of Eq. (5) as presented here. The key to showing mass does not exist begins by rewriting Eq. (5) in terms of \( m_0 \) and substituting for \( m_0 \) in Eq. (1) we obtain the expression

\[ \lambda = h/M\sqrt{c^2 - v^2} \tag{6} \]

But as \( M \) is also a function of velocity \( v \) we have

\[ \lambda = \frac{h}{m_0\sqrt{c^2 - v^2}} \sqrt{1 - \frac{(v/c)^2}{}} \tag{6a} \]

Rather than cancelling terms we observe that if \( v = 0 \) in Eq. (6a) we get the Compton wavelength. Thus when \( v = 0 \) the \( M \) used in Eq. (6) corresponds to \( m_0 \) in Eq. (6a), which leaves the case of \( v > 0 \) where the \( M \) used in Eq. (6) now corresponds to

\[ m_0/\sqrt{1 - (v/c)^2} \tag{#} \]

in Eq. (6a). Therefore, if Eq. (6a) is the Compton wavelength when \( v = 0 \), then holding \( m_0 \) fixed and discarding (\#) when \( v > 0 \) gives us the generalized Compton wavelength

\[ \lambda_{GC} = h/m_0\sqrt{c^2 - v^2} \tag{7} \]

Thus all that remains of Eq. (1) is (\#) whose absence is accounted for below. But first rewriting Eq. (7) in terms of \( m_0 \) and substituting into Eq. (2) we
find a qualitatively different expression where frequency has replaced the concept of rest mass

\[ p = \frac{h f v}{(c^2 - v^2)} \]  \hspace{1cm} (8)

To use Eq. (8) we have only to translate rest mass to frequency as follows

\[ p = m_0 v / \sqrt{1 - (v/c)^2} \]

\[ \lambda_{GC} = \frac{h}{m_0 \sqrt{c^2 - v^2}} \]

\[ p = \frac{hc v}{\lambda_{GC} (c^2 - v^2)} = \frac{h f v}{(c^2 - v^2)} \]

Excluding Eq. (3), then, leaves (#) as the only expression using rest mass, and since (#) in itself is meaningless it is incumbent upon us to account for its absence. Therefore, starting with Eq. (4)

\[ \lambda = \frac{h}{p} \]

and substituting Newton’s definition of momentum \( p = mv \) for \( p \), we get the basis of wave mechanics or de Broglie equation

\[ \lambda = \frac{h}{mv} \]

And substituting (#) for \( m \) puts (#) in context giving the relativistic expression

\[ \lambda = \frac{h \sqrt{1 - (v/c)^2}}{m_0 v} \hspace{1cm} v > 0 \]

Finally, rewriting this expression in terms of \( m_0 \) and substituting into the magnitude of Eq. (2) we find

\[ p = \frac{h v \sqrt{1 - (v/c)^2}}{\lambda v \sqrt{1 - (v/c)^2}} = \frac{h}{\lambda} \]

Therefore, the absence of (#) entails the absence of: 1) de Broglie’s equation, 2) \( p = mv \) and 3) Eq. (2). Consistency now dictates we substitute Eq. (7) into Eq. (3) and then to use the wave vector \( k \) where \( k = 2\pi / \lambda \), the
Dirac constant \( \hbar = h/2\pi \), and the angular frequency \( \omega = 2\pi f = kc \), to give us

\[
\begin{align*}
    p &= \hbar k \\
    E &= \hbar \omega c^2 / (c^2 - v^2) \\
    p &= \hbar \omega v / (c^2 - v^2)
\end{align*}
\]

Thus the absence of Eq. (2) from physics necessitates the use of these irreducible relations and thus the need to translate rest mass to angular frequency \( \omega \) where

\[
\omega = 2\pi c / \lambda_{GC} = m_0 c \sqrt{c^2 - v^2} / \hbar
\]

In the mass-based paradigm to render the photon consistent with a ‘massive’ particle we arbitrarily assigned a zero rest mass to the photon. But now we use the rest mass of a particle to find the angular frequency of that particle. Further, with \( p = Mv \) we are to accept that \( M \to \infty \) as \( v \to c \) contrary to what we would expect by ‘time-dilation’, but with the relations as \( v \to c \) we see that \( \omega \to 0 \) and this is consistent with what we would expect by ‘time-dilation’.

Importantly, as Eq. (2) does not exist we cannot ‘undo’ the relations

\[
\begin{align*}
    p &= \frac{\hbar \omega v}{(c^2 - v^2)} = \frac{m_0 v c \sqrt{c^2 - v^2}}{(c^2 - v^2)} = \frac{m_0 v c}{\sqrt{c^2 - v^2}} = \frac{m_0 v}{\sqrt{1 - (v/c)^2}}
\end{align*}
\]

In contrast, ignoring Eq. (1) and just substituting the Compton wavelength directly into Eq. (2) give us

\[
p = \frac{hf v}{\sqrt{c^2 - v^2}}
\]

However, this expression is meaningless as \( f \) is not velocity-dependent and remains unchanged as \( v \to c \), and ignoring Eq. (1) leaves Eq. (2) intact.

We now conclude that the basis of relativistic dynamics \( (v > 0) \) is consistent with the de Broglie relations \( (v = 0) \), and since Eq. (2) does not exist, it is these relations that are really being confirmed in elementary-particle physics!

Before we continue it is clearly worth mentioning that the algebra of the Lorentz transformations cannot distinguish between the energy and momentum expressions using \( m_0 \) or \( \omega \). But the difference is important and
may only be apparent using Minkowski diagrams in which case the relations necessitate the use of Minkowski space-time.

Now, the coining of the term “photon” by Lewis in 1926 is inappropriate as the term refers to the smallest unit of radiant energy, but the relations just derived imply the term should be extended to include the energy of matter as well. To avoid confusion we shall simply define the Light as the electromagnetic energy and momentum of a particle of radiation, or a particle of matter.

We mentioned above that we would justify our ignoring the negative root in the derivation of

\[ E^* = \pm \sqrt{(m_0 c^2)^2 + (pc)^2} \]

The first thing to note is that no such expression exists now that we have the Light. Nevertheless Dirac’s equation from 1928 assumes that \( E^* \) does exist and associates the negative root with antimatter, which leaves the question of why we don’t observe an equal amount of antimatter in the universe? However, looking at

\[ E = \hbar \omega^2 / (c^2 - v^2) \quad \text{for} \quad v \geq 0 \]

we observe that \( E \) is positive in the interval \( 0 < v < c \) corresponding to matter, and \( E \) is negative in the interval \( c < v < c\sqrt{2} \), which must correspond to antimatter. Thus \( E \) shows the whereabouts of antimatter to be in a mirror-universe reflected by the speed of \( c \). Furthermore, when \( v = 0 \) and \( v = c\sqrt{2} \) we find \( E = \pm \hbar \omega \), which is a difference of \( E = 2\hbar \omega \).

Since matter is ‘massless’ special relativity implies it must be moving at speed \( c \) in its own rest frame, which seems to be a contradiction, for how can matter be both at rest and moving at speed \( c \)? Since matter is the origin in its own rest frame we can extend x-y axes from that ‘origin’ to have a plane. We now represent matter moving at speed \( c \) up a vertical time axis \( t \) with no unit lengths marked, which begins at \( O \). Thus by necessity we have \( ct \). Rotating the axis around \( O \) perpendicularly gives us the time axis \( t' \) where \( i = \sqrt{-1} \) and by symmetry matter’ (with \( x' \)-y’ axes perpendicular to x-y giving a perpendicular plane) moving along this axis at speed \( c \). The relative velocity between these planes is thus \( \sqrt{c^2 + c'^2} = c\sqrt{2} \). However, since the energy of matter’ is in the interval \( c < v \leq c\sqrt{2} \) it must be negative, and thus matter’ corresponds to antimatter. To show this using special relativity we choose a unit of time, say seconds, and note that the number of second’s \( t' \) passing for antimatter relative to the viewpoint of
matter as we count $t = 1$ second passing for matter is given by the reciprocal of the time-dilation formula with $v = c\sqrt{2}$

$$t' = \sqrt{1 - (2c^2/c^2)} = i$$

The problem of how matter can be both at rest and moving at speed $c$ is thus resolved consistent with the existence of matter/antimatter asymmetry. Therefore, generalising the energy of a photon by completely exhausting the equations of relativistic mechanics gives us the Light

<table>
<thead>
<tr>
<th>Matter</th>
<th>$E_+ = \hbar \omega c^2 / (c^2 - v^2)$</th>
<th>$p_+ = \hbar \omega v / (c^2 - v^2)$</th>
<th>$v &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation</td>
<td>$E_0 = \hbar \omega$</td>
<td>$p_0 = \hbar k$</td>
<td>(de Broglie relations)</td>
</tr>
<tr>
<td>Antimatter</td>
<td>$E_- = i\hbar \omega c^2 / (c^2 - v^2)$</td>
<td>$p_- = i\hbar \omega v / (c^2 - v^2)$</td>
<td>$v &gt; 0$</td>
</tr>
</tbody>
</table>

A new mass-free paradigm of Relativity begins with the Light. Does the Light imply CPT asymmetry? We leave this unanswered, and focus upon the fact that the Light entails a revision of the mass-based physics used today.

3. Equivalence Identity

As a particle theory the Standard Model ‘unifies’ electromagnetism, weak, and strong nuclear interactions. Thus the Light entails a fundamental revision of the Standard Model as the Standard Model must be derived anew beginning with the Light in this new paradigm of Relativity. This just leaves the ‘force’ of gravity.

Newton’s second law of motion ($F = ma$) contains all three of Newton’s laws of motion, i.e., the third law ($F = -F$); and first law ($F = 0$), or law of inertia. Inertial frames are those in which the law of inertia holds.

In special relativity inertial frames are considered to be freely falling frames moving with neither acceleration nor gravity where the observer experiences weightlessness and tidal forces are considered ‘non-local’. But suppose it could be shown a priori that $F = ma$ does not exist, then the absence of an inertial frame means the observer could not deny that tidal effects are local. This would have no effect upon the Light as it is defined in the subatomic realm where the nonuniformity of the gravitational field is negligible and
space-time considered flat. *However*, the absence of inertial frames undermines special relativity and thus Maxwell’s equations as theories, and just leaves the constant c as encoded in the Light from Maxwell’s theory.

Consider, then, that as the derivation of the Light also consistently accounts for the absence of \( p = mv \) from physics, this in turn implies the absence of Newton’s second law, for mathematically we have

\[
F = \frac{d(mv)}{dt} = m_ia
\]

where \( F \) is force, \( m_i \) is inertial mass and \( a \) is acceleration. We also have

\[
W = m_Gg
\]

where \( W \) is the weight of a terrestrial body, \( m_G \) is its gravitational mass, and \( g \) is the local acceleration of free fall. If we ignore air resistance, then by Galileo’s empirical law of falling bodies we put \( F = W \) and obtain

\[
a = \frac{m_G}{m_i}g
\]

General relativity necessitates that a body’s acceleration under gravity be independent of its mass. The closest we come to realising this in the existing mass-based paradigm is by *arbitrarily* making the two masses numerically equal by an appropriate choice of units. (This is no different to *arbitrarily* assigning a zero rest mass to the photon.) Now, however, we transcend Newtonian mechanics and mass, for \( m_G = m_i \) *a priori* as the Light entails locally there is only

\[
a = g \quad \text{(Equivalence Identity)}
\]

Therefore, if we assume the gravitational constant, \( G \), is used in this mass-free paradigm of Relativity, then all that remains of Newtonian mechanics is \( G \). In contrast to general relativity, then, Relativity transcends any notion of mass, and necessitates that tidal effects are local whilst leaving the constants \( G \) and \( c \) as the only reminders of the classical physics that preceded it. To this extent it is clear that general relativity did not go far enough.

If this is correct, then there never was anything else but this to understand, and everything that does not follow on from this is erroneous. Since this much could have been discovered before 1925 the misguided advent of non-relativistic quantum mechanics and everything that followed on from that could have been avoided!
That just leaves the classical physics of statistical mechanics and thermodynamics unmentioned. But the nonexistence of Newton’s three ‘laws’ of motion entails a revision of these two theories as well.

4. Bekenstein-Hawking formula

Does Relativity subsume these two theories? The following suggests that it does. Consider the Bekenstein-Hawking formula [3]

\[ S_{BH} = (A/4) \times (k c^3 / G \hbar) \]

where \( k \) is Boltzman’s constant from statistical mechanics, and \( A \) is the surface area of the event horizon. If we assume this formula holds true in Relativity, then we have a consistent unification of \( G, c, k, \hbar \) and entropy to remind us of the mass-based paradigm of classical physics that was.

5. Conclusion

The evolution from classical physics to Relativity was inevitable as mass was an illusion and tidal effects were always local. Herein lies the true significance of Planck’s four-dimensional constant.

References

[1] Spoljaric, R., Hadronic Journal 34, Number 2, 125 (2011)
