On Neutrosophic Semi Alpha Open Sets

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Abstract. In this paper, we presented another concept of neutrosophic open sets called neutrosophic semi-α-open sets and studied their fundamental properties in neutrosophic topological spaces. We also present neutrosophic semi-α-interior and neutrosophic semi-α-closure and study some of their fundamental properties.

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1. Introduction

In 2000, G.B. Navalagi [4] presented the idea of semi-α-open sets in topological spaces. The concept of "neutrosophic set" was first given by F. Smarandache [2,3]. A.A. Salama and S.A. Alblowi [1] presented the concept of neutrosophic topological space (briefly NTS). The objective of this paper is to present the concept of neutrosophic semi-α-open sets and study their fundamental properties in neutrosophic topological spaces. We also present neutrosophic semi-α-interior and neutrosophic semi-α-closure and obtain some of its properties.

2. Preliminaries

Throughout this paper, (U, T) (or simply U) always mean a neutrosophic topological space. The complement of a neutrosophic open set (briefly N-OS) is called a neutrosophic closed set (briefly N-CS) in (U, T). For a neutrosophic set A in a neutrosophic topological space (U, T), Ncl(A), Nint(A) and Aᶜ denote the neutrosophic closure of A, the neutrosophic interior of A and the neutrosophic complement of A respectively.

Definition 2.1:
A neutrosophic subset A of a neutrosophic topological space (U, T) is said to be:
(i) A neutrosophic pre-open set (briefly NP-OS) [7] if A ⊆ Nint(Ncl(A)). The complement of a NP-OS is called a neutrosophic pre-closed set (briefly NP-CS) in (U, T).

(ii) A neutrosophic semi-open set (briefly NS-OS) [6] if A ⊆ Ncl(Nint(A)). The complement of a NS-OS is called a neutrosophic semi-closed set (briefly NS-CS) in (U, T).

(iii) A neutrosophic α-open set (briefly Na-OS) [5] if A ⊆ Nint(Ncl(Nint(A))). The complement of a Na-OS is called a neutrosophic α-closed set (briefly Na-CS) in (U, T).

Definition 2.2:
(i) The neutrosophic pre-interior of a neutrosophic set A of a neutrosophic topological space (U, T) is the union of all NP-OS contained in A and is denoted by PNint(A)[7].

(ii) The neutrosophic semi-interior of a neutrosophic set A of a neutrosophic topological space (U, T) is the union of all NS-OS contained in A and is denoted by SNint(A)[6].

(iii) The neutrosophic α-interior of a neutrosophic set A of a neutrosophic topological space (U, T) is the union of all Na-OS contained in A and is denoted by aNint(A)[5].

Definition 2.3:
(i) The neutrosophic pre-closure of a neutrosophic set A of a neutrosophic topological space (U, T) is the intersection of all NP-CS that contain A and is denoted by PNcl(A)[7].

(ii) The neutrosophic semi-closure of a neutrosophic set A of a neutrosophic topological space (U, T) is the
intersection of all NS-CS that contain $\mathcal{A}$ and is denoted by $S\text{NcI}(\mathcal{A})$[6].

(iii) The neutrosophic $\alpha$-closure of a neutrosophic set $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U}, \mathcal{T})$ is the intersection of all Na-CS that contain $\mathcal{A}$ and is denoted by $\alpha\text{NcI}(\mathcal{A})$[5].

**Proposition 2.4** [5]:

In a neutrosophic topological space $(\mathcal{U}, \mathcal{T})$, the following statements hold, and the equality of each statement are not true:

(i) Every N-OS (resp. N-CS) is a Na-OS (resp. Na-CS).
(ii) Every Na-OS (resp. Na-CS) is a NS-OS (resp. NS-CS).
(iii) Every Na-OS (resp. Na-CS) is a NP-OS (resp. NP-CS).

**Proposition 2.5** [5]:

A neutrosophic subset $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U}, \mathcal{T})$ is a Na-OS if $\mathcal{A}$ is an N-OS and NP-OS.

**Lemma 2.6:**

(i) If $\mathcal{K}$ is a N-OS, then $S\text{NcI}(\mathcal{K}) = N\text{Int}(\text{NcI}(\mathcal{K}))$.

(ii) If $\mathcal{A}$ is a neutrosophic subset of a neutrosophic topological space $(\mathcal{U}, \mathcal{T})$, then $S\text{NcI}(\text{NcI}(\mathcal{A})) = N\text{cI}(\text{NcI}(N\text{Int}(\mathcal{A})))$.

**Proof:** This follows directly from the definition 2.1 and proposition (2.4).

3. Neutrosophic Semi-$\alpha$-Open Sets

In this section, we present and study the neutrosophic semi-$\alpha$-open sets and some of its properties.

**Definition 3.1:**

A neutrosophic subset $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U}, \mathcal{T})$ is called neutrosophic semi-$\alpha$-open set (briefly N$\alpha$-OS) if there exists a Na-$\alpha$-OS $\mathcal{K}$ in $\mathcal{U}$ such that $\mathcal{K} \subseteq \mathcal{A} \subseteq \text{NcI}(\mathcal{K})$ or equivalently if $\mathcal{A} \subseteq \text{NcI}(\alpha\text{NcI}(\mathcal{K}))$. The family of all N$\alpha$-OS of $\mathcal{U}$ is denoted by $\text{N} \alpha \text{O}(\mathcal{U})$.

**Definition 3.2:**

The complement of N$\alpha$-OS is called a neutrosophic semi-$\alpha$-closed set (briefly NS$\alpha$-CS). The family of all NS$\alpha$-CS of $\mathcal{U}$ is denoted by $\text{NS} \alpha \text{C}(\mathcal{U})$.

**Proposition 3.3:**

It is evident by definitions that in a neutrosophic topological space $(\mathcal{U}, \mathcal{T})$, the following hold:

(i) Every N-OS (resp. N-CS) is a NS$\alpha$-OS (resp. NS$\alpha$-CS).
(ii) Every Na-OS (resp. Na-CS) is a N$\alpha$-OS (resp. NSA-OS).

The converse of the above proposition need not be true as seen from the following example.

**Example 3.4:**

Let $\mathcal{U} = \{u\}, \mathcal{A} = \{(u, 0.5, 0.5, 0.4) : u \in \mathcal{U}\}$,

$\mathcal{B} = \{(u, 0.4, 0.5, 0.8) : u \in \mathcal{U}\}, \mathcal{C} = \{(u, 0.5, 0.6, 0.4) : u \in \mathcal{U}\}$.

Then $T = \{0, \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}\}$ is a neutrosophic topology on $\mathcal{U}$.

(i) Let $\mathcal{K} = \{(u, 0.5, 0.1, 0.3) : u \in \mathcal{U}\}, \mathcal{A} \subseteq \mathcal{K} \subseteq \text{NcI}(\mathcal{A}) = \{(u, 0.6, 0.4, 0.2)\}$, the neutrosophic set $\mathcal{K}$ is a N$\alpha$-OS but is not N-OS. It is clear that $\mathcal{K}^c = \{(u, 0.5, 0.9, 0.7) : u \in \mathcal{U}\}$ is a N$\alpha$-CS but not N-OS.

(ii) Let $\mathcal{K} = \{(u, 0.5, 0.1, 0.2) : u \in \mathcal{U}\}, \mathcal{A} \subseteq \mathcal{K} \subseteq \text{NcI}(\mathcal{A}) = \{(u, 0.6, 0.4, 0.2)\}$, the neutrosophic set $\mathcal{K}$ is a N$\alpha$-OS, $\mathcal{K} \subseteq \text{NcI}(\text{NcI}(\mathcal{K})) = \text{NcI}(\text{NcI}(\mathcal{K})) = \text{NcI}(\mathcal{K}) = \{(u, 0.5, 0.5, 0.4)\}$, the neutrosophic set $\mathcal{K}$ is not N$\alpha$-OS. It is clear that $\mathcal{K}^c = \{(u, 0.5, 0.9, 0.8) : u \in \mathcal{U}\}$ is a N$\alpha$-CS but is not N$\alpha$-CS.

**Remark 3.5:**

The concepts of N$\alpha$-OS and NP-OS are independent, as the following examples shows.

**Example 3.6:**

In example (3.4), the neutrosophic set $\mathcal{K} = \{(u, 0.5, 0.1, 0.3) : u \in \mathcal{U}\}$ is a N$\alpha$-OS but is not NP-OS, because $\mathcal{K} \subseteq \text{NcI}(\text{NcI}(\mathcal{K})) = \text{NcI}(\text{NcI}(\mathcal{K})) = \text{NcI}(\mathcal{K}) = \{(u, 0.6, 0.4, 0.2)\} = \{(u, 0.5, 0.5, 0.4)\}$.

**Example 3.7:**

Let $\mathcal{U} = \{(a, b)\}, \mathcal{A} = \{(0.4, 0.8, 0.9), (0.7, 0.5, 0.3)\}, \mathcal{B} = \{(0.5, 0.8, 0.6), (0.8, 0.4, 0.3)\}, \mathcal{C} = \{(0.4, 0.7, 0.9) , (0.6, 0.4, 0.4)\}, \mathcal{D} = \{(0.5, 0.7, 0.5), (0.8, 0.4, 0.6)\}$.

Then $T = \{0, \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}\}$ is a neutrosophic topology on $\mathcal{U}$.

Then the neutrosophic set $\mathcal{K} = \{(1, 1, 0.3 ), (0.7, 0.3, 0.6)\}$ is a NP-OS but is not N$\alpha$-OS.

**Remark 3.8:**

(i) If every N-OS is a N-CS and every nowhere neutrosophic dense set is N-CS in any neutrosophic topological space $(\mathcal{U}, \mathcal{T})$, then every N$\alpha$-OS is a N-OS.

(ii) If every N-OS is a N-CS in any neutrosophic topological space $(\mathcal{U}, \mathcal{T})$, then every N$\alpha$-OS is a N$\alpha$-OS.

**Remark 3.9:**

(i) It is clear that every NS-OS and NP-OS of any neutrosophic topological space $(\mathcal{U}, \mathcal{T})$ is a NS$\alpha$-OS (by proposition (2.5) and proposition (3.3) (ii)).

(ii) A NS$\alpha$-OS in any neutrosophic topological space $(\mathcal{U}, \mathcal{T})$ is a NP-OS if every N-OS of $\mathcal{U}$ is a N-CS (from proposition (2.4) (iii) and remark (3.8) (ii)).

**Theorem 3.10:**

For any neutrosophic subset $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U}, \mathcal{T})$, $\mathcal{A} \in \text{NaO}(\mathcal{U})$ if there exists a N-OS $\mathcal{K}$ such that $\mathcal{K} \subseteq \mathcal{A} \subseteq \text{NcI}(\text{NcI}(\mathcal{K}))$. 

Proof: Let \( A \) be a Ne-OS. Hence \( A \subseteq Nint(Ncl(Nint(A))) \), so let \( U = Nint(A) \), we get \( Nint(A) \subseteq A \subseteq Nint(Ncl(Nint(A))) \). Then there exists a N-OS \( A \) such that \( U \subseteq A \subseteq Nint(Ncl(U)) \), where \( U = Nint(A) \).
Conversely, suppose that there is a N-OS \( H \) such that \( H \subseteq A \subseteq Nint(Ncl(H)) \).
To prove \( A \subseteq \overline{N} \). \( \overline{N} \subseteq \overline{N} \) (since \( \overline{N} \) is the largest N-OS contained in \( A \)).
Hence \( Ncl(U) \subseteq \overline{N} \subseteq Ncl(Ncl(Nint(A))) \) (by theorem (3.10)). Therefore, \( Ncl(Ncl(Nint(A))) \subseteq Ncl(Ncl(Ncl(Nint(A)))) \), implies that \( Ncl(U) \subseteq Ncl(Ncl(Ncl(Ncl(Nint(A)))))) \). Then \( H \subseteq K \subseteq A \subseteq N \subseteq Ncl(Ncl(Ncl(Ncl(Ncl(U))))) \).
To prove \( A \subseteq \overline{N} \) for some \( N \)-OS.

(i) \( A \subseteq \overline{N} \) (since \( \overline{N} \) is the largest N-OS contained in \( A \)).

(ii) Suppose that there exists a N-OS \( H \) such that \( H \subseteq A \subseteq Ncl(Ncl(U)) \) (by theorem (3.10)). Therefore, \( Ncl(Ncl(Nint(A))) \subseteq Ncl(Ncl(Ncl(U))) \) (by hypothesis). Hence \( A \subseteq \overline{N} \) (since \( \overline{N} \) is the largest N-OS contained in \( A \)).

(iii) \( A \subseteq \overline{N} \) (since \( \overline{N} \) is the largest N-OS contained in \( A \)).

Therefore, \( A \subseteq \overline{N} \) (since \( \overline{N} \) is the largest N-OS contained in \( A \)).

Theorem 3.11:
For any neutrosophic subset \( A \) of a neutrosophic topological space \( (U,T) \). The following properties are equivalent:
(i) \( A \subseteq \overline{N} \) (since \( \overline{N} \) is the largest N-OS contained in \( A \)).
(ii) There exists a N-OS \( H \) such that \( H \subseteq A \subseteq Ncl(Ncl(U)) \).
(iii) \( A \subseteq \overline{N} \) (since \( \overline{N} \) is the largest N-OS contained in \( A \)).

Proof:
(i) \( \Rightarrow \) (ii) Let \( A \subseteq \overline{N} \) (since \( \overline{N} \) is the largest N-OS contained in \( A \)).
Hence \( Ncl(U) \subseteq \overline{N} \subseteq Ncl(Ncl(U)) \) (by theorem (3.10)). Therefore, \( Ncl(Ncl(U)) \subseteq Ncl(Ncl(Ncl(U))) \) (by hypothesis).
Therefore, \( A \subseteq Ncl(Ncl(Ncl(U))) \).

Proposition 3.13:
The union of any family of Ne-OS is a Ne-OS.
Proof: Let \( \{ A_i \}_{i \in I} \) be a family of Ne-OS of \( U \). To prove \( \bigcup_{i \in I} A_i \) is a Ne-OS.
Therefore, \( \bigcup_{i \in I} A_i \subseteq Ncl(Nint(U,A_i)) \).
Then \( A_i \subseteq Ncl(Nint(U,A_i)) \), \( \forall i \). Since \( \bigcup_{i \in I} A_i \subseteq Ncl(Nint(U,A_i)) \) and \( \bigcup_{i \in I} A_i \subseteq Ncl(U,A_i) \) hold for any neutrosophic topology.
We have \( \bigcup_{i \in I} A_i \subseteq Ncl(Nint(U,A_i)) \).
Hence \( \bigcup_{i \in I} A_i \) is a Ne-OS.

Theorem 3.14:
The union of any family of Ne-OS is a Ne-OS.
Proof: Let \( \{ A_i \}_{i \in I} \) be a family of Ne-OS. To prove \( \bigcup_{i \in I} A_i \) is a Ne-OS.
Then there is a Ne-OS \( B_i \) such that \( B_i \subseteq A_i \subseteq Ncl(B_i) \), \( \forall i \). Hence \( \bigcup_{i \in I} B_i \subseteq \bigcup_{i \in I} A_i \subseteq \bigcup_{i \in I} Ncl(B_i) \). Therefore, \( \bigcup_{i \in I} B_i \subseteq Ncl(\bigcup_{i \in I} A_i) \).
Hence \( \bigcup_{i \in I} A_i \) is a Ne-OS.
Corollary 3.15:
The intersection of any family of $\text{NS}_\alpha$-CS is a $\text{NS}_\alpha$-CS. 
Proof: This follows directly from the theorem (3.14).

Remark 3.16:
The following diagram shows the relations among the different types of weakly neutrosophic open sets that were studied in this section: 

4. Neutrosophic Semi-\(\alpha\)-Interior and Neutrosophic Semi-\(\alpha\)-Closure

We present neutrosophic semi-\(\alpha\) -interior and neutrosophic semi-\(\alpha\) -closure and obtain some of its properties in this section.

Definition 4.1:
The union of all $\text{NS}_\alpha$-OS in a neutrosophic topological space $(\mathcal{U}, \mathcal{T})$ contained in $\mathcal{A}$ is called neutrosophic semi-\(\alpha\) -interior of $\mathcal{A}$ and is denoted by $\text{SaNint}(\mathcal{A})$, $\text{SaNint}(\mathcal{A}) = \bigcup \{ B : B \subseteq \mathcal{A}, B \text{ is a } \text{NS}_\alpha\text{-OS} \}$.

Definition 4.2:
The intersection of all $\text{NS}_\alpha$ - CS in a neutrosophic topological space $(\mathcal{U}, \mathcal{T})$ containing $\mathcal{A}$ is called neutrosophic semi-\(\alpha\) - closure of $\mathcal{A}$ and is denoted by $\text{SaNcl}(\mathcal{A})$, $\text{SaNcl}(\mathcal{A}) = \bigcap \{ B : B \subseteq \mathcal{A}, B \text{ is a } \text{NS}_\alpha\text{-CS} \}$.

Proposition 4.3:
Let $\mathcal{A}$ be any neutrosophic set in a neutrosophic topological space $(\mathcal{U}, \mathcal{T})$, the following properties are true: 
(i) $\text{SaNint}(\mathcal{A}) = \mathcal{A}$ iff $\mathcal{A}$ is a $\text{NS}_\alpha$-OS.
(ii) $\text{SaNcl}(\mathcal{A}) = \mathcal{A}$ iff $\mathcal{A}$ is a $\text{NS}_\alpha$-CS.
(iii) $\text{SaNint}(\mathcal{A})$ is the largest $\text{NS}_\alpha$-OS contained in $\mathcal{A}$.
(iv) $\text{SaNcl}(\mathcal{A})$ is the smallest $\text{NS}_\alpha$-CS containing $\mathcal{A}$.
Proof: (i), (ii), (iii) and (iv) are obvious.

Proposition 4.4:
Let $\mathcal{A}$ be any neutrosophic set in a neutrosophic topological space $(\mathcal{U}, \mathcal{T})$, the following properties are true:
(i) $\text{SaNint}(1_\mathcal{N} - \mathcal{A}) = 1_\mathcal{N} - (\text{SaNcl}(\mathcal{A}))$.
(ii) $\text{SaNcl}(1_\mathcal{N} - \mathcal{A}) = 1_\mathcal{N} - (\text{SaNint}(\mathcal{A}))$.
Proof: (i) By definition, $\text{SaNcl}(\mathcal{A}) = \bigcap \{ B : \mathcal{A} \subseteq B, B \text{ is a } \text{NS}_\alpha\text{-CS} \}$.
$1_\mathcal{N} - (\text{SaNcl}(\mathcal{A})) = 1_\mathcal{N} - \bigcap \{ B : \mathcal{A} \subseteq B, B \text{ is a } \text{NS}_\alpha\text{-CS} \}$
$= \bigcup \{ 1_\mathcal{N} - B : \mathcal{A} \subseteq B, B \text{ is a } \text{NS}_\alpha\text{-CS} \}$
$= \bigcup \{ \mathcal{H} : \mathcal{H} \subseteq 1_\mathcal{N} - \mathcal{A}, \mathcal{H} \text{ is a } \text{NS}_\alpha\text{-OS} \}$
$= \text{SaNint}(1_\mathcal{N} - \mathcal{A})$.
(ii) The proof is similar to (i).

Theorem 4.5:
Let $\mathcal{A}$ and $\mathcal{B}$ be two neutrosophic sets in a neutrosophic topological space $(\mathcal{U}, \mathcal{T})$. The following properties hold:
(i) $\text{SaNint}(0_\mathcal{N}) = 0_\mathcal{N}, \text{SaNint}(1_\mathcal{N}) = 1_\mathcal{N}$.
(ii) $\mathcal{A} \subseteq \text{SaNcl}(\mathcal{A})$.
(iii) $\mathcal{A} \subseteq \mathcal{B} \Rightarrow \text{SaNint}(\mathcal{A}) \subseteq \text{SaNint}(\mathcal{B})$.
(iv) $\text{SaNint}(\mathcal{A} \cap \mathcal{B}) \subseteq \text{SaNint}(\mathcal{A}) \cap \text{SaNint}(\mathcal{B})$.
(v) $\text{SaNint}(\mathcal{A}) \cup \text{SaNcl}(\mathcal{B}) \subseteq \text{SaNint}(\mathcal{A} \cup \mathcal{B})$.
(vi) $\text{SaNint}(\text{SaNcl}(\mathcal{A})) = \text{SaNcl}(\mathcal{A})$.
Proof: (i) and (ii) are evident.
(iii) By part (ii), $\mathcal{A} \subseteq \text{SaNcl}(\mathcal{B})$. Since $\mathcal{A} \subseteq \mathcal{B}$, we have $\mathcal{A} \subseteq \text{SaNcl}(\mathcal{B})$. But $\text{SaNcl}(\mathcal{B})$ is a $\text{NS}_\alpha$ - CS. Thus $\text{SaNcl}(\mathcal{B})$ is a $\text{NS}_\alpha$-CS containing $\mathcal{A}$. Since $\text{SaNcl}(\mathcal{A})$ is the smallest $\text{NS}_\alpha$-CS containing $\mathcal{A}$, we have $\text{SaNcl}(\mathcal{A}) \subseteq \text{SaNcl}(\mathcal{B})$. Hence, $\mathcal{A} \subseteq \mathcal{B} \Rightarrow \text{SaNint}(\mathcal{A}) \subseteq \text{SaNint}(\mathcal{B})$.
(iv) We know that $\mathcal{A} \cap \mathcal{B} \subseteq \mathcal{A}$ and $\mathcal{A} \cap \mathcal{B} \subseteq \mathcal{B}$.
Therefore, by part (iii), $\text{SaNcl}(\mathcal{A} \cap \mathcal{B}) \subseteq \text{SaNcl}(\mathcal{A})$ and $\text{SaNcl}(\mathcal{A} \cap \mathcal{B}) \subseteq \text{SaNcl}(\mathcal{B})$. Hence $\text{SaNcl}(\mathcal{A} \cap \mathcal{B}) \subseteq \text{SaNcl}(\mathcal{A}) \cap \text{SaNcl}(\mathcal{B})$.
(v) Since $\mathcal{A} \subseteq \mathcal{A} \cup \mathcal{B}$ and $\mathcal{B} \subseteq \mathcal{A} \cup \mathcal{B}$, it follows from part (iii) that $\text{SaNcl}(\mathcal{A}) \subseteq \text{SaNcl}(\mathcal{A} \cup \mathcal{B})$ and $\text{SaNcl}(\mathcal{B}) \subseteq \text{SaNcl}(\mathcal{A} \cup \mathcal{B})$.
(vi) Since $\text{SaNcl}(\mathcal{A})$ is a $\text{NS}_\alpha$-CS, we have by proposition (4.3) part (ii), $\text{SaNcl}(\text{SaNcl}(\mathcal{A})) = \text{SaNcl}(\mathcal{A})$.

Proposition 4.7:
For any neutrosophic subset $\mathcal{A}$ of a neutrosophic topological space $(\mathcal{U}, \mathcal{T})$, then:
(i) $\text{Nint}(A) \subseteq \text{aNint}(A) \subseteq \text{SaNint}(A) \subseteq \text{SaNcl}(A) \subseteq \text{Ncl}(A)$.
(ii) $\text{Nint}(\text{SaNint}(A)) = \text{SaNint}(\text{Nint}(A)) = \text{Nint}(A)$.
(iii) $\text{aNint}(\text{SaNint}(A)) = \text{SaNint}(\text{aNint}(A)) = \text{aNint}(A)$.
(iv) $\text{Ncl}(\text{SaNcl}(A)) = \text{SaNcl}(\text{Ncl}(A)) = \text{Ncl}(A)$.
(v) $\text{aNcl}(\text{SaNcl}(A)) = \text{SaNcl}(\text{aNcl}(A)) = \text{aNcl}(A)$.
(vi) $\text{SaNcl}(A) = A \cup \text{Nint}(\text{Ncl}(\text{SaNint}(A)))$.
(vii) $\text{SaNint}(A) = A \cap \text{Ncl}(\text{Nint}(\text{SaNint}(A)))$.
(viii) $\text{Nint}(\text{Ncl}(A)) \subseteq \text{SaNint}(\text{SaNcl}(A))$.

**Proof:** We shall prove only (ii), (iii), (iv), (vii) and (viii).

(ii) To prove $\text{Nint}(\text{SaNint}(A)) = \text{SaNint}(\text{Nint}(A)) = \text{Nint}(A)$. Since $\text{Nint}(A)$ is a N-OS, then $\text{Nint}(A)$ is a NSa-OS. Hence $\text{Nint}(A) = \text{SaNint}(\text{Nint}(A))$.

(by proposition 4.3). Therefore:

$\text{Nint}(A) = \text{SaNint}(\text{Nint}(A))$........................................(1)

Since $\text{Nint}(A) \subseteq \text{SaNint}(A)$ $\Rightarrow$ Nint$(\text{Ncl}(A)) \subseteq \text{Nint} \left( \text{SaNint}(A) \right)$ $\Rightarrow$ Nint$(\text{SaNint}(A)) \subseteq \text{Nint}(\text{Ncl}(A))$.

Also, $\text{SaNint}(A) \subseteq A$ $\Rightarrow$ $\text{SaNint}(\text{SaNint}(A)) \subseteq \text{Nint}(\text{Ncl}(A))$.

Hence:

$\text{Nint}(A) = \text{SaNint}(\text{Nint}(A))$........................................(2)

Therefore by (1) and (2), we get $\text{Nint}(\text{SaNint}(A)) = \text{SaNint}(\text{Nint}(A)) = \text{Nint}(A)$.

(iii) To prove $\text{aNint}(\text{SaNint}(A)) = \text{SaNint}(\text{aNint}(A)) = \text{aNint}(A)$. Since $\text{aNint}(A)$ is a N-OS, therefore $\text{aNint}(A)$ is a NSa-OS. Therefore by proposition 4.3:

$\text{aNint}(A) = \text{SaNint}(\text{aNint}(A))$........................................(1)

Now, to prove $\text{aNint}(A) = \text{SaNint}(\text{SaNint}(A))$. Since $\text{aNint}(A) \subseteq \text{SaNint}(A)$ $\Rightarrow$ aNint$(\text{Ncl}(A))$ $\subseteq$ aNint$(\text{SaNint}(A))$ $\Rightarrow$ aNint$(\text{SaNint}(A))$ $\subseteq$ aNint$(\text{SaNint}(A))$.

Also, $\text{SaNint}(A) \subseteq A$ $\Rightarrow$ aNint$(\text{SaNint}(A))$ $\subseteq$ aNint$(A)$.

Hence:

$a\text{Nint}(A) = \text{aNint}(\text{SaNint}(A))$........................................(2)

Therefore by (1) and (2), we get $\text{aNint}(\text{SaNint}(A)) = \text{SaNint}(\text{aNint}(A)) = \text{aNint}(A)$.

(iv) To prove $\text{Ncl}(\text{SaNcl}(A)) = \text{SaNcl}(\text{Ncl}(A)) = \text{Ncl}(A)$. We know that $\text{Ncl}(A)$ is a N-CS, so it is a NSa-CS. Hence by proposition 4.3, we have:

$\text{Ncl}(A) = \text{SaNcl}(\text{Ncl}(A))$........................................(1)

To prove $\text{Ncl}(A) = \text{Ncl}(\text{SaNcl}(A))$. Since $\text{SaNcl}(A) \subseteq \text{Ncl}(A)$ (by part (i)).

Then $\text{Ncl}(\text{SaNcl}(A)) \subseteq \text{Ncl}(\text{Ncl}(A)) = \text{Ncl}(A) \Rightarrow \text{Ncl}(\text{SaNcl}(A)) \subseteq \text{Ncl}(A)$. Since $A \subseteq \text{SaNcl}(A) \subseteq \text{Ncl}(\text{SaNcl}(A))$, then $A \subseteq \text{Ncl}(\text{SaNcl}(A))$. Hence

$\text{Ncl}(A) \subseteq \text{Ncl}(\text{Ncl}(\text{SaNcl}(A))) = \text{Ncl}(\text{SaNcl}(A)) \Rightarrow \text{Ncl}(A) \subseteq \text{Ncl}(\text{SaNcl}(A))$ and therefore: $\text{Ncl}(A) = \text{Ncl}(\text{SaNcl}(A))$........................................(2)

Now, by (1) and (2), we get that $\text{Ncl}(\text{SaNcl}(A)) = \text{Ncl}(A)$.

Hence $\text{Ncl}(\text{SaNcl}(A)) = \text{SaNcl}(\text{Ncl}(A)) = \text{Ncl}(A)$.

(vii) To prove $\text{SaNcl}(A) = A \cap \text{Ncl}(\text{SaNint}(\text{Ncl}(\text{SaNint}(A))))$.

Since $\text{SaNcl}(A) \in \text{NSaO}(U)$ $\Rightarrow$ $\text{SaNint}(A) \subseteq \text{Ncl}(\text{SaNint}(A))$.

By (by proposition 4.3) (by part (ii)).

Hence $\text{SaNint}(A) \subseteq \text{Ncl}(\text{SaNint}(A)))$ and also $\text{SaNint}(A) \subseteq \text{SaNint}(A)$.

Then:

$\text{SaNint}(A) \subseteq A \cap \text{Ncl}(\text{SaNint}(\text{Ncl}(\text{SaNint}(A))))$............................................(1)

To prove $A \cap \text{Ncl}(\text{SaNint}(\text{Ncl}(\text{SaNint}(A))))$ is a NSa-OS contained in $A$.

It is clear that $A \cap \text{Ncl}(\text{Ncl}(\text{SaNint}(A)))$ is a NSa-OS (by proposition 4.3). Also, $\cap \text{Ncl}(\text{Ncl}(\text{SaNint}(A)))$ is a NSa-OS.

Hence $A \cap \text{Ncl}(\text{Ncl}(\text{SaNint}(A)))$ is a NSa-OS (by proposition 4.3). Also, $\cap \text{Ncl}(\text{Ncl}(\text{SaNint}(A)))$ is a NSa-OS. Therefore by proposition 4.3:

$\text{SaNint}(A) \subseteq \text{Ncl}(\text{SaNint}(A)))$ (by corollary 3.12). Hence $\text{Ncl}(\text{Ncl}(\text{SaNint}(A)))$ is a NSa-OS (by part (iv)).

Therefore, $\text{SaNint}(\text{Ncl}(\text{SaNint}(A))) \subseteq \text{SaNcl}(A)$ (by part (ii)).

**Theorem 4.8:**

For any neutrosophic subset $A$ of a neutrosophic topological space $(U, T)$. The following properties are equivalent:

(i) $A \in \text{NSaO}(U)$.

(ii) $\mathcal{H} \subseteq A \in \text{Ncl}(\text{Ncl}(\mathcal{H}))$, for some N-OS $\mathcal{H}$.

(iii) $\mathcal{H} \subseteq A \in \text{SNint}(\text{Ncl}(\mathcal{H}))$, for some N-OS $\mathcal{H}$.

(iv) $\mathcal{H} \subseteq \text{SNint}(\text{Ncl}(\mathcal{H}))$.

**Proof:**

(i) $\Rightarrow$ (ii) Let $A \in \text{NSaO}(U)$, then $A \subseteq \text{Ncl}(\text{Ncl}(\text{SaNint}(A)))$ and $\text{Ncl}(A) \subseteq A$. Hence $\mathcal{H} \subseteq A \subseteq \text{Ncl}(\text{Ncl}(\mathcal{H}))$, where $\mathcal{H} = \text{Nint}(A)$.

(ii) $\Rightarrow$ (iii) Suppose $\mathcal{H} \subseteq A \subseteq \text{Ncl}(\text{Ncl}(\mathcal{H}))$, for some N-OS $\mathcal{H}$.
But $SNint(Ncl(\mathcal{H})) = Ncl(Nint(Ncl(\mathcal{H})))$ (by lemma (2.6)).

Then $\mathcal{H} \subseteq \mathcal{A} \subseteq SNint(Ncl(\mathcal{H}))$, for some N-OS $\mathcal{H}$.

(iii) $\Rightarrow$ (iv) Suppose that $\mathcal{H} \subseteq \mathcal{A} \subseteq SNint(Ncl(\mathcal{H}))$, for some N-OS $\mathcal{H}$. Since $\mathcal{H}$ is a N-OS contained in $\mathcal{A}$.

Then $\mathcal{H} \subseteq Nint(\mathcal{A}) \Rightarrow Ncl(\mathcal{H}) \subseteq Ncl(Nint(\mathcal{A})) 
\Rightarrow SNint(Ncl(\mathcal{H})) \subseteq SNint(Ncl(Nint(\mathcal{A}))).$

But $\mathcal{A} \subseteq SNint(Ncl(\mathcal{H}))$ (by hypothesis), then $\mathcal{A} \subseteq SNint(Ncl(Nint(\mathcal{A})))$.

(iv) $\Rightarrow$ (i) Let $\mathcal{A} \subseteq SNint(Ncl(Nint(\mathcal{A})))$. But $SNint(Ncl(Nint(\mathcal{A}))) = Ncl(Nint(Ncl(Nint(\mathcal{A}))))$ (by lemma (2.6)). Hence $\mathcal{A} \subseteq Ncl(Nint(Nint(\mathcal{A}))) 
\Rightarrow \mathcal{A} \in NS\alpha C(U)$.

**Corollary 4.9:**

For any neutrosophic subset $\mathcal{B}$ of a neutrosophic topological space $(U, T)$, the following properties are equivalent:

(i) $\mathcal{B} \in NS\alpha C(U)$.

(ii) $Nint(Ncl(\mathcal{B})) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ N-CS.

(iii) $SNcl(Nint(\mathcal{F})) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ N-CS.

(iv) $SNcl(Ncl(\mathcal{B})) \subseteq \mathcal{B}$.

**Proof:**

(i) $\Rightarrow$ (ii) Let $\mathcal{B} \in NS\alpha C(U) \Rightarrow Nint(Ncl(Ncl(\mathcal{B}))) \subseteq \mathcal{B} \subseteq \mathcal{F}$ (by corollary (3.12)) and $\mathcal{B} \subseteq Ncl(\mathcal{B})$. Hence we get $Nint(Ncl(Ncl(\mathcal{B}))) \subseteq \mathcal{B} \subseteq Ncl(\mathcal{B})$.

Therefore $Nint(Ncl(Nint(\mathcal{F}))) \subseteq \mathcal{B} \subseteq \mathcal{F}$, where $\mathcal{F} = Ncl(\mathcal{B})$.

(ii) $\Rightarrow$ (iii) Let $Nint(Ncl(\mathcal{F})) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ N-CS. But $Nint(Ncl(\mathcal{F})) = SNcl(Nint(\mathcal{F}))$ (by lemma (2.6)). Hence $SNcl(Nint(\mathcal{F})) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ N-CS.

(iii) $\Rightarrow$ (iv) Let $SNcl(Nint(\mathcal{F})) \subseteq \mathcal{B} \subseteq \mathcal{F}$, for some $\mathcal{F}$ N-CS. Since $\mathcal{B} \subseteq \mathcal{F}$ (by hypothesis), hence $Ncl(\mathcal{B}) \subseteq \mathcal{F} \Rightarrow Nint(Ncl(\mathcal{B})) \subseteq Nint(\mathcal{F}) \Rightarrow SNcl(Nint(Ncl(\mathcal{B}))) \subseteq SNcl(Nint(\mathcal{F})) \subseteq \mathcal{B} \Rightarrow SNcl(Nint(Ncl(\mathcal{B}))) \subseteq \mathcal{B}$.

(iv) $\Rightarrow$ (i) Let $SNcl(Nint(Ncl(\mathcal{B}))) \subseteq \mathcal{B}$. But $SNcl(Nint(Ncl(\mathcal{B}))) = Nint(Ncl(Ncl(\mathcal{B})))$ (by lemma (2.6)). Hence $Nint(Ncl(Nint(Ncl(\mathcal{B})))) \subseteq \mathcal{B} \Rightarrow \mathcal{B} \in NS\alpha C(U)$.

**5. Conclusion**

In this work, we have defined new class of neutrosophic open sets called neutrosophic semi-$\alpha$-open sets and studied their fundamental properties in neutrosophic topological spaces. The neutrosophic semi-$\alpha$-open sets can be used to derive a new decomposition of neutrosophic continuity, neutrosophic compactness, and neutrosophic connectedness.

**References**


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