Elastic Collision Between Charged Particles

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An isolated physical system of elastic collision between two identical charged particles is chosen to manifest the physics law, conservation of momentum, in a random inertial reference frame under Lorentz Transformation. In this random reference frame, the center of mass moves at a constant velocity. By applying Lorentz transformation to the velocities of both particles, total momentum during the collision in this random inertial reference frame can be calculated and is expected to remain constant. The calculation shows that conservation of momentum fails to hold under Lorentz Transformation.

I. INTRODUCTION

Elastic collision between two identical charged particles is an excellent physics system for the demonstration of the physics law of conservation of momentum. The collision will be examined in two inertial reference frames, the center of mass (COM) frame and a random inertial reference frame. Conservation of momentum is expected to hold in both reference frames.

Lorentz Transformation[1] transforms the velocities of both particles from COM frame to a random inertial reference frame. The total momentum will be calculated in this random inertial reference frame to verify if conservation of momentum still holds while both particles move toward and away from each other at any velocity

The concept of relativistic mass becomes less popular in modern physics. Relativistic force and relativistic momentum do not share the same relativistic mass. The momentum of an particle is represented by either $\gamma(V) * m(0) * V$ or $m(V) * V$. Both representations are equivalent to each other mathematically. In this paper, $\gamma(V) * m * V$ is chosen to emphasize Lorentz Factor, $\gamma(V)$, in Lorentz Transformation.

$$\frac{dm}{dV} = \frac{dm(0)}{dV} = 0 \tag{1}$$

II. PROOF

Consider one-dimensional motion.

A. Elastic Collision

Two identical charged particles move toward each other to make head-on collision. In the COM frame (Center Of Mass), both particles move at identical speed but opposite direction. The electric force between them is repulsive. Both particles eventually slow down to become stationary. The repulsive force continues to push both particles away.

B. Before Collision

Let a reference frame F_1 be stationary relatively to this COM frame. Let the velocity of one particle be V at a random time in F_1.

TABLE I. Velocity and Momentum in COM Frame

Object	Frame	Value
The velocity of particle 1, O_1, in	F_1	is V
The velocity of particle 2, O_2, in	F_1	is -V
The momentum of O_1 in	F_1	is $\gamma(V) * m * V$
The momentum of O_2 in	F_1	is $\gamma(-V) * m * (-V)$

C. Lorentz Transformation

Let another reference frame F_2 move at the velocity -u relatively to F_1. The velocity of F_1 relative to F_2 is u. According to Lorentz Transformation, the velocity of O_1 in F_2 has to be $\frac{V+u}{1+\frac{V*u}{C^2}}$. The velocity of O_2 in F_2 has to be $\frac{-V+u}{1+\frac{-V*u}{C^2}}$.

TABLE II. Velocity Transformation

Object	Frame	Velocity
The velocity of O_1 in	F_1	is V
The velocity of O_2 in	F_1	is -V
The velocity of F_1 relative to	F_2	is u
The velocity of O_1 in	F_2	is $\frac{V+u}{1+\frac{V*u}{C^2}}$
The velocity of O_2 in	F_2	is $\frac{-V+u}{1+\frac{-V*u}{C^2}}$

D. During Collision

Both particles in F_1 will slow down and come to stand still before moving away. As both particles become stationary in F_1, both particles move at the same velocity in F_2.

TABLE III. Both particles Are Stationary to Each Other

Object	Frame	Velocity
The velocity of O_1 in	F_1	is 0
The velocity of O_2 in	F_1	is 0
The velocity of F_1 relative to	F_2	is u
The velocity of O_1 in	F_2	is u
The velocity of O_2 in	F_2	is u

E. Conservation of Momentum

Let v_1 be the velocity of O_1 in F_2. Let v_2 be the velocity of O_2 in F_2.

$$v_1 = \frac{V+u}{1+\frac{V*u}{C^2}} = \frac{u+V}{1+\frac{u*V}{C^2}} \tag{2}$$

$$v_2 = \frac{-V+u}{1+\frac{-V*u}{C^2}} = \frac{u-V}{1-\frac{u*V}{C^2}} \tag{3}$$

Total momentum P in F_2 is

$$P = \gamma(v_1)*m*v_1 + \gamma(v_2)*m*v_2 \tag{4}$$

Conservation of Momentum demands

$$\frac{dP}{dV} = 0 \tag{5}$$

Total momentum is expected to be constant as particles move at any velocity V.

From equation (4),

$$\frac{dP}{dV} = \gamma(v_1)^3*m*\frac{dv_1}{dV} + \gamma(v_2)^3*m*\frac{dv_2}{dV} \tag{6}$$

From equation (2),

$$\frac{dv_1}{dV} = \frac{1}{1+\frac{u*V}{C^2}}\left(1 - \frac{u+V}{1+\frac{u*V}{C^2}} * \frac{u}{C^2}\right) \tag{7}$$

$$= \frac{v_1}{u+V}\left(1 - v_1 * \frac{u}{C^2}\right) \tag{8}$$

From equation (3),

$$\frac{dv_2}{dV} = \frac{1}{1-\frac{u*V}{C^2}}\left(-1 - \frac{u-V}{1-\frac{u*V}{C^2}} * \frac{-u}{C^2}\right) \tag{9}$$

$$= \frac{v_2}{u-V}\left(-1 + v_2 * \frac{u}{C^2}\right) \tag{10}$$

Equation (5) fails to hold for V=u=$\frac{C}{2}$ when both particles are approaching each other.

$$v_1 = \frac{u+V}{1+\frac{u*V}{C^2}} = \frac{C}{1+\frac{1}{4}} = \frac{4}{5}*C \tag{11}$$

$$v_2 = \frac{u-V}{1-\frac{u*V}{C^2}} = 0 \tag{12}$$

$$\gamma(v_1)^3*m*\frac{dv_1}{dV} = \gamma(\tfrac{4}{5}*C)^3*m*\frac{12}{25} > 0 \tag{13}$$

$$\gamma(v_2)^3*m*\frac{dv_2}{dV} = 0 \tag{14}$$

From equation (6),

$$\frac{dP}{dV} = \gamma(\tfrac{4}{5}*C)^3*m*\frac{12}{25} + 0 > 0 \tag{15}$$

Equation (5) fails to hold for -V=u=$\frac{C}{2}$ when both particles are leaving each other.

$$v_1 = \frac{u+V}{1+\frac{u*V}{C^2}} = 0 \tag{16}$$

$$v_2 = \frac{u-V}{1-\frac{u*V}{C^2}} = \frac{C}{1+\frac{1}{4}} = \frac{4}{5}*C \tag{17}$$

$$\gamma(v_1)^3*m*\frac{dv_1}{dV} = 0 \tag{18}$$

$$\gamma(v_2)^3*m*\frac{dv_2}{dV} = \gamma(\tfrac{4}{5}*C)^3*m*\frac{-12}{25} < 0 \tag{19}$$

From equation (6),

$$\frac{dP}{dV} = 0 - \gamma(\tfrac{4}{5}*C)^3*m*\frac{12}{25} < 0 \tag{20}$$

Total momentum reaches its minimum at V=0.

$$v_1 = \frac{u+V}{1+\frac{u*V}{C^2}} = u \tag{21}$$

$$v_2 = \frac{u-V}{1-\frac{u*V}{C^2}} = u \tag{22}$$

$$\gamma(v_1)^3*m*\frac{dv_1}{dV} = \gamma(u)^3*m*\left(1 - u*\frac{u}{C^2}\right) \tag{23}$$

$$\gamma(v_2)^3*m*\frac{dv_2}{dV} = \gamma(u)^3*m*\left(-1 + u*\frac{u}{C^2}\right) \tag{24}$$

From equation (6),

$$\frac{dP}{dV} = \gamma(v_1)^3*m*\frac{dv_1}{dV} + \gamma(v_2)^3*m*\frac{dv_2}{dV} = 0 \tag{25}$$

From equation (4), minimum total momentum in F_2 is

$$P = 2*\gamma(u)*m*u \tag{26}$$

Under Lorentz Transformation, total momentum in F_2 decreases as both particles move toward each other and increases as both particles move away from each other.

III. CONCLUSION

Lorentz Transformation violates conservation of momentum.

Conservation of momentum fails to hold if Lorentz Transformation is applied to an isolated system of elastic collision. The failure of this physics law is due to the addition of velocity from Lorentz Transformation. The correct formula for velocity addition has been derived by Eric Su in 2018[2][10].

Lorentz Transformation was proposed on the assumption that the speed of light is independent of inertial reference frame.

As the result of this incorrect assumption[3], Lorentz Transformation violates Translation Symmetry[4] in physics. Translation Symmetry requires conservation of simultaneity[5], conservation of distance[6], and conservation of time[7]. All three conservation properties are broken by Lorentz Transformation.

Therefore, Lorentz Transformation is an invalid transformation in physics. Consequently, any theory based on Lorentz Transformation is incorrect in physics. For example, Special Relativity[8][9]

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