EXISTENCE OF PRIME NUMBERS IN SUBSETS OF THE OPPERMANN’S INTERVALS

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ABSTRACT. We suggest that there exists, at least, one prime number in four intervals between $n^2$ and $(n+1)^2$ for any integer $n \geq 2$ such that:
- all intervals are half-open;
- the excluded endpoints are multiples of $n$;
- the number of elements in each interval is equal to the least even upper bound for the biggest prime number strictly less than $n$.

This conjecture is a strong form of Oppermann’s one.

MSC : 11N05
Mathematical statement : conjecture
Keywords: prime numbers, Legendre’s conjecture, Oppermann’s conjecture.

1. INTRODUCTION
For every positive integer $n$, Legendre’s conjecture \[1\] states that there is, at least, one prime number between $n^2$ and $(n+1)^2$. We note that any integer strictly less than $n$ has, at least, two multiples in that interval.

Oppermann’s conjecture \[2\] is a strong form of Legendre’s conjecture because there would be, at least, two prime numbers between $n^2$ and $(n+1)^2$. We also note that any integer strictly less than $n$ has, at least, one multiple between $n^2$ and $n(n+1)$ and another one between $n(n+1)$ and $n(n+2)$.

In this paper, we consider as reference interval the Legendre’s one by separating it into subsets in which any prime number strictly less than $n$ has, at least, one multiple and we suggest the conjecture mentioned below. We then verified the proposed conjecture below 1,193,806,024.

2. RESULTS
Conjecture 2.1. Let $m$ be the biggest prime number strictly less than $n$. Then
\[
\forall \ n > 2 , \ \exists p_1, p_2, p_3, p_4 \text{ primes } \begin{cases} n^2 < p_1 \leq n^2 + (m+1) & \text{[first interval]} \\ \text{and } n(n+1) - (m+1) \leq p_2 < n(n+1) & \text{[second interval]} \\ \text{and } n(n+1) < p_3 \leq n(n+1) + (m+1) & \text{[third interval]} \\ \text{and } n(n+2) - (m+1) \leq p_4 < n(n+2) & \text{[fourth interval]} \end{cases}
\]

We tested the statement and the result shows that the statement holds to $n = 1,700$. Then, by using a table of maximal gaps, the conjecture is verified for all $n$ up to 1,193,806,024.

Examples 2.2.
For $n = 5 : m = 3$ and the first interval is $]25;29]$ which contains the prime 29.
For $n = 8 : m = 7$ and the second interval is $[64;72[$ which contains the primes 67 and 71.
For \( n = 9 \) : \( m = 7 \) and the third interval is \([90;98]\) which contains the prime 97.
For \( n = 10 \) : \( m = 7 \) and the fourth interval is \([112;120]\) which contains the prime 113.

Remark 2.3. The value of \( n \) has to be strictly greater than 2 because no prime number is strictly less than 2.

Remark 2.4. \( p_1 \) and \( p_2 \) may not be necessarily distinct because the intersection of their sets is not empty.

Proof. According to the Bertrand-Chebyshev theorem \([3]\) :

\[
\frac{n}{2} < m < n
\]

\[
\Rightarrow \quad n^2 + \frac{n}{2} < n^2 + m < n^2 + n \quad \text{with } n^2 + n = n(n+1)
\]

\[
\Leftrightarrow \quad n^2 + \frac{n}{2} < n^2 + m < n^2 + (m+1) < n(n+1)
\]

By analogy :

\[
\frac{n}{2} < m < n
\]

\[
\Rightarrow \quad -n < -m < -\frac{n}{2}
\]

\[
\Rightarrow \quad n(n+1) - n < n(n+1) - m < n(n+1) - \frac{n}{2}
\]

\[
\Leftrightarrow \quad n^2 < n(n+1) - m < n(n+1) - \frac{n}{2}
\]

\[
\Leftrightarrow \quad n^2 < n(n+1) - (m+1) < n(n+1) - m < n(n+1) - \frac{n}{2}
\]

Since \( n(n+1) - \frac{n}{2} = n^2 + n - \frac{n}{2} = n^2 + \frac{n}{2} \),
then \( n^2 < n(n+1) - (m+1) < n^2 + \frac{n}{2} < n^2 + (m+1) < n(n+1) \)

Remark 2.5. \( p_3 \) and \( p_4 \) may be the same prime number. This can be proved by the same way we prove that \( p_1 \) and \( p_2 \) may be the same prime numbers.

3. REFERENCES


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