IN-cross Entropy Based MAGDM Strategy under Interval Neutrosophic Set Environment

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Abstract. Cross entropy measure is one of the best way to calculate the divergence of any variable from the priori one variable. We define a new cross entropy measure under interval neutrosophic set (INS) environment, which we call IN-cross entropy measure and prove its basic properties. We also develop weighted IN-cross entropy measure and investigates its basic properties. Based on the weighted IN-cross entropy measure, we develop a novel strategy for multi attribute group decision making (MAGDM) strategy under interval neutrosophic environment. The proposed multi attribute group decision making strategy is compared with the existing cross entropy measure based strategy in the literature under interval neutrosophic set environment. Finally, an illustrative example of multi attribute group decision making problem is solved to show the feasibility, validity and efficiency of the proposed MAGDM strategy.

Keywords: Interval neutrosophic set, IN-cross entropy measure, MAGDM strategy.

1. Introduction
In our daily life we frequently meet with the quantitative measure to take appropriate decision for solving many problems. Entropy measure provides us a quantitative measure of two variables. In 1968, Zadeh [1] introduced fuzzy entropy measure. According to Liu [2], under fuzzy environment, entropy should meet at least three basic following requirements: the entropy of a crisp number is zero; the entropy of an equipossible fuzzy variable is maximum and the entropy is applicable not only to finite and infinite cases but also to discrete and continuous cases. Shang and Jiang [3] proposed a cross entropy measure and symmetric discrimination measure between fuzzy sets. Atanassov [4] introduced intuitionistic fuzzy set (IFS) in 1989, which is the extension of fuzzy set. Some recent applications of IFS are found in [5-11] in the literature. Vlachos and Sergiadis [12] defined cross entropy measure in IFS environment and showed a mathematical connection between the notions of entropy for fuzzy sets and IFSs in terms of fuzziness and intuitionism. In 1998, Smarandache [13] introduced the concept of neutrosophic set (NS) by introducing truth membership, falsity membership and indeterminacy membership functions as independent components and their sum lies (0, 3]). Thereafter, Wang et al. [14] introduced single valued neutrosophic set (SVNS) as a subclass of NS. Thereafter, many researchers paid attention to apply NS and SVNS in many field of research such as conflict resolution [15], clustering analysis [16, 17], decision making [18-47], educational problem [48, 49], image processing [50, 52], medical diagnosis [53], optimization [54-59], social problem [60, 61]. Ye [62] introduced cross entropy measure in SVNS and applied it to multi criteria decision-making (MCDM) problems. Ye [63] defined an improved cross entropy measure for SVNS to overcome drawbacks in [62]. In 2005, Wang et al. [64] introduced interval neutrosophic set (INS) considering truth membership, indeterminate membership and falsity membership as interval number in [0, 1]. Brouni and Smarandache [65] defined correlation coefficient of INS and proved its basic properties. Zhang et al. [66] defined correlation coefficient for
interval neutrosophic number (INN) and applied it iv. MAGDM problems. Zhang et al. [67] presented an out-ranking approach for INS and applied its MCDM problems. Recently, Yu et al. [68] use VIKOR method to solve MAGDM problem with INN. Ye [69] defined similarity measure in INS environment and applied to solve MCDM problem. Pramanik and Mondal [70] extended the single valued neutrosophic grey relational analysis strategy to interval neutrosophic environment and applied it to multi-attribute decision-making (MADM) problems. Zhao et al. [71] proposed a MADM strategy based on generalized weighted aggregation operator with INS. Zhao et al. [72] proposed a MCDM strategy based on two interval neutrosophic number aggregation operators. Sahin [73] defined two cross entropy measures with INS based on fuzzy cross entropy measure and single valued neutrosophic cross entropy measure and applied for solving MCDM problem. Tian et al. [74] proposed a cross entropy measure with INS and TOPSIS for solving MCDM problems.

Sahin [73], Tian et al. [74] proposed cross entropy measures under the interval-valued neutrosophic set environment, which is suitable for single decision maker only. So multiple decision maker cannot participate in their strategies in [73, 74].

The aforementioned applications of cross entropy [63, 73, 74] can be effective in dealing with neutrosophic 1. MADM problems. However, they also bear some limitations, which are outlined below: 2.

1. The strategies [63, 73, 74] are capable of solving neutrosophic MADM problems.
2. In the strategies [73, 74], interval-valued neutrosophic 3. set are transformed to SVNS by suitable transform operators.
3. The strategies [63, 73, 74] have a single decision-making 4. structure, and not enough attention is paid to improving robustness when processing the assessment information.

Research gap:
MAGDM strategy based on cross entropy measure. This study answers the following research questions:

i. Is it possible to define a new cross entropy measure under interval-valued neutrosophic set environment that is free from asymmetrical phenomena?
ii. Is it possible to define a new weighted cross entropy measure under interval-valued neutrosophic set that is free from asymmetrical phenomena?
iii. Is it possible to develop a new MAGDM strategy based on the proposed cross entropy measure under interval-valued neutrosophic set environment?

Is it possible to develop a new MAGDM strategy based on the proposed weighted cross entropy measure under interval-valued neutrosophic set environment?

Motivation:
The above-mentioned analysis describes the motivation behind proposing a novel IN-cross entropy-based strategy for tackling MAGDM under the interval-valued neutrosophic environment. This study develops a novel IN-cross entropy-based MAGDM strategy that can deal with multiple decision-makers and free from the drawbacks that exist in [63, 72, 73].

The objectives of the paper are:
1. To define a new cross entropy measure under interval-valued neutrosophic set environment without using any transformation operator and prove its basic properties.
2. To define a new weighted cross measure and prove its basic properties.
3. To develop a new MAGDM strategy based on weighted cross entropy measure under interval-valued neutrosophic set environment.

To fill the research gap, we propose IN-cross entropy-based MAGDM, which is capable of dealing with multiple decision-makers.

The main contributions of this paper are summarized below:

i. We define a new IN-cross entropy measure and prove its basic properties. It is straightforward symmetric.
ii. We define a new weighted IN-cross entropy measure in the single-valued neutrosophic set environment and prove its basic properties. It is straightforward symmetric.
iii. In this paper, we develop a new MAGDM strategy based on weighted IN cross entropy to solve MAGDM problems.
iv. In this paper, we solve a MAGDM problem based on the proposed MAGDM strategy.

The paper unfolds as follows: In section 2, we describe the basic definitions and operations of SVNS, INS. In section 3, we present the definition of proposed IN-cross entropy measure, weighted IN-cross entropy measure and their basic properties. In section 4, we develop a MAGDM strategy with the proposed IN-cross entropy measure. In section 5, we solve a MAGDM problem to show the feasibility, validity and efficiency of the proposed strategy. In section 6, we present conclusion and future direction of this study.
2. Preliminaries

2.1 Definition: Single valued neutrosophic set (SVNS) [14]

Assume that $U$ be a space of points (objects) with generic elements $u \in U$. A SVNS $H$ in $U$ is characterized by a truth-membership function $T_U(u)$, an indeterminacy-membership function $I_U(u)$, and a falsity-membership function $F_U(u)$, where $T_U(u)$, $I_U(u)$, $F_U(u) \in [0, 1]$ for each point $u$ in $U$. Therefore, a SVNS $A$ can be expressed as $H = \{ u, T_H(u), I_H(u), F_H(u) \mid u \in U \}$, whereas, the sums of $T_U(u)$, $I_U(u)$ and $F_U(u)$ satisfy the condition $0 \leq T_U(u) + I_U(u) + F_U(u) \leq 3$.

2.2 Definition: Interval neutrosophic sets (INSs) [64]

Assume that $U$ be a space of points (objects) with generic elements $u \in U$. An INSs $J$ in $U$ is characterized by a truth-membership measure $T_U(u)$, an indeterminacy-membership measure $I_U(u)$, and a falsity-membership measure $F_U(u)$, where,

\[ T_J(u) = [T_J^L(u), T_J^R(u)], \quad I_J(u) = [I_J^L(u), I_J^R(u)], \quad F_J(u) = [F_J^L(u), F_J^R(u)] \]

for each point $u$ in $U$. Therefore, a INSs $J$ can be expressed as $J = \{ u, [T_J(u), T_J^L(u)], [I_J(u), I_J^L(u)], [F_J(u), F_J^L(u)] \mid u \in U \}$. Where,

\[ T_J(u) = [T_J^L(u), T_J^R(u)], \quad I_J(u) = [I_J^L(u), I_J^R(u)], \quad F_J(u) = [F_J^L(u), F_J^R(u)] \leq [0, 1]. \]

2.3 Definition: Inclusion of two INSs [64]

Let \( J_1 = \{ u, [T_{J_1}^L(u), T_{J_1}^R(u)], [I_{J_1}^L(u), I_{J_1}^R(u)], [F_{J_1}^L(u), F_{J_1}^R(u)] \mid u \in U \} \) and \( J_2 = \{ u, [T_{J_2}^L(u), T_{J_2}^R(u)], [I_{J_2}^L(u), I_{J_2}^R(u)], [F_{J_2}^L(u), F_{J_2}^R(u)] \mid u \in U \} \) be any two INSs in $U$, then \( J_1 \subseteq J_2 \) iff \( T_{J_1}(u) \leq T_{J_2}(u), \quad I_{J_1}(u) \leq I_{J_2}(u), \quad F_{J_1}(u) \geq F_{J_2}(u) \) for all $u \in U$.

2.4 Definition: Complement of an INS [64]

The complement \( J' \) of an INS $J$ is \( \{ u, [1-T_J^L(u), 1-T_J^R(u)], [1-I_J^L(u), 1-I_J^R(u)], [1-F_J^L(u), 1-F_J^R(u)] \mid u \in U \} \) is defined as follows:

\[ J' = \{ u, \quad [1-T_J^L(u), 1-T_J^R(u)], \quad [1-I_J^L(u), 1-I_J^R(u)], \quad [1-F_J^L(u), 1-F_J^R(u)] \mid u \in U \} \]

2.5 Definition: Equality of two INSs [64]

Let \( J_1 = \{ u, [T_{J_1}^L(u), T_{J_1}^R(u)], [I_{J_1}^L(u), I_{J_1}^R(u)], [F_{J_1}^L(u), F_{J_1}^R(u)] \mid u \in U \} \) and \( J_2 = \{ u, [T_{J_2}^L(u), T_{J_2}^R(u)], [I_{J_2}^L(u), I_{J_2}^R(u)], [F_{J_2}^L(u), F_{J_2}^R(u)] \mid u \in U \} \) be any two INSs in $U$, then \( J_1 = J_2 \) iff \( T_{J_1}(u) = T_{J_2}(u), \quad I_{J_1}(u) = I_{J_2}(u), \quad F_{J_1}(u) = F_{J_2}(u) \) for all $u \in U$.

3. Definition: IN-cross-entropy measure

Let $J_1$ and $J_2$ be any two INSs in $U = \{ u_1, u_2, u_3, ..., u_n \}$. Then, the interval neutrosophic cross-entropy measure of $J_1$ and $J_2$ is denoted by $CE_{IN}(J_1, J_2)$ and defined as follows:

\[
CE_{IN}(J_1, J_2) = \frac{1}{4} \left( \frac{2[T_{J_1}(u) - T_{J_2}(u)]}{\sqrt{1 + T_{J_1}(u)^2} + \sqrt{1 + T_{J_2}(u)^2}} + \frac{2[I_{J_1}(u) - I_{J_2}(u)]}{\sqrt{1 + I_{J_1}(u)^2} + \sqrt{1 + I_{J_2}(u)^2}} + \frac{2[F_{J_1}(u) - F_{J_2}(u)]}{\sqrt{1 + F_{J_1}(u)^2} + \sqrt{1 + F_{J_2}(u)^2}} \right)
\]

\[
\left[ \frac{2[T_{J_1}(u) - T_{J_2}(u)]}{\sqrt{1 + T_{J_1}(u)^2} + \sqrt{1 + T_{J_2}(u)^2}} + \frac{2[I_{J_1}(u) - I_{J_2}(u)]}{\sqrt{1 + I_{J_1}(u)^2} + \sqrt{1 + I_{J_2}(u)^2}} + \frac{2[F_{J_1}(u) - F_{J_2}(u)]}{\sqrt{1 + F_{J_1}(u)^2} + \sqrt{1 + F_{J_2}(u)^2}} \right]
\]

Theorem 1.

Interval-valued neutrosophic cross entropy $CE_{IN}(J_1, J_2)$ for any two INSs $J_1$ and $J_2$ of $U$, satisfies the following properties:

i) $CE_{IN}(J_1, J_2) \geq 0$.

ii) $CE_{IN}(J_1, J_2) = 0$ if and only if $T_{J_1}(u) = T_{J_2}(u), \quad I_{J_1}(u) = I_{J_2}(u), \quad F_{J_1}(u) = F_{J_2}(u)$ for all $u \in U$.

iii) $CE_{IN}(J_1, J_2) = CE_{IN}(J_2', J_1')$.

iv) $CE_{IN}(J_1, J_2) = CE_{IN}(J_2, J_1)$.

Proof:

...
For all values of \( u_i \in U \), 
\[
\begin{align*}
|T_1(u_i)| + |T_2(u_i)| &\geq 0, \quad |T_1(u_i)| + |T_2(u_i)| \geq 0, \\
\left(1 - T_1(u_i)\right) + \left(1 - T_2(u_i)\right) &\geq 0, \quad \left(1 - T_1(u_i)\right) + \left(1 - T_2(u_i)\right) \geq 0, \\
\left(1 - T_1(u_i) - \left(1 - T_2(u_i)\right)\right) &\geq 0, \quad \left(1 - T_1(u_i) - \left(1 - T_2(u_i)\right)\right) \geq 0, \\
\left(\sqrt{1 + \frac{2}{\sqrt{1 + \left(1 - T_1(u_i)\right)^2}} - \sqrt{1 + \left(1 - T_2(u_i)\right)^2}}\right) &\geq 0
\end{align*}
\]

and
\[
\left|T_1(u_i)\right| + \left|T_2(u_i)\right| \geq 0, \quad \left|T_1(u_i)\right| - \left|T_2(u_i)\right| \geq 0, \\
\left(1 - T_1(u_i) - \left(1 - T_2(u_i)\right)\right) \geq 0, \quad \left(1 - T_1(u_i) - \left(1 - T_2(u_i)\right)\right) \geq 0, \\
\left(\sqrt{1 + \frac{2}{\sqrt{1 + \left(1 - T_1(u_i)\right)^2}} - \sqrt{1 + \left(1 - T_2(u_i)\right)^2}}\right) \geq 0
\]

Similarly, we can show that
\[
\begin{align*}
2\left|T_1(u_i) - T_2(u_i)\right| &\geq 0, \quad 2\left|T_1(u_i) - T_2(u_i)\right| \geq 0, \\
2\left|T_1(u_i) - T_2(u_i)\right| &\geq 0, \quad 2\left|T_1(u_i) - T_2(u_i)\right| \geq 0, \\
2\left|T_1(u_i) - T_2(u_i)\right| &\geq 0, \quad 2\left|T_1(u_i) - T_2(u_i)\right| \geq 0, \\
2\left|T_1(u_i) - T_2(u_i)\right| &\geq 0, \quad 2\left|T_1(u_i) - T_2(u_i)\right| \geq 0
\end{align*}
\]

and
\[
\begin{align*}
\left[2\left|T_1(u_i) - T_2(u_i)\right|\right] &\geq 0, \quad \left[2\left|T_1(u_i) - T_2(u_i)\right|\right] \geq 0, \\
\left[2\left|T_1(u_i) - T_2(u_i)\right|\right] &\geq 0, \quad \left[2\left|T_1(u_i) - T_2(u_i)\right|\right] \geq 0, \\
\left[2\left|T_1(u_i) - T_2(u_i)\right|\right] &\geq 0, \quad \left[2\left|T_1(u_i) - T_2(u_i)\right|\right] \geq 0, \\
\left[2\left|T_1(u_i) - T_2(u_i)\right|\right] &\geq 0, \quad \left[2\left|T_1(u_i) - T_2(u_i)\right|\right] \geq 0
\end{align*}
\]

Hence, we can conclude that \( CE_{IN}(J_1, J_2) \geq 0 \).

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\[
\Leftrightarrow F_i^s(u_i) = F_i^s(u_i)
\]

So, \( CE_{IN}(J_1, J_2) = 0 \) if and only if

\[
T_i^s(u_i) = T_i^s(u_i), \quad T_i^s(u_i) = T_i^s(u_i), \quad \Gamma_i^s(u_i) = \Gamma_i^s(u_i), \quad F_i^s(u_i) = F_i^s(u_i), \quad F_i^s(u_i) = F_i^s(u_i) \quad \forall u_i \in U.
\]

Hence complete the proof.

iii). Using definition (2.4), we obtain the following expression:

\[
CE_{IN}(J_1, J_2) = \frac{1}{4} \sum_{i=1}^{n} \left( \frac{2|1-T_i^s(u_i) - 0| - 2|F_i^s(u_i) - T_i^s(u_i)|}{\sqrt{1 + |1-T_i^s(u_i)|^2 + \sqrt{1 + |0-T_i^s(u_i)|^2}}} + \frac{2|0-F_i^s(u_i) - 0| - 2|F_i^s(u_i) - T_i^s(u_i)|}{\sqrt{1 + |0-F_i^s(u_i)|^2 + \sqrt{1 + |1-F_i^s(u_i)|^2}}} \right)
\]

Hence complete the proof.

iv). Using definition (2.4), we obtain the following expression:

\[
CE_{IN}(J_1, J_2) = \frac{1}{4} \sum_{i=1}^{n} \left( \frac{2|T_i^s(u_i) - T_i^s(u_i)|}{\sqrt{1 + |T_i^s(u_i)|^2 + \sqrt{1 + |0-T_i^s(u_i)|^2}}} + \frac{2|0-F_i^s(u_i) - 0| - 2|F_i^s(u_i) - T_i^s(u_i)|}{\sqrt{1 + |0-F_i^s(u_i)|^2 + \sqrt{1 + |1-F_i^s(u_i)|^2}}} \right)
\]

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Neutrosophic Sets and Systems, 18/2017

3.1 Definition: Weighted IN-cross-entropy measure

We consider the weight \( w_i \) (i = 1, 2, 3, ..., n) of \( u_i \) (i = 1, 2, 3, ..., n) with \( \sum_{i=1}^{n} w_i = 1 \).

Then the weighted cross entropy between \( J_1 \) and \( J_2 \) can be defined as follows:

\[
CE_{w}^{IN} (J_1, J_2) = \frac{1}{4} \sum_{i=1}^{n} w_i \left( \frac{2[F_{J_1}(u_i) - T_{J_2}(u_i)]}{\sqrt{1 + [F_{J_1}(u_i)]^2 + [T_{J_2}(u_i)]^2}} + \frac{2[1 - F_{J_1}(u_i) - T_{J_2}(u_i)]}{\sqrt{1 + [1 - F_{J_1}(u_i)]^2 + [1 - T_{J_2}(u_i)]^2}} \right)
\]

\[
= CE_{IN} (J_1, J_2).
\]

Hence complete the proof.

Theorem 2.

Interval neutrosophic weighted cross-entropy measure \( CE_{w}^{IN} (J_1, J_2) \) satisfies the following properties:

i). \( CE_{w}^{IN}(J_1, J_2) \geq 0 \)

ii). \( CE_{w}^{IN}(J_1, J_2) = 0 \), if and only if \( T_{J_1}(u_i) = T_{J_2}(u_i) \), \( F_{J_1}(u_i) = F_{J_2}(u_i) \), for all \( u_i \in U \)

iii). \( CE_{w}^{IN}(J_1, J_2) = CE_{w}^{IN}(J_1', J_2') \)

iv). \( CE_{w}^{IN}(J_1, J_2) = CE_{w}^{IN}(J_2, J_1) \)

Proof:

i). For all values of \( u_i \in U \), \( [T_{J_1}(u_i)] \geq 0 \), \( [F_{J_2}(u_i)] \geq 0 \),

\[
\frac{2[F_{J_1}(u_i) - T_{J_2}(u_i)]}{\sqrt{1 + [F_{J_1}(u_i)]^2 + [T_{J_2}(u_i)]^2}} \geq 0,
\]

\[
\frac{2[1 - F_{J_1}(u_i) - T_{J_2}(u_i)]}{\sqrt{1 + [1 - F_{J_1}(u_i)]^2 + [1 - T_{J_2}(u_i)]^2}} \geq 0.
\]

\[
= \frac{2[F_{J_1}(u_i) - T_{J_2}(u_i)]}{\sqrt{1 + [F_{J_1}(u_i)]^2 + [T_{J_2}(u_i)]^2}} + \frac{2[1 - F_{J_1}(u_i) - T_{J_2}(u_i)]}{\sqrt{1 + [1 - F_{J_1}(u_i)]^2 + [1 - T_{J_2}(u_i)]^2}} \geq 0
\]

and \( T_{J_1}^*(u_i) \geq 0 \), \( T_{J_2}^*(u_i) \geq 0 \), \( [T_{J_1}(u_i)] - [T_{J_2}(u_i)] \geq 0 \),

\[
\frac{1 + [T_{J_1}(u_i)]}{\sqrt{1 + [F_{J_2}(u_i)]^2}} \geq 0,
\]

\[
\frac{1 - [T_{J_2}(u_i)]}{\sqrt{1 + [1 - F_{J_2}(u_i)]^2}} \geq 0.
\]

\[
= \frac{2[F_{J_1}(u_i) - T_{J_2}(u_i)]}{\sqrt{1 + [F_{J_1}(u_i)]^2 + [T_{J_2}(u_i)]^2}} + \frac{2[1 - F_{J_1}(u_i) - T_{J_2}(u_i)]}{\sqrt{1 + [1 - F_{J_1}(u_i)]^2 + [1 - T_{J_2}(u_i)]^2}} \geq 0.
\]

Similarly, we can show that

\[
\frac{2[F_{J_2}(u_i) - T_{J_1}(u_i)]}{\sqrt{1 + [F_{J_1}(u_i)]^2 + [T_{J_2}(u_i)]^2}} + \frac{2[1 - F_{J_2}(u_i) - T_{J_1}(u_i)]}{\sqrt{1 + [1 - F_{J_1}(u_i)]^2 + [1 - T_{J_2}(u_i)]^2}} \geq 0.
\]

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and
\[
\begin{bmatrix}
\frac{1}{\sqrt{1+T_{11}(u_i)^2 + \sqrt{1+T_{12}(u_i)^2}}} & \frac{2}{1+T_{11}(u_i)^2 - 1 - T_{12}(u_i)^2}
\end{bmatrix}
\geq 0
\]
Since \( w_i \in [0,1], \sum_{i=1}^{n} w_i = 1 \), we have, \( CE_{IN}(I_1, J_2) \geq 0 \). Hence complete the proof.

ii).
\[
\begin{bmatrix}
\frac{1}{\sqrt{1+T_{11}(u_i)^2 + \sqrt{1+T_{12}(u_i)^2}}} & \frac{2}{1+T_{11}(u_i)^2 - 1 - T_{12}(u_i)^2}
\end{bmatrix}
\geq 0
\]
\( \Rightarrow T_{11}(u_i) = T_{12}(u_i) \)
\[
\begin{bmatrix}
\frac{1}{\sqrt{1+T_{11}(u_i)^2 + \sqrt{1+T_{12}(u_i)^2}}} & \frac{2}{1+T_{11}(u_i)^2 - 1 - T_{12}(u_i)^2}
\end{bmatrix}
\geq 0
\]
\( \Rightarrow I_1(u_i) = I_2(u_i) \)
\[
\begin{bmatrix}
\frac{1}{\sqrt{1+T_{11}(u_i)^2 + \sqrt{1+T_{12}(u_i)^2}}} & \frac{2}{1+T_{11}(u_i)^2 - 1 - T_{12}(u_i)^2}
\end{bmatrix}
\geq 0
\]
\( \Rightarrow F_1(u_i) = F_2(u_i) \)
\[
\begin{bmatrix}
\frac{1}{\sqrt{1+T_{11}(u_i)^2 + \sqrt{1+T_{12}(u_i)^2}}} & \frac{2}{1+T_{11}(u_i)^2 - 1 - T_{12}(u_i)^2}
\end{bmatrix}
\geq 0
\]
For all values of \( u_i \in U \).
Since, \( w_i \in [0,1], \sum_{i=1}^{n} w_i = 1 \), we can show that
\[
CE_{IN}(I_1, J_2) = 0 \iff T_{11}(u_i) = T_{12}(u_i), \ I_1(u_i) = I_2(u_i), \ F_1(u_i) = F_2(u_i), \ T_1(u_i) = T_2(u_i) \) for all \( u_i \in U \).

iii).
Using definition (2.4), we obtain the following expression:
Since, \( I_1(u_i) - I_2(u_i) = I_2(u_i) - I_1(u_i) \),
\( F_1(u_i) - F_2(u_i) = F_2(u_i) - F_1(u_i) \),
\( (1^{-T_1_1(u_i)}) - (1^{-T_2_1(u_i)}) = (1^{-T_2_1(u_i)}) - (1^{-T_1_1(u_i)}) \),
\( (1^{-F_1_1(u_i)}) - (1^{-F_2_1(u_i)}) = (1^{-F_2_1(u_i)}) - (1^{-F_1_1(u_i)}) \).

Then, we obtain
\[
\begin{align*}
1 + (1^{-T_1_1(u_i)}) + 1 + (1^{-T_2_1(u_i)}) = 1 + (1^{-F_1_1(u_i)}) + 1 + (1^{-F_2_1(u_i)}) = 1 + (1^{-T_2_1(u_i)}) + 1 + (1^{-T_1_1(u_i)}) = 1 + (1^{-F_2_1(u_i)}) + 1 + (1^{-F_1_1(u_i)}) = \sqrt{1 + (1^{-F_1_1(u_i)}) + 1 + (1^{-F_2_1(u_i)}) + \sqrt{1 + (1^{-F_1_1(u_i)}) + 1 + (1^{-F_2_1(u_i)})} ,
\end{align*}
\]
\[ \forall u_i \in U. \]

Similarly,
\[
\begin{align*}
\sqrt{1 + (1^{-F_1_1(u_i)}) + 1 + (1^{-F_2_1(u_i)})} = \sqrt{1 + (1^{-F_1_1(u_i)}) + 1 + (1^{-F_2_1(u_i)})} ,
\end{align*}
\]
\[ \forall u_i \in U. \]

So, \( CE_{IN}^m(J_1, J_2) = CE_{IN}^m(J_2, J_1). \) Hence complete the proof.

4. Multi attribute group decision making strategy using IN-cross entropy measure in interval neutrosophic set environment

In this section we develop a novel MAGDM strategy based on proposed IN- cross entropy measure.

The MAGDM problem can be consider as follows:
Let \( A = \{ A_1, A_2, A_3, \ldots, A_n \} \) and \( G = \{ G_1, G_2, G_3, \ldots, G_n \} \) be the discrete set of alternatives and attribute respectively. Let \( W = \{ w_1, w_2, w_3, \ldots, w_n \} \) be the weight vector of attributes \( G_j \) \((j = 1, 2, 3, \ldots, n)\), where \( w_j \geq 0 \) and \( \sum_{j=1}^{n} w_j = 1 \). Let \( E = \{ E_1, E_2, E_3, \ldots, E_p \} \) be the set of decision makers who are employ to evaluate the alternative. The weight vector of the decision makers \( E_k \) \((k = 1, 2, 3, \ldots, p) \) is \( \lambda = (\lambda_1, \lambda_2, \ldots, \lambda_p) \) (where, \( \lambda \geq 0 \) and \( \sum_{k=1}^{p} \lambda_k = 1 \)), which can be determined according to the decision makers expertise, judgment quality and decision making knowledge.
Now, we describe the steps of the proposed MAGDM strategy (See Figure 1.) using weighted IN-cross entropy measure.

**MAGDM strategy using IN-cross entropy measure**

**Step 1. Formulate the decision matrices**

For MAGDM with INSs information, the rating values of the alternatives $A_i$ ($i=1, 2, 3, \ldots, m$) on the basis of criteria $G_j$ ($j=1, 2, 3, \ldots, n$) by the k-th decision maker can be expressed in INN as $a_{ik} = ([T_{i1}^k, T_{i2}^k], [T_{i3}^k, T_{i4}^k], \ldots, [T_{in}^k, T_{in}^k])$ ($i = 1, 2, 3, \ldots, m; j = 1, 2, 3, \ldots, n; k = 1, 2, 3, \ldots, p$). We arrange these rating values of alternatives provided by the decision makers in matrix form as follows:

$$
M^k = \begin{pmatrix}
G_1 & G_2 & \ldots & G_n \\
A_1 & a_{11} & a_{12} & \ldots & a_{1n} \\
A_2 & a_{21} & a_{22} & \ldots & a_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_m & a_{m1} & a_{m2} & \ldots & a_{mn}
\end{pmatrix} \quad \text{…………(3)}
$$

**Step 2. Formulate the weighted aggregated decision matrix**

For obtaining one group decision, we aggregate all individual decision matrices ($M^k$) to an aggregated decision matrix ($M$) using interval-valued neutrosophic weighted averaging (INNWA) operator ([72]) as follows:

$$
\begin{align*}
\lambda_1 a_{ij} \oplus \lambda_2 a_{ij} \oplus \lambda_3 a_{ij} \oplus \cdots \oplus \lambda_p a_{ij} &= \\
\left< [1-\prod_{k=1}^p (1-T_{ij1}^k), 1-\prod_{k=1}^p (1-T_{ij2}^k)], [\prod_{k=1}^p (1-T_{ij1}^k)^{\lambda_1}, \prod_{k=1}^p (1-T_{ij2}^k)^{\lambda_1}] \right>
\end{align*}
$$

$$
\left< [1-\prod_{k=1}^p (1-T_{ij1}^k), 1-\prod_{k=1}^p (1-T_{ij2}^k)], [\prod_{k=1}^p (1-T_{ij1}^k)^{\lambda_1}, \prod_{k=1}^p (1-T_{ij2}^k)^{\lambda_1}] \right>
$$

(4)

$$
(i = 1, 2, 3, \ldots, m; j = 1, 2, 3, \ldots, n; k = 1, 2, 3, \ldots, p).
$$

Therefore, the aggregated decision matrix is defined as follows:

$$
M = \begin{pmatrix}
G_1 & G_2 & \ldots & G_n \\
A_1 & a_{11} & a_{12} & \ldots & a_{1n} \\
A_2 & a_{21} & a_{22} & \ldots & a_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_m & a_{m1} & a_{m2} & \ldots & a_{mn}
\end{pmatrix} \quad \text{…………(5)}
$$

**Step 3. Formulate priori/ ideal decision matrix**

In the MAGDM processes, the priori decision matrix is used to select the best alternatives among the set of collected feasible alternatives. In this decision making processes we use the following decision matrix as prior decision matrix.

$$
P = \begin{pmatrix}
G_1 & G_2 & \ldots & G_n \\
A_1 & a_{11} & a_{12} & \ldots & a_{1n} \\
A_2 & a_{21} & a_{22} & \ldots & a_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_m & a_{m1} & a_{m2} & \ldots & a_{mn}
\end{pmatrix}
$$

**Step 4. Formulate the weighted IN-cross entropy matrix**

Using equation (2), we calculate weighted cross entropy value between aggregate matrix and priori matrix. The cross entropy value can be present in matrix form as follows:

$$
\text{IN} M^w = \begin{pmatrix}
\text{CE}_{IN}^w (A_1) \\
\text{CE}_{IN}^w (A_2) \\
\vdots \\
\text{CE}_{IN}^w (A_m)
\end{pmatrix}
$$

(7)

**Step 5. Rank the priority**

Smaller value of the cross entropy reflect that an alternative is closer to the ideal alternative. Therefore, the priority order of all the alternatives can be determined according to the increasing order of the cross entropy values $\text{CE}_{IN}^w (A_i)$ ($i = 1, 2, 3, \ldots, m$). Smallest cross entropy value indicates the best alternative and greatest cross entropy value indicates the worst alternative.
5. Illustrative example

In this section, we provide an illustrative example of MAGDM problems to reflect the validity and efficiency of our proposed strategy under INSs environment.

Now, we solve an illustrative example adapted from [9] for cultivation and analysis. A venture capital firm intends to make evaluation and selection to five enterprises with the investment potential:

1) Automobile company ($A_1$)
2) Military manufacturing enterprise ($A_2$)
3) TV media company ($A_3$)
4) Food enterprises ($A_4$)
5) Computer software company ($A_5$)

On the basis of four attributes namely:

1) Social and political factor ($G_1$)
2) The environmental factor ($G_2$)
3) Investment risk factor ($G_3$)
4) The enterprise growth factor ($G_4$).

The investment firm makes a panel of three decision makers $E = \{E_1, E_2, E_3\}$ having their weights vector...
\( \lambda = \{0.42, 0.28, 0.30\} \) and weight vector of attributes is \( W = \{0.24, 0.25, 0.23, 0.28\} \).

The steps of decision making strategy to rank alternatives are presented below:

**Step: 1. Formulate the decision matrices**

Using equation (4), the aggregated decision matrix is presented below:

\[
M^2 = \begin{bmatrix}
G_1 & G_2 & G_3 & G_4 \\
A_1 & 1,1,2,3,4 & 1,6,7,3,4 & 1,6,7,3,4 & 1,6,7,3,4 \\
A_2 & 1,6,7,3,4 & 1,6,7,3,4 & 1,6,7,3,4 & 1,6,7,3,4 \\
A_3 & 1,6,7,3,4 & 1,6,7,3,4 & 1,6,7,3,4 & 1,6,7,3,4 \\
A_4 & 1,6,7,3,4 & 1,6,7,3,4 & 1,6,7,3,4 & 1,6,7,3,4 \\
A_5 & 1,6,7,3,4 & 1,6,7,3,4 & 1,6,7,3,4 & 1,6,7,3,4 \\
\end{bmatrix} ^{\ldots}(8)
\]

**Step: 2. Formulate the weighted aggregated decision matrix**

Using equation (4), the aggregated decision matrix is presented below:

\[
M = \begin{bmatrix}
G_1 & G_2 & G_3 & G_4 \\
A_1 & 1,6,8,3,4 & 1,6,8,3,4 & 1,6,8,3,4 & 1,6,8,3,4 \\
A_2 & 1,6,8,3,4 & 1,6,8,3,4 & 1,6,8,3,4 & 1,6,8,3,4 \\
A_3 & 1,6,8,3,4 & 1,6,8,3,4 & 1,6,8,3,4 & 1,6,8,3,4 \\
A_4 & 1,6,8,3,4 & 1,6,8,3,4 & 1,6,8,3,4 & 1,6,8,3,4 \\
A_5 & 1,6,8,3,4 & 1,6,8,3,4 & 1,6,8,3,4 & 1,6,8,3,4 \\
\end{bmatrix} ^{\ldots}(11)
\]

**Step: 3. Formulate priori/ ideal decision matrix**

Priori/ ideal decision matrix

\[
M^4 = \begin{bmatrix}
G_1 & G_2 & G_3 & G_4 \\
A_1 & 1,1,1,0,0,0 & 1,1,1,0,0,0 & 1,1,1,0,0,0 & 1,1,1,0,0,0 \\
A_2 & 1,1,1,0,0,0 & 1,1,1,0,0,0 & 1,1,1,0,0,0 & 1,1,1,0,0,0 \\
A_3 & 1,1,1,0,0,0 & 1,1,1,0,0,0 & 1,1,1,0,0,0 & 1,1,1,0,0,0 \\
A_4 & 1,1,1,0,0,0 & 1,1,1,0,0,0 & 1,1,1,0,0,0 & 1,1,1,0,0,0 \\
A_5 & 1,1,1,0,0,0 & 1,1,1,0,0,0 & 1,1,1,0,0,0 & 1,1,1,0,0,0 \\
\end{bmatrix} ^{\ldots}(12)
\]

**Step: 4. Calculate the weighted IN-cross entropy matrix**

Using equation (2), we calculate the interval neutrosophic weighted cross entropy values between ideal matrixes (12) and weighted aggregated decision matrix (11).
Step: 5. Rank the priority
The position of cross entropy values of alternatives arranging in increasing order is 0.77 < 0.78 < 0.86 < 0.90 < 0.95. Since, smallest values of cross entropy indicate the alternative is closer to the ideal alternative. Thus the ranking priority of alternatives is \( A_2 > A_3 > A_1 > A_5 > A_4 \). Hence, military manufacturing enterprise \( (A_2) \) is the best alternative for investment.

In Figure 2, we draw a bar diagram to represent the cross entropy values of alternatives which shows that \( A_2 \) is the best alternative according to our proposed strategy.

![Figure 2. Bar diagram of alternatives versus cross entropy values of alternatives](image)

2. Conclusion
In this paper we have defined IN-cross entropy measure in INS environment which is free from all the drawback of existence cross entropy measures under interval neutrosophic set environment. We have proved the basic properties of the cross entropy measures. We have also defined weighted IN-cross entropy measure and proved its basic properties. Based on the weighted IN-cross entropy measure, we have proposed a novel MAGDM strategy. Finally, we solve a MAGDM problem to show the feasibility and efficiency of the proposed MAGDM making strategy. The proposed IN-cross entropy based MAGDM strategy can be employed to solve a variety of problems such as logistics center selection, teacher selection, renewable energy selection, fault diagnosis, etc.

References


Received: November 15, 2017. Accepted: December 4, 2017.