

# Elastic Collision Between Charged Particles

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An isolated physical system of elastic collision between two identical charged particles is chosen to manifest the physics law, conservation of momentum, in a random inertial reference frame under Lorentz Transformation. In this random reference frame, the center of mass moves at a constant velocity. By applying Lorentz transformation to the velocities of both particles, total momentum during the collision in this random inertial reference frame can be calculated and is expected to remain constant. The calculation shows that conservation of momentum fails to hold under Lorentz Transformation.

## I. INTRODUCTION

Elastic collision between two identical charged particles is an excellent physics system for the demonstration of the physics law of conservation of momentum. The collision will be examined in two inertial reference frames, the center of mass (COM) frame and a random inertial reference frame. Conservation of momentum is expected to hold in both reference frames.

Lorentz Transformation[1] transforms the velocities of both particles from COM frame to a random inertial reference frame. The total momentum will be calculated in this random inertial reference frame to verify if conservation of momentum still holds while both particles move toward and away from each other at any velocity

The concept of relativistic mass becomes less popular in modern physics. Relativistic force and relativistic momentum do not share the same relativistic mass. The momentum of an particle is represented by either  $\gamma(V) * m(0) * V$  or  $m(V) * V$ . Both representations are equivalent to each other mathematically. In this paper,  $\gamma(V) * m * V$  is chosen to emphasize Lorentz Factor,  $\gamma(V)$ , in Lorentz Transformation.

$$\frac{dm}{dV} = \frac{dm(0)}{dV} = 0 \quad (1)$$

## II. PROOF

Consider one-dimensional motion.

### A. Elastic Collision

Two identical charged particles move toward each other to make head-on collision. In the COM frame (Center Of Mass), both particles move at identical speed but opposite direction. The electric force between them is repulsive. Both particles eventually slow down to become stationary. The repulsive force continues to push both particles away.

### B. Before Collision

Let a reference frame  $F_1$  be stationary relatively to this COM frame. Let the velocity of one particle be  $V$  at a random time in  $F_1$ .

TABLE I. Velocity and Momentum in COM Frame

Object	Frame	Value
The velocity of particle 1, $O_1$ , in	$F_1$	is $V$
The velocity of particle 2, $O_2$ , in	$F_1$	is $-V$
The momentum of $O_1$ in	$F_1$	is $\gamma(V) * m * V$
The momentum of $O_2$ in	$F_1$	is $\gamma(-V) * m * (-V)$

### C. Lorentz Transformation

Let another reference frame  $F_2$  move at the velocity  $-u$  relatively to  $F_1$ . The velocity of  $F_1$  relative to  $F_2$  is  $u$ . According to Lorentz Transformation, the velocity of  $O_1$  in  $F_2$  has to be  $\frac{V+u}{1+\frac{V*u}{C^2}}$ . The velocity of  $O_2$  in  $F_2$  has to be  $\frac{-V+u}{1+\frac{-V*u}{C^2}}$ .

TABLE II. Velocity Transformation

Object	Frame	Velocity
The velocity of $O_1$ in	$F_1$	is $V$
The velocity of $O_2$ in	$F_1$	is $-V$
The velocity of $F_1$ relative to	$F_2$	is $u$
The velocity of $O_1$ in	$F_2$	is $\frac{V+u}{1+\frac{V*u}{C^2}}$
The velocity of $O_2$ in	$F_2$	is $\frac{-V+u}{1+\frac{-V*u}{C^2}}$

### D. During Collision

Both particles in  $F_1$  will slow down and come to stand still before moving away. As both particles become stationary in  $F_1$ , both particles move at the same velocity in  $F_2$ .

TABLE III. Both particles Are Stationary to Each Other

	Object	Frame	Velocity
	The velocity of $O_1$ in	$F_1$	is 0
	The velocity of $O_2$ in	$F_1$	is 0
The velocity of $F_1$ relative to	The velocity of $O_1$ in	$F_2$	is u
	The velocity of $O_2$ in	$F_2$	is u
	The velocity of $O_2$ in	$F_2$	is u

### E. Conservation of Momentum

Let  $v_1$  be the velocity of  $O_1$  in  $F_2$ . Let  $v_2$  be the velocity of  $O_2$  in  $F_2$ .

$$v_1 = \frac{V+u}{1+\frac{V*u}{C^2}} = \frac{u+V}{1+\frac{u*V}{C^2}} \quad (2)$$

$$v_2 = \frac{-V+u}{1-\frac{V*u}{C^2}} = \frac{u-V}{1-\frac{u*V}{C^2}} \quad (3)$$

Total momentum P in  $F_2$  is

$$P = \gamma(v_1) * m * v_1 + \gamma(v_2) * m * v_2 \quad (4)$$

Conservation of Momentum demands

$$\frac{dP}{dV} = 0 \quad (5)$$

Total momentum is expected to be constant as particles move at any velocity V.

From equation (4),

$$\frac{dP}{dV} = \gamma(v_1)^3 * m * \frac{dv_1}{dV} + \gamma(v_2)^3 * m * \frac{dv_2}{dV} \quad (6)$$

From equation (2),

$$\frac{dv_1}{dV} = \frac{1}{1+\frac{u*V}{C^2}} \left(1 - \frac{u+V}{1+\frac{u*V}{C^2}} * \frac{u}{C^2}\right) \quad (7)$$

$$= \frac{v_1}{u+V} \left(1 - v_1 * \frac{u}{C^2}\right) \quad (8)$$

From equation (3),

$$\frac{dv_2}{dV} = \frac{1}{1-\frac{u*V}{C^2}} \left(-1 - \frac{u-V}{1-\frac{u*V}{C^2}} * \frac{-u}{C^2}\right) \quad (9)$$

$$= \frac{v_2}{u-V} \left(-1 + v_2 * \frac{u}{C^2}\right) \quad (10)$$

Equation (5) fails to hold for  $V=u=\frac{C}{2}$  when both particles are approaching each other.

$$v_1 = \frac{u+V}{1+\frac{u*V}{C^2}} = \frac{C}{1+\frac{1}{4}} = \frac{4}{5} * C \quad (11)$$

$$v_2 = \frac{u-V}{1-\frac{u*V}{C^2}} = 0 \quad (12)$$

$$\gamma(v_1)^3 * m * \frac{dv_1}{dV} = \gamma\left(\frac{4}{5} * C\right)^3 * m * \frac{12}{25} > 0 \quad (13)$$

$$\gamma(v_2)^3 * m * \frac{dv_2}{dV} = 0 \quad (14)$$

From equation (6),

$$\frac{dP}{dV} = \gamma\left(\frac{4}{5} * C\right)^3 * m * \frac{12}{25} + 0 > 0 \quad (15)$$

Equation (5) fails to hold for  $-V=u=\frac{C}{2}$  when both particles are leaving each other.

$$v_1 = \frac{u+V}{1+\frac{u*V}{C^2}} = 0 \quad (16)$$

$$v_2 = \frac{u-V}{1-\frac{u*V}{C^2}} = \frac{C}{1+\frac{1}{4}} = \frac{4}{5} * C \quad (17)$$

$$\gamma(v_1)^3 * m * \frac{dv_1}{dV} = 0 \quad (18)$$

$$\gamma(v_2)^3 * m * \frac{dv_2}{dV} = \gamma\left(\frac{4}{5} * C\right)^3 * m * \frac{-12}{25} < 0 \quad (19)$$

From equation (6),

$$\frac{dP}{dV} = 0 - \gamma\left(\frac{4}{5} * C\right)^3 * m * \frac{12}{25} < 0 \quad (20)$$

Total momentum reaches its maximum at  $V=0$ .

$$v_1 = \frac{u+V}{1+\frac{u*V}{C^2}} = u \quad (21)$$

$$v_2 = \frac{u-V}{1-\frac{u*V}{C^2}} = u \quad (22)$$

$$\gamma(v_1)^3 * m * \frac{dv_1}{dV} = \gamma(u)^3 * m * \left(1 - u * \frac{u}{C^2}\right) \quad (23)$$

$$\gamma(v_2)^3 * m * \frac{dv_2}{dV} = \gamma(u)^3 * m * \left(-1 + u * \frac{u}{C^2}\right) \quad (24)$$

From equation (6),

$$\frac{dP}{dV} = \gamma(v_1)^3 * m * \frac{dv_1}{dV} + \gamma(v_2)^3 * m * \frac{dv_2}{dV} = 0 \quad (25)$$

From equation (4), maximum total momentum in  $F_2$  is

$$P = 2 * \gamma(u) * m * u \quad (26)$$

Under Lorentz Transformation, total momentum in  $F_2$  increases as both particles move toward each other and decreases as both particles move away from each other.

### III. CONCLUSION

Lorentz Transformation violates conservation of momentum.

Conservation of momentum fails to hold if Lorentz Transformation is applied to an isolated system of elastic collision. The failure of this physics law is due to the addition of velocity from Lorentz Transformation. The

correct formula for velocity addition has been derived by Eric Su in 2018[2][10].

Lorentz Transformation was proposed on the assumption that the speed of light is independent of inertial reference frame.

As the result of this incorrect assumption[3], Lorentz Transformation violates Translation Symmetry[4] in

physics. Translation Symmetry requires conservation of simultaneity[5], conservation of distance[6], and conservation of time[7]. All three conservation properties are broken by Lorentz Transformation.

Therefore, Lorentz Transformation is an invalid transformation in physics. Consequently, any theory based on Lorentz Transformation is incorrect in physics. For example, Special Relativity[8][9]

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