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Solving the Incompletely Predictable problems of Riemann hypothesis, Polignac's and Twin prime conjectures

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Abstract L-functions form an integral part of the 'L-functions and Modular Forms Database' with far-reaching implications. In perspective, Riemann zeta function is the simplest example of an L-function. Riemann hypothesis refers to the 1859 proposal by Bernhard Riemann whereby all nontrivial zeros are [mathematically] conjectured to lie on the critical line of this function. This proposal is equivalently stated in this research paper as all nontrivial zeros are [geometrically] conjectured to exactly match the 'Origin' intercepts of this function. Deeply entrenched in number theory, prime number theorem entails analysis of prime counting function for prime numbers. Solving Riemann hypothesis would enable complete delineation of this important theorem. Involving proposals on the magnitude of prime gaps and their associated sets of prime numbers, Twin prime conjecture deals with prime gap = 2 (representing twin primes) and is thus a subset of Polignac's conjecture which deals with all even number prime gaps = 2, 4, 6,... (representing prime numbers in totality except for the first prime number '2'). Both nontrivial zeros and prime numbers are Incompletely Predictable entities allowing us to employ our novel Virtual Container Research Method to solve the associated hypothesis and conjectures.

Keywords Information-Complexity conservation · Gram points · Polignac's conjecture · Real Container Research Method · Riemann hypothesis · Sigma-Power Laws · Twin prime conjecture · Virtual Container Research Method

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1 Introduction

This research paper refers to the intractable open problems in number theory of Riemann hypothesis, Polignac's & Twin prime conjectures. Comparatively, the celebrated 1859-dated

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Riemann hypothesis proposed over 150 years ago by German mathematician Bernhard Riemann (September 17, 1826 – July 20, 1866) is often regarded as the more prominent problem. Our engineered proofs on all three problems based on convoluted but self-consistent mathematical arguments must be absolutely correct when subjected to rigorous analysis.

With values traditionally given by imaginary part parameter t, nontrivial zeros in Riemann zeta function consist of the countable infinite set (CIS) of irrational (transcendental) numbers [rounded off to six decimal places] 14.134725, 21.022040, 25.010858, 30.424876, 32.935062, 37.586178,.... Prime numbers consist of the CIS of rational (whole) numbers 2, 3, 5, 7, 11, 13,.... Both nontrivial zeros and prime numbers [with exception of the very first and only even prime number '2' which is regarded in this setting as a Completely Predictable entity obeying Simple Elementary Fundamental Laws] are Incompletely Predictable entities obeying Complex Elementary Fundamental Laws. We now outline two 'Exact Criteria' of great significance required to usefully define the sets of nontrivial zeros and prime numbers.

Exact Criterion required to define the set of nontrivial zeros: The three sets of nontrivial zeros (or Gram[x=0,y=0] points), Gram[y=0] points and Gram[x=0] points together will respectively constitute all 'Origin' intercepts, [positive] x-axis intercepts and [positive] y-axis intercepts of Riemann zeta function except for the very first [and only negative] x-axis intercept (or negative Gram[y=0] point) which is classified as not belonging to any of these three sets. Thus the sets of Incompletely Predictable nontrivial zeros, Gram[y=0] points (with exception of the negative Gram[y=0] point endowed with t = 0 rational number value, which is regarded in this setting as a Completely Predictable number) and Gram[x=0] points are complementary to each other since Total Axes Intercepts = Gram[x=0,y=0] points + Gram[y=0] points + Gram[x=0] points + Solitary negative Gram[y=0] point; and this can only happen when they are stipulated by this Criterion. It is these so-called x-axis intercepts resulting in our Gram[y=0] points which are classically referred to as the traditional/usual 'Gram points'.

Exact Criterion required to define the set of prime numbers: The two sets of prime numbers (2, 3, 5, 7, 11, 13,....) and composite numbers (4, 6, 8, 9, 10, 12,....) together will constitute the set of natural numbers (1, 2, 3, 4, 5, 6,....) except for the very first natural number '1' which is classified as not belonging to any of these two sets. Thus the sets of Incompletely Predictable prime numbers [with the exception of the very first and only even prime number '2' which is regarded in this setting as a Completely Predictable number] and composite numbers are complementary to each other since Natural Numbers = Prime Numbers + Composite Numbers + Solitary Number '1'; and this can only happen when they are stipulated by this Criterion. Whole Numbers = Natural Numbers + Solitary Number '0'. The Completely Predictable numbers '0' & '1' in this setting are deemed to be special numbers as they are neither prime nor composite because they represent nothingness (zero) and wholeness (one), and the idea of having factors for '0' & '1' does not make sense.

The choice of the index 'n' chosen for Gram[y=0] points is historically crudely chosen in such a manner that the first Gram[y=0] point, which is by convention n = 0, corresponds to the first value which is larger than the smallest positive zero at t = 14.134725 (viz, the first nontrivial zero) of Riemann zeta function on the critical line. Thus when strictly defined to include all x-axis intercepts of Riemann zeta function, the first six Gram[y=0] points should occur at n = -3, -2, -1, 0, 1 and 2 with parameter t having the following values [rounded off to six decimal places]: at n = -3, t = 0; at n = -2, t = 3.436218; at n = -1, t = 9.666908; at n = 0, t = 17.845599; at n = 1, t = 23.170282; at n = 2, t = 27.670182. Note that at n = -3 (which is classified by the first Exact Criterion above as "not belonging to any of the three sets of nontrivial zeros, Gram[y=0] points and Gram[x=0] points"), inputting its associated parameter t = 0 into Riemann zeta function will result in the calculated $\zeta(\frac{1}{2})$ =

-1.4603545 value, an Incompletely Predictable irrational (transcendental) number [rounded off to seven decimal places]. This value, derived from the Completely Predictable t = 0 [rational number], is our negative x-axis intercept or negative Gram[y=0] point; and it can be calculated as a limit similar to the limit for Euler-Mascheroni constant or Euler gamma.

Likened to the Completely Predictable first and only even prime number '2' belonging to the set of prime numbers in totality; one could artificially assign the Completely Predictable first and only negative Gram[y=0] point as belonging to the set of nontrivial zeros in totality. This is validly achieved by arbitrarily redefining nontrivial zeros more broadly as "The set of nontrivial zeros will mathematically consist of all 'Origin' and negative x-axis intercepts". The other two alternative options involve artificially assigning our negative Gram[y=0] point as belonging to either the set of Gram[y=0] points or the set of Gram[x=0] points by arbitrarily redefining these sets more broadly to (additionally) include the term "...and negative x-axis intercepts".

Riemann hypothesis involves analysis on nontrivial zeros in Riemann zeta function – the study of which will, in addition, achieve the secondary objective of providing complete explanations for the existence of its closely related Gram points, or more specifically Gram[x=0] and Gram[y=0] points, in this same function. Both Polignac's and [its subset] Twin prime conjectures involve analysis on even number prime gaps and their associated sets of odd prime numbers. A broad overview on how we execute our plans to derive the rigorous proofs on these three open problems is communicated next. Stemming from this 'Executive Summary', we notice differences between Riemann hypothesis versus Polignac's and Twin prime conjectures. The former demands exact solutions for the *conjectured single line location* of all nontrivial zeros which are known to be of infinite magnitude while the later demands exact solutions for each set of odd prime numbers *conjectured to be of infinite magnitude* which is generated by each of the even number prime gaps *conjectured again to be of infinite magnitude*.

Executive Summary:

I. Riemann hypothesis Riemann hypothesis refers to the proposal whereby all nontrivial zeros of Riemann zeta function are conjectured to be located on the critical line. Our required proof is overall dependent on simultaneously satisfying two mutually inclusive conditions: Condition 1. All nontrivial zeros in Riemann zeta function will be shown to be located on the critical line when this function rigidly complies with Dimensional analysis homogeneity which is associated with $\sigma = \frac{1}{2}$. Condition 2. None of the nontrivial zeros in Riemann zeta function will be shown to be located on the critical line when this function rigidly comply with Dimensional analysis non-homogeneity which is associated with $\sigma \neq \frac{1}{2}$.

II. Polignac's and Twin prime conjectures Twin prime conjecture involves analysis of prime gap = 2 which is the very first even number prime gap representing all twin primes. This is a subset of Polignac's conjecture involving analysis of prime gaps = 2, 4, 6, 8, 10,... which are all the even number prime gaps representing prime numbers in totality[a] except for the very first (and only) even prime number '2' representing the very first (and only) odd number prime gap = 1. Together these two conjectures are unambiguously represented by two closely related mini-proposals: Mini-proposal 1. Even number prime gaps are infinite (and arbitrarily large[b]) in magnitude. Mini-proposal 2. Each individual even number prime gap will generate odd prime numbers which are again infinite in magnitude. Our required proof is overall dependent on simultaneously satisfying two mutually inclusive conditions: Condition 1. The "quantitive" aspect to the existence of the set of even number prime gaps and their associated sets of odd prime numbers, all as sets of infinite magnitude, will be shown to be correct by utilizing concepts derived from Set theory (such as incorporating the cardinality of a set with the 'well-ordering principle' application and arguments based on the 'pigeonhole principle'). Condition 2. The "qualitative" aspect to the existence of the set of even number prime gaps and their associated sets of odd prime numbers, all as sets of infinite magnitude, will be shown to be correct by the 'Plus-Minus Composite Gap 2 Number Alternating Law' which is applicable to all even number prime gaps apart from the special case of the first even number prime gap = 2 for twin primes. The prime gap = 2 situation will obey the 'Plus Composite Gap 2 Number Continuous Law'. In this paper, the 'Composite Gap 2 Number' occurrences are present on the finite scale mathematical landscape through direct observation and the infinite scale mathematical landscape through indirect logical deductions. In essence, these laws are Laws of Continuity[c] based on the entity 'Composite Gap 2 Number' inferring underlying intrinsic driving mechanisms that enable the infinity magnitude association for both the set of even number prime gaps and their associated sets of odd prime numbers to co-exist. By the same token, these laws must have the all-important implication that they will be perpetually applicable for all relevant even number prime gaps and their associated sets of odd prime numbers.

Footnote a: *We hereby alude to the fact that the stand-alone phrase "countable infinite set of all even number prime gaps representing prime numbers [2, 3, 5, 7, 11,...] in totality" (or similarly worded phrases in relevant parts of this paper) is not completely correct because this only encompass all known odd prime numbers [3, 5, 7, 11, 13,...] associated with all even number prime gaps = 2, 4, 6, 8, 10,... but without including the very first and only known even prime number '2' (which is associated with the very first and only known odd number prime gap = 1). In other words, prime numbers in totality consist of multiple odd prime numbers of infinite magnitude & the single even prime number of finite magnitude.*

Footnote b: *Simply representing the expression of Mini-proposal 1 in full, the dissimilar looking phrases 'Even number prime gaps are infinite in magnitude' and 'Even number prime gaps are arbitrarily large in magnitude' with their different intrepreted meanings will, of course, be equally valid when this mini-proposal in relation to the magnitude of prime gaps is proven to be true. Seemingly defying logic, this action is justifiable with the proviso that the term 'arbitrarily large' here must be clarified to denote the following caveats: The cummulative sum total of prime gaps is relatively much slower to attain the 'infinite in magnitude' status when compared to the cummulative sum total of prime numbers which will rapidly attain this status. The simple reason for this situation to exist is that every one of the even number prime gaps should generate (at the very least) more than one odd prime number, if not an unique set of odd prime numbers of infinite magnitude [which will happen when Mini-proposal 2 in relation to 'Each individual even number prime gap generating odd prime numbers which are infinite in magnitude' is proven to be true]. Failure to comply with the above situation will obviously occur just once at prime gap = 1 which represent the solitary even prime number '2'.*

Footnote c: *Introduced in 1701 by Gottfried Leibniz based on earlier work by Nicholas of Cusa & Johannes Kepler, the Law of Continuity is a heuristic principle that "whatever succeeds for the finite, also succeeds for the infinite". It is implemented through the transfer principle in the context of hyperreal numbers & is used to lay the groundwork for infinitesimal calculus by extending concepts such as arithmetic operations from ordinary numbers to infinitesimals. For the Plus-Minus Composite Gap 2 Number Alternating Law & Plus Composite Gap 2 Number Continuous Law, the heuristic principle equates to "these two laws based on Composite Gap 2 Number succeed for the finite, also succeed for the infinite".*

We will now render all our claims and statements in this paper meaningful by explaining the Hybrid method of Integer Sequence classification with its associated Ratio Study; defining the terms 'Completely Predictable entities' & 'Incompletely Predictable entities'; and providing a hierarchical classification on the terms 'Simple Elementary Fundamental Laws' & 'Complex Elementary Fundamental Laws'.

Hybrid method of Integer Sequence classification We tentatively advocate for the Hybrid method of Integer Sequence classification to act as a simple mathematical tool enabling meaningful division of all integer sequences into either Hybrid integer sequences or non-Hybrid integer sequences. In regards to usage of the terms 'Hybrid integer sequence' and 'non-Hybrid integer sequence', we now mention here the curious A228186 integer sequence [1]. It is the first ever novel [infinite length] Hybrid integer sequence artificially synthesized from Combinatorics Ratio [constituting an inequality criteria according to 'Ratio Study']. In the 'Position i' notation, let i = 0, 1, 2, 3,... be the set of natural numbers of infinite magnitude. A228186 "Greatest k $>$ n such that ratio R $<$ 2 is a maximum rational number with R = (Combinations with repetition)/(Combinations without repetition)" is equal to [infinite length] non-Hybrid (usual "garden-variety") integer sequence A100967 [2] except for the finite number of 21 'exceptional' terms at Positions 0, 11, 13, 19, 21, 28, 30, 37, 39, 45, 50, 51, 52, 55, 57, 62, 66, 70, 73, 77, and 81 with their values given by the relevant A100967 term plus 1. It was previously published by us on The On-line Encyclopedia of Integer Sequences website in 2013. The first 49 terms [from Position 0 to Position 48] of A100967 "Least k such that binomial(2k+1, k-n) $\geq$ binomial(2k, k)" are listed below: 3, 9, 18, 29, 44, 61, 81, 104, 130, 159, 191, 225, 263, 303, 347, 393, 442, 494, 549, 606, 667, 730, 797, 866, 938, 1013, 1091, 1172, 1255, 1342, 1431, 1524, 1619, 1717, 1818, 1922, 2029, 2138, 2251, 2366, 2485, 2606, 2730, 2857, 2987, 3119, 3255, 3394, and 3535. Then for those 21 'exceptional terms': at Position 0, A228186 (= 4) is given by A100967 (= 3) + 1; at Position 11, A228186 (= 226) is given by A100967 (= 225) + 1; at Position 13, A228186 (= 304) is given by A100967 (= 303) + 1; at Position 19, A228186 (= 607) is given by A100967 (= 606) + 1; etc. Commencing from Position 0 onwards, we can usefully envision the following attractive idea: "In the limit" that i approaches 82, A228186 becomes (and is identical to) A100967 for all i$\geq$82 values used to denote Position i. In relevant parts of this paper below, we encounter these unique concepts stemming from Integer Sequence classification and Ratio Study mainly during the derivation of our rigorous proof for Riemann hypothesis.

Completely Predictable & Incompletely Predictable entities We can arbitrarily create three useful groups of entities [with each group containing entities of infinite magnitude] – Group I: Completely Predictable entities, Group II: Incompletely Predictable entities, and Group III: Completely Unpredictable entities. Group III could also be appropriately coined Completely Chaotic or Completely Indeterministic entities.

Only certain correctly selected and naturally occurring physical processes can ever give rise to true [measured] random numbers in Group III since these physical processes are totally indeterministic or chaotic, and thus they can consequently be regarded entirely as Completely Unpredictable entities. In this sense, the [computational] pseudorandom number generators based solely on deterministic logic can never be regarded as sources for true random numbers (or true Completely Unpredictable entities). Any given set of *computed* numbers derived from Group I and Group II will always be "fixed" and will always be perfectly reproducible as the main distinguishing feature [at one end of a spectrum] when compared to any given set of *measured* numbers derived from Group III which will never be "fixed" and will never be perfectly reproducible [at the opposite end of a spectrum].

Prime numbers behave pseudorandomly with a strange mixture of order (structure) and chaos. It is with this pseudorandomly behavior that prime [and composite] numbers are regarded as 'Pseudorandom numbers'. At the outset here, we allude readers to the point that usage of the term 'Pseudorandom number' is deemed to be synonymous with usage of our devised term 'Incompletely Predictable number' in this paper.

Performing numerical analysis on a selected Completely Predictable number (or entity) and a selected Incompletely Predictable number (or entity) will allow us to fully comprehend them. Each can respectively be defined as a number (or entity) whose position is able to be fully specified without, and with, needing to know the associated details of the positions of all related preceding numbers (or entities) in its neighborhood. This property is demonstrated using the example of odd number '99' (calculated by using a 'simple' formula and is a Completely Predictable number obeying Simple Elementary Fundamental Laws) and prime number '97' (computated by using a 'complicated' algorithm and is a Incompletely Predictable number obeying Complex Elementary Fundamental Laws).

Randomly picked odd number '99': Can we completely predict its associated details (i) specifying whether it could be classified as an odd number and (ii) whether its precise position could be specified without needing to know the positions of all preceding odd numbers? '99' satisfy the Odd number Label "Always end with a digit of 1, 3, 5, 7, or 9" and hence is truly an odd number. Its precise i^{th} position can be calculated as follows: i = (99+1)/2 = 50. This implies that 99 is the 50^{th} odd number [and also the 73^{rd} composite number – computed (indirectly) using the Sieve of Eratosthenes]. Note that '99' is odd and composite but not prime as it consists of factors derived as 99 = 3 X 33 = 3 X 3 X 11; and '99' in this setting when contextually treated as a composite number is an Incompletely Predictable number. Therefore the answer is affirmative to both questions, and '99' as an odd number is a Completely Predictable number.

Randomly picked prime number '97': Can we completely predict its associated details (i) specifying whether it could be classified as a prime number and (ii) whether its precise position could be specified without needing to know the positions of all preceding prime numbers? '97' satisfy the Prime number Label "Always evenly divisible only by 1 or itself & must be whole numbers greater than 1" as confirmed by the trial division method resulting in 97 = 1 X 97. Hence '97' is truly prime. However, its precise position can only be determined by computing the positions of all preceding 24 prime numbers using the Sieve of Eratosthenes to eventually conclude that 97 is the 25^{th} prime number. Therefore the answer is affirmative to the first question but negative to the second question, and '97' as a prime number is a Incompletely Predictable number. {We would have already realize that '97' is also the [i = (97+1)/2] 49^{th} odd number as it satisfy the Odd number Label "Always end with a digit of 1, 3, 5, 7, or 9". Note that '97' in this setting when contextually treated as an odd number is a Completely Predictable number and will conform to similar numerical analysis conducted in the previous paragraph for odd number '99'.}

Thus with the exception of the very first and only even prime number '2' which is a Completely Predictable prime number, the Incompletely Predictable number property is applicable to all the other remaining Incompletely Predictable prime numbers [and to all known composite numbers by default]. We emphasize that this Incompletely Predictable number property is equally applicable to, for instance, any Incompletely Predictable algebraic or transcendental irrational number. We importantly discern that all Incompletely Predictable nontrivial zeros of Riemann zeta function as transcendental numbers will manifest the Incompletely Predictable number property twice, firstly in the countable infinite set of all nontrivial zeros, and secondly in the countable infinite set of all numerical digits after the decimal point in each nontrivial zero. In particular, the exact position of a nominated nontrivial zero and the exact position of a nominated numerical digit after the decimal point in each nontrivial zero, will not be known without calculating the positions of all relevant preceding nontrivial zeros and the positions of all relevant preceding numerical digits after the decimal point of that nontrivial zero. Immediately one would intuitively sense that any mathematical algorithm or equation required to deal with, for instance, "discrete-type" Incompletely Predictable prime numbers as opposed to, for instance, "discrete-type" (and "continuous-like") Incompletely Predictable nontrivial zeros would manifest features, respectively, of a "discrete" nature as opposed to features of a "continuous" nature. We point out here that, with the exception of the very first negative Gram[y=0] point endowed with the Completely Predictable t = 0 value, this double manifestation of Incompletely Predictable number property will similarly occur for all Gram[x=0] and Gram[y=0] points of Riemann zeta function since all their endowed t values will also consist of transcendental numbers.

Hierarchical classification on Fundamental Laws In the grand scheme of things, this paper manifests the classically encountered phenomenon that pure & applied mathematics during, and resulting from, derivation of many mathematical proofs are largely inseparable. Some of the less conventional and intuitively useful scientific ideas resulting from the applied mathematics in this paper are contained in our proposed hierarchical classification on Fundamental Laws. This classification is depicted from biologist-to-physicist point of view with highest-to-lowest decreasing hierarchical order as follows:

I. Complex 'Living Things' obeying Complex Emergent Fundamental Laws

II. Simple 'Living Things' obeying Simple Emergent Fundamental Laws

III. Complex 'Nonliving Things' obeying Complex Elementary Fundamental Laws

IV. Simple 'Nonliving Things' obeying Simple Elementary Fundamental Laws

In this context, the Incompletely Predictable problems of Riemann hypothesis, Polignac's & Twin prime conjectures are considered complicated 'Nonliving Things' obeying Complex Elementary Fundamental Laws. 'Living Things' are traditionally deemed to arise from 'Nonliving Things' via either Evolution process [as per atheist belief] or Creation process [as per religious belief]. Speculatively, could the concocted expression "Living Things seem to exist at the edge of Chaos and Fractals" be mathematically equivalent to the statement[d,e,f,g] "Living Things must be made up of a combination of Completely Predictable entities, Incompletely Predictable entities, &/or Completely Unpredictable entities"?

Footnote d: *The irrationality measure (or irrationality exponent or approximation exponent or Liouville-Roth constant) of any real number is a measure of how "closely" it can be approximated by rationals. For a rational number, the irrationality measure is 1. The Thue-Siegel-Roth theorem states that for an algebraic irrational number, viz. real but not rational number, then the irrationality measure is 2. Transcendental irrational numbers have irrationality measure 2 or greater; for instance, the transcendental Euler's number e (= 2.718281828459...) has irrationality measure equal to 2. The [seemingly] simplistic-looking Liouville numbers is typified by Liouville's constant, sometimes also called Liouville's number, a real number defined by* $L \equiv \sum_{n=1}^{\infty} 10^{-n!} = 0.110001000000000000000001...$ *with '!' denoting factorial. These numbers are irrational numbers of [the relatively more "complex"] transcendental types instead of [the relatively less "complex"] algebraic types; and their numerical make-up consist of just '0' and '1' digits. Despite this apparently simple-looking numerical make-up of Liouville numbers (as opposed to more complicated-looking numerical make-up of e), they are precisely those numbers [paradoxically] having infinite irrationality measure. Based on concepts from our classification on Fundamental Laws, we would assign all [Completely Predictable] rational numbers to obeying Simple Elementary Fundamental Laws, and all [Incompletely Predictable] irrational numbers to obeying Complex Elementary Fundamental Laws.*

Footnote e: *Rigorous mathematical proofs must obviously be associated with 100% certainty. This can only apply to Simple & Complex Elementary Fundamental Laws on 'Nonliving Things'. Mathematical proofs for Simple & Complex Emergent Fundamental Laws on 'Living Things' can never be rigorous since they cannot be associated with perfect 100% certainty simply because we are dealing with "ALIVE" Living Things with dynamic spatial and temporal properties that could not be totally predictable. In this setting, the proofs for Simple cases [e.g. physiologically modeling Cardiac Output (CO) equals to Heart Rate (HR) multiplied by Stroke Volume (SV) in Cardiovascular System (CVS)] will comparatively be less challenging to derive than Complex cases [e.g. physiologically modeling complex Human Brain functions using Neural Networks in Central Nervous System (CNS)].*

Footnote f: *The terms 'Elementary' and 'Emergent' are used here in the preceding and subsequent footnotes to, respectively, denote 'Nonliving Things' and 'Living Things'. In the real world situation for 'Living Things', there will always be the perpetual presence of infinitesimally tiny and unpredictable "Chaos and Fractals physiological variability", for instance, in the Simple Emergent Fundamental Law CO = HR X SR. This variability phenomenon will inevitably occur even in the most relaxed state of a person in deep sleep whereby dynamic processes such as intrinsic neuro-endocrine continuous signal input to the heart must occur on a permanent basis thus giving rise to this variability.*

Footnote g: *The mathematical sciences behind Evidence based Medicine (EBM) and Evidence based Practice (EBP) in the medical domain imply that both must comply with either Simple or Complex Emergent Fundamental Laws on Living Things (namely, Human Beings in this scenario). EBM is typically depicted pictorially as a 'Pyramidal hierarchy of Literature Review' classifying available medical research materials into [the most powerful] Systematic Reviews down to [the least powerful] Expert Opinion. Then EBP = Clinician Experience + Patient Expectation + Best Practice; with Best Practice being roughly equated with EBM. Doctors and medical researchers confronted daily with responsibly abiding to and improving up-to-date EBP and EBM must be familiar with most statistical tools employed in medical research. The classic example is research hypothesis expressed as a null hypothesis [the "devil's advocate" position] and alternative hypothesis. The level of statistical significance for hypothesis testing is often expressed as the so-called p-value. Whilst there is relatively little justification why a [cut-off] significance level of 0.05 is widely used in academic research [rather than 0.01 or 0.10]; we could be particularly more confident in our results by setting a more stringent level of (say) 0.01 [a 1% chance or less; 1 in 100 chance or less]. Despite this experimental/research tactic, we could strive to, but never, achieve perfect or 100% confidence in our results by setting ever more stringent levels.*

1.1 On the Real and Virtual Container Research Method

The common denominator 'Completely Predictable entities being present in all Completely Predictable mathematical problems' will always permit the convenient grouping of these problems, and in allowing the employment of our mathematical tool coined 'Real Container Research Method' (RCRM) on these problems to solve them. Counterintuitively, the common denominator 'Incompletely Predictable entities being present in all three of our open problems' will be a decisive asset in permitting the convenient grouping of, and in allowing the employment of our novel mathematical tool coined 'Virtual Container Research Method' (VCRM) on, these three challenging problems. The recognition of this crucial idea is paramount in permissively allowing the VCRM approach to literally act as foundation for the elegant mathematical framework that enables successful completion of the assigned monumental task to solve those three open problems.

The key ideas behind RCRM & VCRM are described by two scenarios: Scenario 1. Completely Predictable mathematical problems involving Completely Predictable entities of infinite magnitude are associated with Completely Predictable properties that are amendable to treatment by conventional mathematical tools such as Calculus to solve these problems. Scenario 2. Incompletely Predictable mathematical problems involving Incompletely Predictable entities of infinite magnitude are associated with Incompletely Predictable properties that are not amendable to treatment by conventional mathematical tools such as Calculus to solve these problems. In Scenario 2, we need to initially derive certain Completely Predictable "meta-properties" out of the Incompletely Predictable properties. Only then can these problems in the guise of Completely Predictable "meta-properties" be amendable to treatment by conventional mathematical tools such as Calculus [and additional less commonly used ones such as Dimensional analysis and concepts from Set theory] so that they can finally be solved. Thus it is by indirectly deriving certain Completely Predictable "meta-properties" from Incompletely Predictable properties associated with the common denominator 'Incompletely Predictable entities being present in all three of our open problems' that will eventually allow us to successfully solve them.

Remark 1.1. Real Container Research Method & Virtual Container Research Method can, respectively, be applied to Completely Predictable entities & Incompletely Predictable entities (both of infinite magnitude).

We now provide clarification on the coined 'Real Container' concept in RCRM and 'Virtual Container' concept in VCRM. Our invented RCRM is the applied mathematics for solving the 'General-Class-of-Mathematical-Problems with Multiple-Proof-Solutions' (or simply the 'Completely Predictable problems') containing Completely Predictable entities of infinite magnitude, and our invented VCRM is the applied mathematics for solving the 'Special-Class-of-Mathematical-Problems with Solitary-Proof-Solution' (or simply the 'Incompletely Predictable problems') containing Completely Predictable entities of infinite magnitude. RCRM & VCRM are schematically depicted below.

RCRM: Completely Predictable problems
$\longrightarrow$ Completely Predictable entities of infinite magnitude with Completely Predictable properties
$\longrightarrow$ Solutions for Completely Predictable problems

VCRM: Incompletely Predictable problems
$\longrightarrow$ Incompletely Predictable entities of infinite magnitude with Incompletely Predictable properties
$\longrightarrow$ Completely Predictable meta-properties
$\longrightarrow$ Solutions for Incompletely Predictable problems

We now demonstrate RCRM applicable to problems involving Group I Completely Predictable entities which we classify as 'Completely Predictable problems' using "discrete" even & odd numbers as two nominated examples and "continuous" y = 2x & y = 2x - 1 equations as another two nominated examples; and VCRM applicable to Group II Incompletely Predictable entities which we classify as 'Incompletely Predictable problems' using "discrete" prime & composite numbers as two nominated examples and "continuous" Riemann zeta function as another solo nominated example.

Consider x for all real number values ≥ 1. Let y be the set of real numbers such that y = 2x. Then this y = 2x "continuous" linear equation is literally the Real Container mathematically "containing" the [complete set] straight line of infinite length commencing from the Cartesian point (x=1, y=2). This straight line will fully represent the y = 2x output real number values for all the specified x $\geq$ 1 input real number values. Computing y = 2x values an infinite number of times will not *per se* result in obtaining the gradient or slope of 2 for this equation. This gradient can be obtained by utilizing more than one method – either via trigonometrically calculating the tangent of the y = 2x straight line [which equals to 2] or via mathematically analyzing the intrinsic property of the y = 2x equation using Differential Calculus [viz. dy/dx = d(2x)/dx = 2]. As a side note, we observe that by applying Integral Calculus together with Fundamental Theorem of Calculus to the continuous y = 2x equation for the interval [1, +∞], viz. $\int_1^\infty(2x)dx = [x^2 + C]_1^\infty = (\infty^2 + C) - (1^2 + C) = \infty$, will result in the "area of infinite size enclosed by the [straight line] curve and the x-axis".

We can carry out an identical treatment to the y = 2x - 1 "continuous" equation for the same x $\geq$ 1 real number values to obtain the infinite length straight line but commencing this time from the different Cartesian point (x=1, y=1). Its gradient of 2 can similarly be obtained either using the tangent method or the dy/dx = d(2x-1)/dx Differential Calculus method. As a side note, we observe in a similar fashion that by applying Integral Calculus together with Fundamental Theorem of Calculus to the continuous y = 2x - 1 equation for the interval [1, +∞], viz. $\int_1^\infty(2x - 1)dx = [x^2 - x + C]_1^\infty = (\infty^2 - \infty + C) - (1^2 - 1 + C) = \infty$, will result in the "area of infinite size enclosed by the [straight line] curve and the x-axis".

By carrying out this identical treatment using the same y = 2x and y = 2x - 1 as "discrete" equations by considering x for all integer number values $\geq$ 1 [instead of x for all real number values $\geq$ 1], we easily obtain (respectively) the complete set of even and odd numbers [with both sets of infinite magnitude in size]. These "discrete" equations are the Real Containers "containing" all known even and odd numbers. Computing even and odd numbers infinitely often will not *per se* enable us to ever conclude that the gap between any two consecutive even numbers (even gap) and any two consecutive odd numbers (odd gap) will both always equal to 2. This "gradient-equivalent" even gaps and odd gaps can simply be obtained by transforming those equations from their "discrete" formats into the equivalent "continuous" formats [viz. "discrete" $\Delta x = 1 \longrightarrow$ "continuous" $\Delta x = 0$] to obtain their gradients either using the tangent method or Differential Calculus method (as outlined in the preceding two paragraphs). Then the even and odd gaps, both equal to 2, is numerically identical and mathematically equivalent to the relevant obtained gradients, both also equal to 2. Similar in nature to our mentioned process of transforming equations from their "discrete" formats into the equivalent "continuous" formats; we importantly point out here that one of the many crucial steps, as depicted in the relevant parts of this paper below, required to successfully solve Riemann hypothesis will involve applying Riemann integral to the "discrete-like" Riemann-Dirichlet Ratio (in its summation format) in order to obtain the "continuous-like" Riemann-Dirichlet Ratio (in its integral format).

Two crucial points to note here are (i) the two equations y = 2x and y = 2x - 1 in both their "discrete" and "continuous" formats are totally independent of each other as we can successfully obtain their respective gradient or gap values by individually analyzing each relevant equation by itself, and (ii) there are more than one way to obtain those gradient or gap values as clearly illustrated above using the tangent method or Differential Calculus method. Specifically by this two points, we imply that Completely Predictable entities will always belong to the 'General-Class-of-Mathematical-Problems with Multiple-Proof-Solutions' [or simply the 'Completely Predictable problems'].

With exceptions of the Completely Predictable first & only negative Gram[y=0] point and the Completely Predictable first & only even prime number '2', we can now logically apply in an analogous manner similar treatment using Virtual Containers to (a) nontrivial zeros & its closely related two types of Gram points – all computationally obtained directly from the "continuous" Riemann zeta function; and (b) prime & composite numbers – all computationally obtained directly & indirectly from the "discrete" Sieve of Eratosthenes algorithm.

They are all of infinite magnitude in size and are typical representations of Incompletely Predictable entities. For the "continuous" Riemann zeta function, the axes intercepts at the 'Origin' (viz. nontrivial zeros or Gram[x=0,y=0] points), the x-axis (viz. usual/traditional 'Gram points' or Gram[y=0] points) and the y-axis (viz. Gram[x=0] points) – all consisting of transcendental numbers – are the Incompletely Predictable entities of interest that we wish to study. As we shall subsequently observe in all of our obtained proofs for our three open problems based on Riemann zeta function and Sieve of Eratosthenes, both (a) the three axes intercepts sets of infinite nontrivial zeros, infinite Gram[y=0] points and infinite Gram[x=0] points, and (b) the two numerical sets of infinite prime numbers & infinite composite numbers, are totally dependent on each other in the following sense. There is just one solitary way to solve those open problems as we can only suceed in rigorously obtaining the relevant proofs when (a) nontrivial zeros, Gram[y=0] points and Gram[x=0] points are all simultaneously analyzed together and "contained" using the relevant Virtual Container largely represented by Sigma-Power Laws derived via certain non-negotiable mathematical steps (see the relevant **'Riemann hypothesis mathematical foot-prints'** located towards the end of Section 6 below) being correctly undertaken, and (b) prime numbers & composite numbers are simultaneously analyzed together using the relevant Virtual Container largely represented by Information-Complexity conservation derived via certain non-negotiable mathematical steps (see the relevant **'Polignac's and Twin prime conjectures mathematical foot-prints'** located just after the proof for Proposition 9.1 in Section 9 below) being correctly undertaken to "contain" them. In other words, we are only able to solve those open problems when (i) the axes intercepts of Riemann zeta function are dependently analyzed together using the derived 'solitary-style' relevant Virtual Container that "contains" them, and prime numbers & composite numbers are dependently analyzed together to derive this 'solitary-style' relevant Virtual Container to "contain" them, and (ii) these representative Virtual Containers can only be derived via certain non-negotiable mathematical steps being correctly undertaken. In a nutshell, satisfying criteria (i) and (ii) is the *sine qua non* of the requirements to fulfill the condition that solving the three open problems of Riemann hypothesis, Polignac's and Twin prime conjectures – all endowed with Incompletely Predictable entities – will always be classified as solving the 'Special-Class-of-Mathematical-Problems with Solitary-Proof-Solution' [or simply the 'Incompletely Predictable problems'].

We intuitively sense that to ultimately solve open problems related to nontrivial zeros (& the two types of Gram points) of Riemann zeta function and prime numbers (& composite numbers) generated from the Sieve of Eratosthenes would subsequently/concurrently require the correct analysis of certain [finite number of] intrinsic properties and behaviors arising from those representative Virtual Containers. In particular, we are dealing with entities such as Incompletely Predictable 'varying gaps' [which is the equivalent of Incompletely Predictable 'varying gradients'] between consecutive prime numbers (prime gaps) & between consecutive composite numbers (composite gaps); and our hereby conjured-for-illustration-purpose Incompletely Predictable 'varying gaps' [which is the equivalent of Incompletely Predictable 'varying gradients'] between consecutive nontrivial zeros (dubbed nontrivial zero gaps), between consecutive Gram[y=0] points (dubbed Gram[y=0] points gaps) & between consecutive Gram[x=0] points (dubbed Gram[x=0] points gaps).

Of utmost significance, these Incompletely Predictable 'varying gaps' or 'varying gradients' of infinite magnitude are *de novo* natural phenomena arising out of Incompletely Predictable function or algorithm – which in these cases refer to Riemann zeta function and Sieve of Eratosthenes algorithm. They will inevitably never be amendable to direct or conventional treatments, for example, by using tangent method or Differential Calculus method which can only ever be validly applied to Completely Predictable functions or algorithms

Table 1 The Completely Predictable problem of Even-odd number pairing

Set of even number
——————————————-CP Straight line INTERFACE
Set of odd number

Table 2 The Incompletely Predictable problem of Prime-composite number pairing

Set of composite number
vvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvIP Jagged line INTERFACE
Set of prime nuumber

Table 3 The Completely Predictable problem of y = sin x function

Set of x-axis intercepts in y = sin x
——————————————-CP Straight line INTERFACE
Set of 'Origin' intercept in y = sin x

Table 4 The Incompletely Predictable problem of Riemann zeta function

Set of Gram[y=0] points ('usual' Gram points)
vvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvIP Jagged line INTERFACE
Set of Gram[x=0,y=0] points (nontrivial zeros)
vvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvIP Jagged line INTERFACE
Set of Gram[x=0] points

[such as those associated with the above mentioned y = 2x & y = 2x - 1 equations, which are both endowed with 'non-varying gaps' or 'non-varying gradients' of infinite magnitude].

We can now deduce that a critical step in using our VCRM technique to successfully obtain the proofs for Riemann hypothesis, Polignac's and Twin prime conjectures [which are respectively connected with Riemann zeta function, and Sieve of Eratosthenes algorithm] will consist of analyzing the finite number of Completely Predictable "meta-properties" that are (intrinsically) already present in the relevant Incompletely Predictable function and algorithm [instead of analyzing the infinite number of Incompletely Predictable entities (extrinsically) generated by these same function and algorithm]. Then the tell-tale sign indicating Virtual Container use in this paper is epitomized by relevant statements or sentences incorporating expressions with wordings such as "...containing each and every conceivable nontrivial zeros [but not its actual identity]..." or "...containing each and every conceivable prime number [but not its actual identity]...".

We diagrammatically depict Group I Completely Predictable entities (giving rise to Completely Predictable problems), symbolized by the Completely Predictable "——CP Straight line INTERFACE" for "discrete" even-odd numbers (in Table 1) and "continuous" y = sin x function (in Table 3); and Group II Incompletely Predictable entities (giving rise to Incompletely Predictable problems), symbolized by the Incompletely Predictable "vvvvvvIP Jagged line INTERFACE" for "discrete" prime-composite numbers (in Table 2) and "continuous" Riemann zeta function (in Table 4). There is a finite number of intrinsic properties with special characteristics that will manifest themselves infinitely often at both 'numerical relationship interface' and 'axes intercepts relationship interface' for only Group II Incompletely Predictable entities. As elaborated below, it is these intrinsic properties with special characteristics – which are our so-called Completely Predictable "meta-properties" – observed at those relevant interfaces that can be perceived as automatic sequelae arising from various "interactions" between/amongst the two dependent sets of

[Incompletely Predictable] prime & composite numbers and the solitary [Completely Predictable] even prime number '2' (all derived by utilizing the Sieve of Eratosthenes); and between/amongst the three dependent sets of [Incompletely Predictable] axes intercepts viz. Gram[y=0], Gram[x=0] & Gram[x=0,y=0] points and the solitary [Completely Predictable] negative Gram[y=0] point (all being present in Riemann zeta function). A simple *overall* Completely Predictable "meta-properties" could then be perceived as the perpetual presence of [solitary] critical line location of all nontrivial zeros of Riemann zeta function (indicating Riemann hypothesis to be true); and as the perpetual presence of [dual] infinity magnitude size of the set consisting all even number prime gaps and infinity magnitude sizes of the individual sets of odd prime numbers that are derived from each and every even number prime gaps (together indicating Polignac's and Twin prime conjectures to be true).

Numerical relationship interface For the Completely Predictable problem of even-odd number pairing and the Incompletely Predictable problem of prime-composite number pairing, the following collective statements are valid:

For the Completely Predictable problem of even-odd number pairing in Table 1: Set of natural number = Set of even number + Set of odd number. Set of even number = Set of odd number equality relationship is an "exact non-varying" Completely Predictable relationship. The numerical relationship interface for this even-odd number pairing is symbolized by the Straight line INTERFACE.

For the Incompletely Predictable problem of prime-composite number pairing (involving the Sieve of Eratosthenes) in Table 2: Set of natural number = Set of prime number + Set of composite number + The number '1'. Set of composite number $>$ Set of prime number inequality relationship is an "exact varying" Incompletely Predictable relationship. The numerical relationship interface for this prime-composite number pairing is symbolized by the Jagged line INTERFACE. Classical examples of intrinsic properties with special characteristics ("meta-properties") arising out of the "interactions" between the two dependent sets of prime and composite numbers are the 'Plus-Minus Composite Gap 2 Number Alternating Law' and the 'Plus Composite Gap 2 Number Continuous Law'. Outlined in the relevant parts of this paper below, these are essentially Completely Predictable laws that are computationally applicable [albeit with Incompletely Predictable timing on an eternal basis] to certain designated prime numbers (which are respectively generated from all even number prime gaps apart from prime gap = 2 in the first law, and from even number prime gap = 2 in the second law).

Axes intercepts relationship interface For the Completely Predictable problem of y = sin x function and the Incompletely Predictable problem of Riemann zeta function, the following collective statements are valid:

For the Completely Predictable problem of y = sin x function in Table 3: Set of all axes intercepts = Set of x-axis intercepts + Set of 'Origin' intercept. Set of [infinite] x-axis intercepts $>>$ Set of [single/finite] 'Origin' intercept inequality relationship is an "exact non-varying" Completely Predictable relationship. The axes intercepts relationship interface for this even-odd number pairing is symbolized by the Straight line INTERFACE.

For the Incompletely Predictable problem of Riemann zeta function (involving the three axes intercepts) in Table 4: Set of all axes intercepts = Set of x-axis intercepts + Set of y-axis intercepts + Set of 'Origin' intercepts + The solitary 'negative x-axis intercept'. The Set of [infinite] x-axis intercepts = Set of [infinite] y-axis intercepts = Set of [infinite] 'Origin' intercepts equality relationship is an "exact varying" Incompletely Predictable relationship. The axes intercepts relationship interface for this Riemann zeta function is symbolized by the Jagged line INTERFACE. Classical examples of intrinsic properties with special characteristics ("meta-properties") arising out of the "interactions" between the two dependent sets of usual/traditional 'Gram' points (x-axis intercepts) and nontrivial zeros ('Origin' intercepts) are the Completely Predictable periodic, albeit Incompletely Predictable timing of, occurrances of Gram's Law and its violation, Gram block, etc on an eternal basis. These phenomena are outlined in the relevant part of this paper below.

Unless stated otherwise in this paper, the symbol 'log' refer to natural logarithm. We will generally outline materials on proof for Riemann hypothesis ahead of materials on proofs for Polignac's & Twin prime conjectures. The term 'hypothesis' is often taken to connote a 'conjecture' once it has been rigorously proven to be true. Then the traditionally-dubbed 'Riemann hypothesis' should strictly be previously labeled 'Riemann conjecture' because chronologically this reference to 'Riemann hypothesis' has [ideally] not been used correctly in the era prior to rigorous proof being obtained for this conjecture. Therefore utilizing more accurate terms & metaphorically speaking in relevant parts of this paper, we aim to produce easy-to-understand [initially] geometrical-format-version & [subsequently] mathematical-format-version of the wider proposed-in-2018 Dirichlet-Gram-Riemann conjecture which also encompasses the proposed-in-1859 Riemann conjecture; with both conjectures, once proven, being able to be denoted by (respectively) Dirichlet-Gram-Riemann hypothesis and Riemann hypothesis. On the other hand, this designated conjecture-to-hypothesis terminology usage with Polignac's & Twin prime conjectures is appropriate whereby the adjoined term 'conjecture' can now be replaced by the term 'hypothesis' to give rise to Polignac's & Twin prime hypotheses once these conjectures are proven to be true.

1.2 Brief overview of Riemann hypothesis

Riemann hypothesis refers to the famous 1859 conjecture explicitly equivalent to the mathematical statement that the critical line [which is a vertical straight line of infinite length defined by $\sigma = \frac{1}{2}$] in the critical strip [which is a vertical rectangular region of infinite area defined by $0 < \sigma < 1$] of Riemann zeta function is the location for all nontrivial zeros. At the most rudimentary level, Riemann hypothesis simply and graphically refers to the generated curves of Riemann zeta function simultaneously intersecting both x- and y-axes [formally named the 'Origin'] an infinite number of times. Thus these 'Origin' intercepts (or Gram[x=0,y=0] points) corresponding exactly to nontrivial zeros are Incompletely Predictable entities constituting a countable infinite set (CIS) of irrational [transcendental] numbers which forms a mathematical subset of the uncountable infinite set (UIS) of generated curves. In an identical manner, the usual/traditional 'Gram' points (or Gram[y=0] points) and Gram[x=0] points can similarly be depicted as two CIS of irrational [transcendental] numbers forming two respective mathematical subsets of the UIS of generated curves.

Remark 1.2. Computationally checking for nontrivial zeros to be correctly located on the critical line implies [but does not rigorously prove] that Riemann hypothesis is true.

In Riemann hypothesis, it was postulated that nontrivial zeros of Riemann zeta function must (mathematically) all lie on the critical line ($\sigma = \frac{1}{2}$) or [equivalently stated in this paper] must (geometrically) all exactly match the 'Origin' intercepts. This has previously been computationally checked for the first 10,000,000,000,000 identities. It has already been shown that there are infinitely many nontrivial zeros on this critical line by Hardy in 1914 [3] and Hardy & Littlewood in 1921 [4] via considering moments of certain functions related to Riemann zeta function. However, this discovery by Hardy and Littlewood does not constitute rigorous proof for Riemann hypothesis because they have not mathematically exclude the possible existence of nontrivial zeros which are located away from the critical line. As we shall see, use of our Virtual Container [essentially embodied in the relevant Theorem {Riemann} I to IV below] **in "containing" all nontrivial zeros but without needing to know their true identities** is paramount in allowing us to accomplish the feat of convincingly proving Riemann hypothesis via subsequent/concurrent "correct analysis" of this Virtual Container. This is predominantly achieved by using our mathematical tool Sigma-Power Laws and concepts from Hybrid method of Integer Sequence classification.

In addition to nontrivial zeros, Riemann zeta function also contains trivial zeros. Resulting points generated from the "discrete" CIS Output of trivial zeros are subsets of the overall "continuous" UIS Output of the generated curves in Riemann zeta function. All negative even (integer) number values in the σ parameter constituting [and defining] these "discrete-type" trivial zeros are Completely Predictable entities whereas all the irrational [transcendental] number values for "discrete-type" (and "continuous-like") nontrivial zeros, x- & y-axes intercepts [with the exception of the first and only negative x-axis intercept endowed with t = 0 rational number value] are Incompletely Predictable entities. This Input and Output concepts for Riemann zeta function are succinctly summarized below.

Input: Continuous-type real number values straight lines denoted by σ and t variables –> Black Box: Unique Equations related to Riemann zeta function (with alternating + and - summation carried over all integers n = 1, 2, 3,..., ∞ in the critical strip specified by its surrogate Dirichlet eta function) –> Output: Continuous-type real number values curves/spirals/loops denoted by $Re\{\zeta(\frac{1}{2}+\imath t)\}$ and $Im\{\zeta(\frac{1}{2}+\imath t)\}$ parameters. The discrete-type integer number values of trivial zeros (via Riemann functional equation) and the discrete-type (and continuous-like) real number values of nontrivial zeros are, respectively, graphical intercepts at the negative part of horizontal x-axis and at the 'Origin'. Excluding the first negative Gram[y=0] point, there are two other possible discrete-type (and continuous-like) real number values of graphical intercepts occurring at the remaining parts of horizontal x- & vertical y-axes coined respectively as the usual/traditional 'Gram points' (or Gram[y=0] points) & Gram[x=0] points. Then nontrivial zeros are dubbed Gram[x=0,y=0] points.

Thus all the Completely Predictable trivial zeros occur at σ = -2, -4, -6,... (all negative even numbers) and all the Incompletely Predictable nontrivial zeros are conjectured to occur at $\sigma = \frac{1}{2}$ with the locations given by parameter t [rounded off to six decimal places] = 14.134725, 21.022040, 25.010858, 30.424876, 32.935062, 37.586178,.... In the relevant section below, we begin our initial mission to prove the phenomenal Riemann hypothesis (and explain its closely related two types of Gram points) with the starting geometrical principle "Any given 2-variable equation able to be computationally depicted by a 2-dimensional graph with its x- and y-axes relevantly defined often have point(s) of intersection on (i) x-axis, and/or (ii) y-axis, and/or (iii) both x- and y-axes (formally known as the 'Origin')" to eventually provide equivalent geometrical-format-version of this hypothesis – itself deemed to logically constitute one of three components of the 'glorified' Dirichlet-Gram-Riemann conjecture. Note that the three components of Dirichlet-Gram-Riemann conjecture refer to the study of x-axis, y-axis, and both x- & y-axes intercepts; and when this conjecture is proven to be true, it can be appropriately named Dirichlet-Gram-Riemann hypothesis.

1.3 Brief overview of Polignac's and Twin prime conjectures

The ancient Euclid's proof on the infinitude of prime numbers predominantly by *reductio ad absurdum* (proof by contradiction), occurring well over 2000 years ago (c. 300 BC), is the earliest known but not the only possible proof for this simple problem in number theory. Since then dozens of proofs have been devised to show that prime numbers in totality are indeed infinite in magnitude such as the three chronologically listed below with the strangest candidate likely to be Furstenberg's Topological Proof.

1. Goldbach's Proof using Fermat numbers (written in a letter to Swiss mathematician Leonhard Euler, July 1730)

2. Furstenberg's Topological Proof in 1955 [5]

3. Filip Saidak's Proof in 2006 [6]

One could subjectively consider the ancient Euclid's proof to subtly be the "more" or "most" elementary proof on the infinitude of prime numbers with all the other dozens of proofs being relatively "lesser" or "least" elementary proofs. Ultimately, all these "least", "lesser", "more" or "most" elementary proofs, or for that matter any proofs [including our proposed relatively elementary proofs for Riemann hypothesis, Polignac's and Twin prime conjectures], must regardless and objectively always be rigorous mathematical proofs. We now provide a brief synopsis pertaining to important treatises on prime numbers.

Solving Polignac's & Twin prime conjectures demands the rigorous proof that each and every single one of the infinitely many odd prime numbers is derived from the infinite (and arbitrarily large) magnitude of even number prime gaps = 2, 4, 6,... with each even number prime gap generating its own (unique) infinite magnitude of odd prime numbers. The exception to this trend occur at the very first & only even prime number '2' which is associated with the solitary odd number prime gap = 1. These two conjectures can equivalently be stated explicitly as they demanding the rigorous proof that each and every single one of the infinitely many composite numbers is derived from the finite magnitude of odd number composite gap = 1 & even number composite gap = 2. There is intricate unification of prime numbers & Riemann hypothesis in that a crucial primary or direct by-product arising out of the rigorous proof for Riemann hypothesis is theorized to result in complete formalization of prime number theorem which relates to prime counting function for prime numbers.

The Incompletely Predictable prime numbers will consist of prime numbers with 'small gaps' and prime numbers with 'large gaps' alike. Our lemmas, propositions & theorems in relation to Polignac's and Twin prime conjectures in this paper are comparatively simple and prominently based on compulsorily using Virtual Containers **to "contain" the infinite magnitude of all known prime numbers but without needing to know their true identities** simply because otherwise we have to infinitely often prove (without complete certainty) that every single prime number will comply with certain mathematical properties (satisfying Condition X or Y or Z). This inevitable mathematical snag could be perceived as the classical equivalent of persistently encountering the (fatal) mathematical error "undefined" when dividing a non-zero number N by zero; viz. N÷0 = "infinitely large arbitrary number" is undefined, whereas the reciprocal 0÷N = 0 is clearly defined. [The analogy of this (fatal) mathematical error "undefined" will similarly be encountered by all other Incompletely Predictable entities such as nontrivial zeros of Riemann zeta function whereby every single nontrivial zero will need to comply with certain mathematical properties (satisfying Condition X or Y or Z).] Phrased in another way: Once successfully obtained, our Virtual Containers can then be used to rigorously prove Polignac's and Twin prime conjectures (with complete certainty) via the required finite steps of subsequent/concurrent correct analysis on various derived mathematical "meta-properties" (once again satisfying Condition X or Y or Z) obtained from these Virtual Containers.

Furthermore, the Virtual Containers for prime numbers must be endowed with the following key properties and behaviors. They must mathematically (i) incorporate the ability to accurately and completely "contain" the relevant prime (& composite) numbers without being "contaminated" by non-prime (& non-composite) numbers entities, and (ii) not utilize the ability to either fully or partially "calculate" identities of relevant prime (& composite) numbers in an intrinsic manner.

The first & only even prime number '2' associated with prime gap = 1 is not included in Polignac's conjecture but it is a Completely Predictable number. Thus our main goal is to solve Polignac's conjecture which can be succinctly stated as whether even number prime gaps are infinite (and arbitrarily large) in magnitude with each individual even number prime gap generating prime numbers which are again infinite in magnitude. Polignac's conjecture involves analysis of all possible even number prime gaps = 2, 4, 6,... which slowly become infinitely large at prime number examination on larger ranges (in the opposite direction to that of the smallest possible prime gap = 2). Bearing in mind that Twin prime conjecture involves analysis of even number prime gap = 2 (for twin primes), we can regard this conjecture as a mathematical subset of Polignac's conjecture. We use our Virtual Container Research Method, which neatly incorporates the novel mathematical tool coined Information-Complexity conservation, to solve those conjectures. Having obtained the relevant Virtual Container [which is essentially embodied in the relevant Theorem {Polignac-Twin prime} I to IV below], subsequent/concurrent correct analysis of this Virtual Container will result in the rigorous proofs for these two conjectures to materialize. Prime and composite numbers are intimately related simply because the complementary set of composite numbers constitutes the set of natural numbers with the exact set of prime numbers (and the solitary number '1') excluded in its entirety. The Information-Complexity conservation has its core foundation based on this [complete] prime-composite number relationship. In addition, a key mathematical law dubbed 'Plus-Minus Composite Gap 2 Number Alternating Law' (and its companion law dubbed 'Plus Composite Gap 2 Number Continuous Law') arising naturally from applying this Information-Complexity conservation form vital mathematical bridges in achieving those rigorous proofs.

In 2013, Yitang Zhang proved a spectacular landmark mathematical result showing that there is some unknown even number 'N' smaller than 70 million such that there are infinitely many pairs of primes that differ by 'N' [7]. Without going into specific details concerning optimizing Zhang's bound, subsequent Polymath Project collaborative efforts employing a new refinement of the GPY sieve in 2013 lowered 'N' to 246; and assuming the Elliott-Halberstam conjecture and its generalized form have managed to further lower 'N' to 12 and 6, respectively. Thus 'N' has (intuitively) more than one valid values such that there are infinitely many pairs of primes that differ by each of those 'N' values. No matter what, we can only theoretically lower 'N' to 2 (in regards to prime numbers with 'small gaps'), and unfortunately there are still an infinite number of even number prime gaps (in regards to prime numbers with 'large gaps') that will require "the proof that each will generate a set of infinite prime numbers".

On the [dual] infinity of the set of even number prime gaps and of each associated set of prime numbers derived from individual even number prime gap, we can see that the potential mathematical source(s) of prime number infiniteness could feasibly arise in two ways via (i) one or more than one or all of the even number prime gap(s) with those nominated even number prime gap(s) each generating an infinite magnitude of distinct prime numbers &/or (ii) the infinite (and arbitrarily) large magnitude of even number prime gaps collectively able to generate an infinite magnitude of prime numbers such that the criterion in (ii) will hold true even if none of the individual even number prime gap were to ever generate an infinite magnitude of prime numbers. Stated differently for the "none of the individual even number prime gaps were to ever generate an infinite magnitude of prime numbers" scenario, we are alleging that groups of these [imaginary] finite prime numbers derived from each even number prime gap are [wrongly] classified as countable finite sets (CFS) but this arrangement will still culminate in producing a [incomplete] countable infinite set (CIS) of prime numbers as long as there are infinitely many even number prime gaps. We now arrive at the realization that rigorously proving Polignac's and Twin prime conjectures in essence would entail the need to show that (i) the solitary set of all even number prime gaps and (ii) the infinite sets of prime numbers (comprising of prime numbers in totality apart from prime number '2') arising from all even number prime gaps, must all be CIS. We next outline some interesting properties of prime numbers.

English mathematician John Horton Conway coined the term 'jumping champion' in 1993. An integer n is a jumping champion if n is the most frequently occurring difference (prime gap) between consecutive prime numbers $< x$ for some x. Example: for any x with $7 < x < 131$, n = 2 (indicating twin prime numbers) is the jumping champion. It has been conjectured that (i) the only jumping champions are 1, 4 and the primorials 2, 6, 30, 210, 2310, 30030,... and (ii) jumping champions tend to infinity. Their required proofs will likely need the proof of the k-tuple conjecture. [For i = 1, 2, 3, 4, 5, 6,...; primordial $P_i\#$ is the analog of the usual factorial for prime numbers (2, 3, 5, 7, 11, 13,...). For instance $P_1\#$ = 2, $P_2\#$ = 2 X 3 = 6, $P_3\#$ = 2 X 3 X 5 = 30, $P_4\#$ = 2 X 3 X 5 X 7 = 210, $P_5\#$ = 2 X 3 X 5 X 7 X 11 = 2310, $P_6\#$ = 2 X 3 X 5 X 7 X 11 X 13 = 30030, etc.]

We now look at the data of all prime numbers obtained when extrapolated out over a wide range of x values. Generally speaking, as the sequence of prime numbers carries on, prime numbers with ever larger prime gaps will tend to appear. For the given range of x values, we say that prime gap = n_2 is a 'maximal prime gap' if prime gap = $n_1 <$ prime gap = n_2 for all $n_1 < n_2$. In other words, the largest such prime gaps in the sequence are called maximal prime gaps. The ratio n_2 : log (prime number associated with prime gap = n_2) is called the 'merit' of prime gap = n_2 (maximal prime gap). Thus the merit of a prime gap is a normalized number representing how "soon" in the sequence a maximal prime gap appears relative to the natural logarithm of the associated larger prime number of interest. To the best of our knowledge, there is no clear-cut correlation between the largest known merit value and either the relative size of the relevant prime number with prime gap = n_2 or the relative size of that prime gap = n_2.

The term 'first occurrence prime gaps' commonly refers to first occurrences of maximal prime gaps whereby maximal prime gaps can also be perceived here as prime gaps of "at least of this length". The CIS of 'maximal prime gaps' and the (complementary) CIS of 'non-maximal prime gaps' can be fully derived and depicted as below. We endorse non-maximal prime gaps with the interesting nickname 'slow jumpers' in this paper. We coin the term 'slow jumpers' here because non-maximal prime gaps always lag behind their maximal prime gaps counterparts for their onset appearances in the prime number sequence. This is tabulated for the first 17 prime gaps in Table 5 consisting of maximal prime gaps and non-maximal [slow jumper] prime gaps.

Note that the progressive resultant prime numbers generated here in Table 5 solidly represent only a single prime number for each prime gap and this will always be less than the complete set of all prime numbers generated from, for instance, the Sieve of Eratosthenes. The initial seven of the [majority] "missing" prime numbers are 5, 11, 13, 17, 19, 29, 31,...; and they all belong to the subset of prime numbers with 'residual' prime gaps which must be the potential source of prime numbers in relation to the proposal that each of the even number prime gaps of 2, 4, 6, 8, 10,... will generate its specific CIS of prime numbers.

Remark 1.3. Maximal and non-maximal prime gaps supply crucial indirect evidence to intuitively support, but does not prove, the mathematical statement "Each even number prime gap will generate an infinite magnitude of prime numbers on its own accord".

Table 5 First 17 prime gaps depicted in the format utilizing maximal prime gaps [depicted with the asterisk symbol (*)] and non-maximal prime gaps [depicted without this symbol].

Prime gap	Following the prime number
1*	2
2*	3
4*	7
6*	23
8*	89
10	139
12	199
14*	113
16	1831
18*	523
20*	887
22*	1129
24	1669
26	2477
28	2971
30	4297
32	5591

From the above brief analysis on prime number distribution, we easily deduce that [predominantly the groups of] prime numbers with jumping champion prime gaps, [the individual/groups of] prime numbers with maximal prime gaps, and [the individual/groups of] prime numbers with non-maximal prime gaps would seem to make perpetual repeating appearances amongst the complete CIS of prime numbers. A vitally crucial observation is that all prime numbers generated by (i) non-maximal (slow jumper) prime gaps and (ii) maximal prime gaps, will still not generate the complete CIS of prime numbers. This is simply because, apart from the one-off prime gap = 1 associated with the very first prime number '2', all other [infinite magnitude] prime gaps 2, 4, 6, 8, 10,... must each generate more than one, if not a CIS of, prime numbers in order to account for all prime numbers. This clear-cut observation constitutes indirect evidence to intuitively support [but does not prove] the proposition that each even number prime gap will likely generate an infinite magnitude of prime numbers on its own accord.

Although not crucial for the purpose of this paper, we could potentially study exciting Incompletely Predictable behaviors from the subset of prime numbers with 'residual' prime gaps as obtained when the subset of prime numbers with maximal prime gaps and the subset of prime numbers with non-maximal prime gaps are progressively removed from the complete set of prime numbers derived from all known prime gaps. This can mathematically be visualized as: Complete set of prime numbers with all prime gaps = Subset of prime numbers with maximal prime gaps + Subset of prime numbers with non-maximal prime gaps + Subset of prime numbers with 'residual' prime gaps. In addition, prime numbers with 'residual' prime gaps must include all the correctly selected (odd) prime numbers representating all even number prime gaps 2, 4, 6, 8, 10.... [except the one-off very first odd number prime gap = 1 representing the very first (and only even) prime number '2'].

2 The eight categories of intercepts for 2-Variable Equations

In the Class of n-Variable Equations with n = 2 [which translate to 2-Variable Equations], when computationally depicted by 2-dimensional graphs with their x- and y-axes relevantly

defined; they often have one or more points of intersection on (i) x-axis, and/or (ii) y-axis, and/or (iii) both x- and y-axes [formally known as the 'Origin']. The Origin, often labeled with capital letter 'O', is defined as the point where the vertical y-axis and the horizontal x-axis intersect each other. Not all functions, though, will have intercepts; which are where the graph crosses either the x-axis (viz. the x-axis intercept, often referred to as "zeros or roots of the equation"), or the y-axis (viz. the y-axis intercept), or both the x- and y-axes (viz. the Origin intercept). There are eight possible Categories of Intercepts for 2-Variable Equations, as detailed below:
Category I Intercept: comprising of nil intercept
Category II Intercept: comprising of single x-axis intercept(s) only
Category III Intercept: comprising of single y-axis intercept(s) only
Category IV Intercept: comprising of single Origin intercept(s) only
Category V Intercept: comprising of double x- and y-axes intercept(s)
Category VI Intercept: comprising of double x-axis and Origin intercept(s)
Category VII Intercept: comprising of double y-axis and Origin intercept(s)
Category VIII Intercept: comprising of triple x-, y-axes and Origin intercept(s)

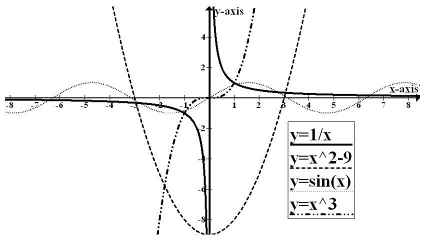


Fig. 1 Sample graphs for Category I Intercept using $y = \frac{1}{x}$ with nil intercept; Category V Intercept using $y = (x+3)(x-3) = x^2 - 9$ with two x-axis intercepts and one y-axis intercept; Category VI Intercept using $y = \sin(x)$ with one Origin intercept and infinite number of x-axis intercepts; and Category VIII Intercept using $y = x^3$ with one Origin intercept.

Combinatorics language wise, the eight categories are from the sum total of (a) choosing zero item from three as in Category I; (b) choosing one item from three as in Category II, III, and IV; (c) choosing two items from three as in Category V, VI, and VII; and (d) choosing three items from three as in Category VIII. Theoretically, the various intercepts could numerically consist of whole numbers from $0, 1, 2, ..., \infty$. Permutational wise, this observation would apparently result in (not unexpectedly) limitless, or almost limitless, 'exotic-flavored' equations; typified (for instance) by the simple case of 2-Variable Equation $y = \sin(x)$ (shown as part of Figure 1 and discussed under Remark 1.1 in Section 1.1 above as an example of 'Axes intercepts relationship interface'), which is easily seen as belonging to Category VI Intercept containing a Completely Predictable solitary Origin intercept combined with a Completely Predictable infinite number of x-axis intercepts.

3 Riemann zeta and Dirichlet eta functions

The iconic Riemann zeta function, $\zeta(s)$, is a function of complex variable s (= $\sigma \pm \imath t$) that analytically continues the sum of an infinite series $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots$. The arguments of this function are traditionally denoted by two letters: sigma (σ) for the real part, and t for the imaginary part where $i = \sqrt{-1}$ is the imaginary number. Alternatively stated, $\zeta(s)$ is the famous complex number infinite series constituting a real and an imaginary part determined by its complex variable s; whereby s itself is further constituted by a real part σ, and an imaginary part t. In practice, the positive ($0 < t < +\infty$) and numerically equal to the negative ($-\infty < t < 0$), counterpart of the conjugate pairs for x-axis, y-axis, and Origin intercepts is usually quoted or employed for calculation purposes.

The Dirichlet eta function, $\eta(s)$, also known as the alternating zeta function, must act as the proxy/surrogate for $\zeta(s)$ in the critical strip ($0 < \sigma < 1$) whereby the critical line ($\sigma = \frac{1}{2}$) lies. This is because $\zeta(s)$ only converges when $\sigma > 1$, implying that it is essentially undefined to the left of this region [viz. $0 < \sigma < 1$] which then requires its proxy $\eta(s)$ representation instead. Their mathematical relationship is defined by $\zeta(s) = \gamma.\eta(s)$, whereby the proportionality factor γ is defined as $\gamma = \frac{1}{(1-2^{1-s})}$ and $\eta(s) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s} = \frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \cdots$.

The above paragraphs are further discussed in terms of Simplicity and Complexity. The concept of Simplexity [as opposed to Simplicity] could be amplified as the process by which nature strives towards simple ends by complex or complicated means. In other words, Simplexity may be defined as the combination of Simplicity and Complexity within the context of a dynamic relationship between means and ends. We observe the extra presence of alternating + and - signs in $\eta(s)$ conferring an extra layer of Complexity as opposed to just the + sign present in $\zeta(s)$ with relative Simplicity – bearing in mind that there are in general "Subjective to Semi-objective to Objective views on the Simplicity to Complexity continuum-spectrum range". Stated in a slightly different manner, the words 'Simplicity' and 'Complexity' can be seen to roughly progress along the following equivalent but opposing continuum-spectrum range.

Left————————————>Right

Decreasing Order from Left to Right for

Maximal Simplicity to Minimal Simplicity range

should correspond to:

Increasing Order from Left to Right for

Minimal Complexity to Maximal Complexity range

Left————————————->Right

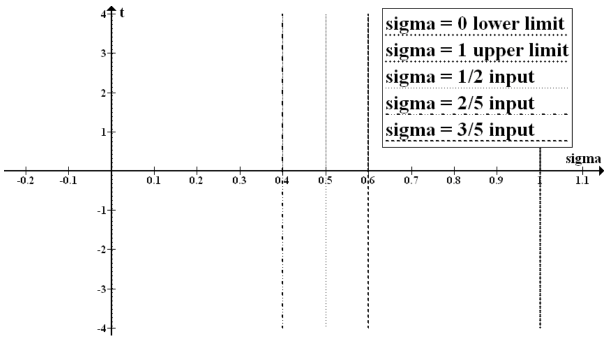


Fig. 2 INPUT for $\sigma = \frac{1}{2}$, $\frac{2}{5}$, and $\frac{3}{5}$. For Riemann zeta function, the set of zeros, or roots, in ζ(s) consist of the easily identifiable (Completely Predictable) trivial zeros located at σ = negative even numbers -2, -4, -6, -8, -10,... and the not-so-easily identifiable (Incompletely Predictable) nontrivial zeros located at σ = $\frac{1}{2}$ for various computed t values. Both trivial and nontrivial zeros are of infinite magnitude.

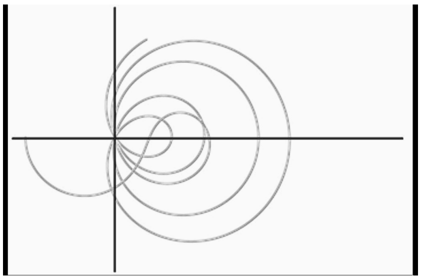


Fig. 3 OUTPUT for $\sigma = \frac{1}{2}$. Schematically depicted polar graph of $\zeta(\frac{1}{2} + \imath t)$ with plot of $\zeta(s)$ along the critical line for real values of t running from 0 to 34, horizontal axis: $Re\{\zeta(\frac{1}{2} + \imath t)\}$, and vertical axis: $Im\{\zeta(\frac{1}{2} + \imath t)\}$. The presence of Origin intercepts is clearly shown on this figure.

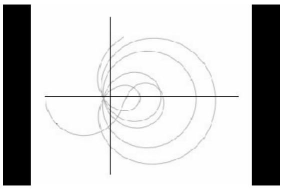


Fig. 4 OUTPUT for $\sigma = \frac{2}{5}$ with identical axes definitions as that used in Figure 3.

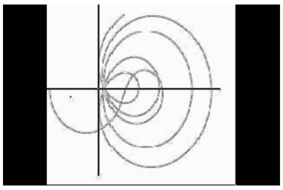


Fig. 5 OUTPUT for $\sigma = \frac{3}{5}$ with identical axes definitions as that used in Figure 3.

$$
\begin{aligned}
\zeta(s) &= \sum_{n=1}^{\infty} \frac{1}{n^s} \qquad (1)\\
&= \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots \\
&= \Pi_{p\ prime} \frac{1}{(1-p^{-s})} \\
&= \frac{1}{(1-2^{-s})} \cdot \frac{1}{(1-3^{-s})} \cdot \frac{1}{(1-5^{-s})} \cdot \frac{1}{(1-7^{-s})} \cdot \frac{1}{(1-11^{-s})} \cdots \frac{1}{(1-p^{-s})} \cdots
\end{aligned}
$$

Eq. (1) can only be defined for the $1 < \sigma < \infty$ region whereby $\zeta(s)$ is absolutely convergent. There are no zeros located in this region. The equivalent Euler product formula with product over prime numbers [instead of summation over natural numbers] can also be used to represent Riemann zeta function.

$$
\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) .\Gamma(1-s).\zeta(1-s) \qquad (2)
$$

With $\sigma = \frac{1}{2}$ as a symmetry line of reflection, Eq. (2) is the Riemann's functional equation fully satisfying $-\infty < \sigma < \infty$ and can be used to find all trivial zeros on the horizontal line at $\imath t = 0$ and σ = -2, -4, -6, -8, -10,..., ∞ [all negative even numbers] whereby $\zeta(s) = 0$ because the factor $\sin(\frac{\pi s}{2})$ vanishes. Γ is the gamma function, an extension of the factorial function [a product function denoted by the ! notation; n! = n(n-1)(n-2)...(n-(n-1))] with its argument shifted down by 1, to real and complex numbers. That is, if n is a positive integer, $\Gamma(n) = (n-1)!$

$$\zeta(s) = \frac{1}{(1-2^{1-s})} \cdot \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s} = \frac{1}{(1-2^{1-s})} \cdot \left(\frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \cdots \right) \tag{3}$$

Eq. (3) is defined for all $\sigma > 0$ except for a simple pole at σ = 1. As just alluded to above, ζ(s) without the $\frac{1}{(1-2^{1-s})}$ proportionality factor, viz. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^s}$ is also known as Dirichlet eta (η) or alternating zeta function. This $\eta(s)$ function is a holomorphic function of s as defined by analytic continuation and can mathematically be seen to be defined at σ = 1 whereby an analogous trivial zeros [with presence only] for η(s) [and not for ζ(s)] on the vertical straight line σ = 1 are obtained at $s = 1 \pm i.\frac{2\pi k}{\log(2)}$ where k = 1, 2, 3, ..., ∞. All nontrivial zeros are conjectured to be located on the critical line ($\sigma = \frac{1}{2}$) in the critical strip ($0 < \sigma < 1$) of this region.

Any given function or equation including our $\zeta(s)$ can be supplied with an INPUT and resulting in an OUTPUT. Figure 2 pictorially depict complex variable s (= $\sigma \pm \imath t$) as INPUT with x-axis denoting the real part Re{s}, equating to σ; and y-axis denoting the imaginary part Im{s}, equating to t. Figures 3, 4, and 5 schematically depict (respectively) $\zeta(s)$ as OUTPUT for real values of t running from 0 to 34 at $\sigma = \frac{1}{2}$ (critical line), $\sigma = \frac{2}{5}$, and $\sigma = \frac{3}{5}$ with x-axis denoting the real part Re$\{\zeta(s)\}$ and y-axis denoting the imaginary part Re$\{\zeta(s)\}$. Riemann hypothesis can be computationally visualized as the appearance of infinite number of Origin intercepts by its generated spirals/curves occurring only when $\sigma = \frac{1}{2}$. There will be an infinite types-of-spirals possibilities associated with each and every σ value arising from all possible infinite σ values in the critical strip.

From this, we derive the following two "interim" propositions for $\zeta(s)$ with their proofs which will be shown to naturally arise out of the "other relevant proofs" obtained for subsequent lemmas, propositions and theorems below. This ambitious statement will be shown to be correct simply because those "other relevant proofs" [mathematically] culminates in the action of rigorously proving Riemann hypothesis viz. proving the single line location of all nontrivial zeros (of infinite magnitude); and those nontrivial zeros can [geometrically] be simultaneously seen to be totally equivalent to all Origin intercepts (of infinite magnitude).

Proposition 3.1. Only at $\sigma = \frac{1}{2}$ value will the generated single [finite] type-of-spiral have Category VIII Intercept comprising of triple x-axis, y-axis and Origin intercepts and with all these intercepts consisting of transcendental numbers having Incompletely Predictable properties and be of infinite magnitude.

Proposition 3.2. For all other $\sigma \neq \frac{1}{2}$ (viz. $0 < \sigma < \frac{1}{2}$ and $\frac{1}{2} < \sigma < 1$) values, the generated multiple [infinite] types-of-spirals have Category V Intercept comprising of double x-axis and y-axis intercepts and with all these intercepts consisting of transcendental numbers having Incompletely Predictable properties and be of infinite magnitude.

The conceptually important mathematical ideas depicted under Remark 1.2 in subsection 1.2 above are that the 'varying gap' between two consecutive nontrivial zeros (dubbed nontrivial zeros gap), two consecutive 'usual' Gram points (dubbed Gram points gap) and two consecutive Gram[x=0] points (dubbed Gram[x=0] points gap) can be seen as the equivalent 'varying gradient' dubbed, respectively, nontrivial zeros gradient, 'usual' Gram points gradient and Gram[x=0] points gradient. We can now geometrically visualize these 'varying gaps' or 'varying gradients' as being related to the size and shape of those spirals/loops present in Figure 3. The foundation mathematics targeted to concisely prove the infinite magnitude existence of all intercepts associated with both propositions above would naturally belong to the realm of rigorously proving Dirichlet-Gram-Riemann conjecture which literally consists of Gram[x=0] conjecture, Gram[y=0] or 'usual' Gram conjecture, and Gram[x=0,y=0] conjecture (Riemann hypothesis). The expanded explanations on each respective conjecture are elaborated in the relevant sections below. But, firstly, preliminary $\zeta(s)$ and $\eta(s)$ nomenclature materials with interpretational meanings are provided by the following six clear-cut correlation points.

(A) At the one specific $\sigma = \frac{1}{2}$ value whereby the term Gram points is understood to denote the "Critical line-Gram points" official notation;

Point 1. The Origin intercepts are synonymous with all Gram[x=0,y=0] points or the traditionally denoted 'nontrivial zeros'. The associated Riemann hypothesis is synonymous with Gram[x=0,y=0] conjecture.

Point 2. The x-axis intercepts are synonymous with all Gram[y=0] points or the traditionally denoted 'Gram points' (plus the very first and only negative Gram[y=0] point). This is associated with Gram[y=0] conjecture.

Point 3. The y-axis intercepts are synonymous with all Gram[x=0] points. This is associated with Gram[x=0] conjecture.

(B) For all other infinite $\sigma \neq \frac{1}{2}$ (viz. $0 < \sigma < \frac{1}{2}$ and $\frac{1}{2} < \sigma < 1$) values whereby the term 'near-identical' (virtual) Gram points is understood to denote the "Non-critical lines-Gram points" official notation;

Point 4. The Origin intercepts are non-existent.

Point 5. The x-axis intercepts are synonymous with all 'near-identical' (virtual) Gram[y=0] points (plus the very first and only negative (virtual) Gram[y=0] point). These points have totally different numerical values to the Gram[y=0] points mentioned in Point 2.

Point 6. The y-axis intercepts are synonymous with all 'near-identical' (virtual) Gram[x=0] points. These points have totally different numerical values to the Gram[x=0] points mentioned in Point 3.

4 {Geometrical-format-version} conjectures on Riemann zeta function

Our Gram[y=0] {geometrical-format-version} conjecture is explicitly equivalent to the statement that the infinite number of x-axis intercepts / Gram[y=0] points (plus the solitary negative Gram[y=0] point) derived from the infinite number of spirals generated by Riemann zeta function in the critical strip occurs only on the critical line denoted by $\sigma = \frac{1}{2}$.

Our Gram[x=0] {geometrical-format-version} conjecture is explicitly equivalent to the statement that the infinite number of y-axis intercepts / Gram[x=0] points derived from the infinite number of spirals generated by Riemann zeta function in the critical strip occurs only on the critical line denoted by $\sigma = \frac{1}{2}$.

From previous reasoning above, we can now justifiably coin the 1859 Riemann {geometrical-format-version} hypothesis as Gram[x=0,y=0] {geometrical-format-version} conjecture; which is explicitly equivalent to the statement that the infinite number of Origin intercepts / Gram[x=0,y=0] points / nontrivial zeros derived from the infinite number of spirals generated by Riemann zeta function in the critical strip occurs only on the critical line denoted by $\sigma = \frac{1}{2}$. In descriptive terms, this is seen in Figure 3 as Riemann zeta function in the critical strip generating an infinite number of spirals graphically intersecting the Origin an infinite number of times only on the critical line which is denoted by $\sigma = \frac{1}{2}$.

Then, as honor and tribute to the three famous namesake mathematicians, it is pure common sense to create the 'glorified' Dirichlet-Gram-Riemann {geometrical-format-version} conjecture which is compatibly equivalent to the statement that the infinite number of all Gram[x=0] points, all Gram[y=0] points (plus the solitary negative Gram[y=0] point), and all Gram[x=0,y=0] points derived from the infinite number of spirals generated by Riemann zeta function in the critical strip occurs only on the critical line denoted by $\sigma = \frac{1}{2}$.

5 Prerequisite lemma, corollary and propositions for Gram[x=0,y=0] conjecture (Riemann hypothesis)

We treat and closely analyze Riemann zeta and Dirichlet eta functions as unique mathematical objects looking for key intrinsic properties and behaviors. As original true equations containing all possible x-axis, y-axis and Origin intercepts, Riemann zeta and Dirichlet eta functions by themselves viz. *without* computationally supplying "input information" as depicted in Figure 2 [with horizontal axis: σ and vertical axis: t] so as *not* to generate the necessary "output complexity" as depicted in Figure 3 [with horizontal axis: $Re\{\zeta(\frac{1}{2}+\imath t)\}$ and vertical axis: $Im\{\zeta(\frac{1}{2}+\imath t)\}$]; these two functions will both intrinsically incorporate the **actual presence [but not the actual locations]** of the complete set of Gram[x=0] points, Gram[y=0] points, and Gram[x=0,y=0] points. We use the typical case of nontrivial zeros or Gram[x=0,y=0] points to obtain the required lemma, corollary, propositions and their proofs prior to ultimately proving the relevant Theorem {Riemann} I to IV below. The lemma, corollary, propositions and proofs associated with the other two cases of Gram[x=0] and Gram[y=0] points will simply reflect "slight mathematical variations to the theme for nontrivial zeros". These are outlined in Appendix A.

Lemma 5.1. The Riemann-Dirichlet Ratio is derived from Riemann zeta or Dirichlet eta function and thus has the resultant capability to incorporate the actual presence [but not the actual locations] of the complete set of nontrivial zeros.

Proof. Euler formula is commonly stated as $e^{\imath x} = \cos x + \imath.\sin x$. The magnificent Euler identity (where $x = \pi$) is $e^{\imath\pi} = \cos\pi + \imath.\sin\pi = -1+0$, commonly stated as $e^{\imath\pi}+1 = 0$. The n^s of Riemann zeta function can be expanded to $n^s = n^{(\sigma+\imath t)} = n^{\sigma}.e^{t.\log(n).\imath}$ since $n^t = e^{t.\log(n)}$. Apply the Euler formula to n^s will result in $n^s = n^{\sigma}.(\cos(t.\log(n)) + \imath.\sin(t.\log(n))$ – designated here with the short-hand notation $n^s(Euler)$ – whereby n^{σ} is the modulus and $t.\log(n)$ is the polar angle.

Apply $n^s(Euler)$ to Eq. (1), we have $\zeta(s) = Re\{\zeta(s)\} + \imath.Im\{\zeta(s)\}$ whereby $Re\{\zeta(s)\} = \sum_{n=1}^{\infty} n^{-\sigma}.\cos(t.\log(n))$ and $Im\{\zeta(s)\}$ = i.$\sum_{n=1}^{\infty} n^{-\sigma}.\sin(t.\log(n))$. As Eq. (1) is defined only for $\sigma > 1$ where zeros never occur, we will not carry out further treatment related to this subject area.

Apply $n^s(Euler)$ to Eq. (3), we have $\zeta(s) = \gamma.\eta(s) = \gamma.[Re\{\eta(s)\} + \imath.Im\{\eta(s)\}]$ whereby

$$Re\{\eta(s)\} = \sum_{n=1}^{\infty}((2n-1)^{-\sigma}.\cos(t.\log(2n-1)) - (2n)^{-\sigma}.\cos(t.\log(2n)))$$ and

$$Im\{\eta(s)\} = \imath.\sum_{n=1}^{\infty}((2n)^{-\sigma}.\sin(t.\log(2n)) - (2n-1)^{-\sigma}.\sin(t.\log(2n))).$$ Here γ is the proportionality factor $\frac{1}{(1-2^{1-s})}$.

Apply the trigonometry identity $\cos(x) - \sin(x) = \sqrt{2}.\sin\left(x + \frac{3}{4}\pi\right)$ to $\sum ReIm\{\eta(s)\} = Re\{\eta(s)\} + Im\{\eta(s)\}$. Then,

$$\begin{aligned}
&\sum ReIm\{\eta(s)\} \\
&= \sum_{n=1}^{\infty}[(2n-1)^{-\sigma}.\cos(t.\log(2n-1))\mathbf{TAG}:+\cos\mathbf{2n-1}(\mathbf{Re}) - (2n-1)^{-\sigma}.\sin(t.\log(2n-1)) \\
&\qquad \mathbf{TAG}:-\sin\mathbf{2n-1}(\mathbf{Im}) - (2n)^{-\sigma}.\cos(t.\log(2n))\mathbf{TAG}:-\cos\mathbf{2n}(\mathbf{Re}) \\
&\qquad + (2n)^{-\sigma}.\sin(t.\log(2n))\mathbf{TAG}:+sin\,\mathbf{2n}(\mathbf{Im})] \\
&= \sqrt{2}\sum_{n=1}^{\infty}[(2n-1)^{-\sigma}.\sin(t.\log(2n-1) + \frac{3}{4}\pi)\mathbf{TAG}:+\cos\mathbf{2n-1}(\mathbf{Re})\& - \sin\mathbf{2n-1}(\mathbf{Im}) \\
&+ -(2n)^{-\sigma}.\sin(t.\log(2n) + \frac{3}{4}\pi)\mathbf{TAG}:-\cos\mathbf{2n}(\mathbf{Re})\& + \sin\mathbf{2n}(\mathbf{Im})]
\end{aligned} \tag{4}$$

Note our self-explanatory **TAG** legend used to illustrate where each term in the equations above originated from. It can easily be seen that both terms in the final equation consist of a mixture of real and imaginary portions. As Riemann hypothesis on nontrivial zeros based on $\zeta(s)$ is identical to that based on its proxy $\eta(s)$, then it is satisfied when

$$\sum ReIm\{\eta(s)\} = Re\{\eta(s)\} + Im\{\eta(s)\} = 0 \tag{5}$$

Ignoring the $\sqrt{2}$ term temporarily and applying Eq. (5), Eq. (4) becomes

$$\sum_{n=1}^{\infty}(2n-1)^{-\sigma}.\sin(t.\log(2n-1) + \frac{3}{4}\pi) = \sum_{n=1}^{\infty}(2n)^{-\sigma}.\sin(t.\log(2n) + \frac{3}{4}\pi) \tag{6}$$

We note from the above sequential mathematical derivation of Eq. (6) that this equation will completely and intrinsically fulfills the 'presence of the complete set of nontrivial zeros without knowing their actual location' criteria.

$$\frac{\sum_{n=1}^{\infty}\sin(t.\log(2n) + \frac{3}{4}\pi)}{\sum_{n=1}^{\infty}\sin(t.\log(2n-1) + \frac{3}{4}\pi)} = \frac{\sum_{n=1}^{\infty}(2n)^{\sigma}}{\sum_{n=1}^{\infty}(2n-1)^{\sigma}} \tag{7}$$

Eq. (7) will also abide to this specified criteria as it is simply the result of rearranging the terms in Eq. (6) thus giving rise to our desired Riemann-Dirichlet Ratio. *The proof is now complete for Lemma 5.1*□.

In Eq. (7), denote the left hand side ratio as Ratio R1 of a 'cyclical' nature (generating cyclical outputs due to the presence of sine function) and the right hand side ratio as Ratio R2 of a 'non-cyclical' nature (generating non-cyclical outputs). Riemann-Dirichlet Ratio can be deemed to be represented by a more complicated 'dynamic' version of [infinite length] Hybrid integer sequence in that besides consisting of a particular 'Class function' expressed in Ratio R1's numerator and denominator, this first Ratio R1 is again given as an equality to another seemingly different looking Ratio R2 whose numerator and denominator are expressed by yet another different 'Class function'. One may intuitively think of a non-Hybrid integer sequence to metaphorically arise from a Hybrid integer sequence "in the limit" these two different 'Class functions' in Hybrid integer sequence becomes one same

'Class function' in the new non-Hybrid integer sequence. Note the absence and presence of σ variable in Ratio R1 and R2 respectively.

Riemann-Dirichlet Ratio calculations, valid for all continuous real number values of t, would theoretically result in infinitely many non-Hybrid integer sequences [here arbitrarily] for the $0 < \sigma < 1$ critical strip region of interest with n = 1, 2, 3,..., ∞ being discrete integer number values, or n being continuous real numbers from 1 to ∞ with Riemann integral applied in the interval from 1 to ∞. This infinitely many integer sequences can geometrically be interpreted to representatively cover the entire plane of the critical strip bounded by σ values of 0 and 1, thus (at least) allowing our proposed proof to be of a 'complete' nature.

Proposition 5.2. The Sigma-Power Laws can be rigorously derived from Riemann-Dirichlet Ratio.

Proof. In Calculus, integration is defined as the reverse process of differentiation. Integration is geometrically viewed as the area enclosed by the curve of the function and the axis. Using the definite integral *I* between the points a and b (i.e. in the interval [a, b] where a <b) and computing the value when $\Delta x \longrightarrow 0$, we get $I = \lim_{\Delta x \longrightarrow 0} \sum_{i=1}^{n} f(x_i)\Delta x_i = \int_a^b f(x)dx$ – this is the Riemann integral of function f(x) in the interval [a, b]. We apply Riemann integral to the four continuous functions of Ratio R1 and Ratio R2 in Eq. (7) thus depicting Riemann-Dirichlet Ratio in integral forms – see subsequent Eq. (12) below.

Thereafter, step-by-step we derive the closely related Dirichlet σ-Power Law [expressed in real numbers] and the Riemann σ-Power Law [expressed in real and complex numbers]. Due to the resemblance to various power-law functions in that the σ variable from s (= $\sigma + \imath t$) being the exponent of a power function n^{σ}, the log scale use, and the harmonic ζ(s) series connection in Zipf's law; we explain here why we have elected to endow our newly derived formula with the name Sigma-Power Law. Its Dirichlet and Riemann versions are directly related to each other via Dirichlet η(s) being the equivalence of Riemann ζ(s) but without the $\frac{1}{(1-2^{1-s})}$ proportionality factor. We stress the underlying mathematically-consistent properties of *symmetry* and *constraints* arising from this power law that allows our most direct, basic and elementary proof for Riemann hypothesis to mature. An important characteristic to note of σ-Power Law is that its exact formula expression in the usual mathematical language [$y = f(x_1, x_2)$ format description for a 2-variable function] consists of $y = \{2n\}$ or $\{2n-1\} = f(t, \sigma)$ with n = 1, 2, 3,..., ∞ or n = 1 to ∞ with Riemann integral application; -∞ < t < +∞; and σ being of real number values $0 < \sigma < 1$ corresponding to the [arbitrarily defined] critical strip of interest in this particular case scenario.

For the, initially, $\{2n\}$ parameter integration of R1, $\int_1^{\infty} \sin\left(t.\log(2n) + \frac{3}{4}\pi\right).dn$

Use integration by u-substitution technique to obtain $u = t.\log(2n) + \frac{3}{4}\pi, n = \frac{1}{2}e^{\frac{1}{t}\left(u-\frac{3}{4}\pi\right)}, \frac{du}{dn} = \frac{2t}{2n} = \frac{t}{n}, du = t.\frac{dn}{n}, dn = 2n.\frac{du}{2t} = n.\frac{du}{t}$
$\int_1^{\infty} \sin(u).\frac{n}{t}.du = \int_1^{\infty} \sin(u).\frac{1}{t}.\frac{1}{2}.e^{\frac{1}{t}\left(u-\frac{3}{4}\pi\right)}.du = \frac{1}{2t}.e^{\frac{3}{4}\pi} \int_1^{\infty} \sin(u).e^{\frac{1}{t}u}.du$

Use the Products of functions proportional to their second derivatives, namely the indefinite integral

$\int \sin(a.u).e^{b.u}du = \frac{e^{bu}}{a^2+b^2}(b.\sin(a.u) - a.\cos(a.u)) + C.$

Then a = 1, $b = \frac{1}{t}$, and temporarily ignore the $\frac{1}{2t}e^{\frac{3}{4}\pi}$ term, we have

$$\int_1^\infty \sin(u).e^{\frac{1}{t}u}.du$$
$$= [(e^{\frac{1}{t}u})/(1+\frac{1}{t^2})).(\frac{1}{t}.\sin(u) - \cos(u)) + C]_1^\infty$$
$$= [(t^2.e^{\frac{1}{t}u})/(t^2+1).(\frac{1}{t}.\sin(u) - \cos(u)) + C]_1^\infty$$

Now apply the non-linear combination of sine and cosine functions identity, namely $a.\sin(u) + b.\cos(u) = c.\sin(u+\varphi)$ where $c = \sqrt{(a^2+b^2)}$ and $\varphi = atan2(b,a)$.

Here $a = \frac{1}{t}, b = -1, c = \sqrt{\left((\frac{1}{t})^2 + 1\right)} = \frac{\sqrt{(t^2+1)}}{t}$. Then we have

$$\int_1^\infty \sin(u).e^{\frac{1}{t}u}.du$$
$$= [(t^2.e^{\frac{1}{t}u})/(t^2+1).\frac{\sqrt{(t^2+1)}}{t}.\sin(u + atan2(b,a)) + C]_1^\infty$$
$$= [(t.e^{\frac{1}{t}u})/\sqrt{(t^2+1)}).\sin(u + \arctan(t)) + C]_1^\infty$$

But there was a $\frac{1}{2t}.e^{\frac{3}{4}\pi}$ term in front of this integral as can be seen above. Then after substituting this term and simplifying, the integral

$$\int_1^\infty \sin(u).e^{\frac{1}{t}u}.du$$
$$= [\left(e^{\frac{1}{t}u - \frac{3}{4}\pi}\right)/2\sqrt{(t^2+1)}.\sin(u - \arctan(t)) + C]_1^\infty$$

But $u = t.\log(2n) + \frac{3}{4}\pi$. Reverting back to the n variable, and incorporating $\sqrt{2}$ originating from the beginning during Eq. (6) derivation, the equation for the $\{2n\}$ parameter finally becomes

$$\sqrt{2}\int_1^\infty \sin(t.\log(2n) + \frac{3}{4}\pi).dn$$
$$= [\sqrt{2}.(\{2n\}.e^{\frac{1}{t}\cdot\frac{3}{4}\pi})/(2\sqrt{(t^2+1)}.e^{\frac{3}{4}\pi}.\sin(t.\log(2n) + \frac{3}{4}\pi - \arctan(t)) + C)]_1^\infty \quad (8)$$

In a similar manner integration for the $\{2n\text{-}1\}$ parameter, this equation becomes

$$[\sqrt{2}.(\{2n-1\}.e^{\frac{1}{t}\cdot\frac{3}{4}\pi})/(2\sqrt{(t^2+1)}.e^{\frac{3}{4}\pi}.\sin(t.\log(2n-1) + \frac{3}{4}\pi - \arctan(t)) + C)]_1^\infty \quad (9)$$

In R2 using $\{2n\}$ parameter,

$$\int_1^\infty (2n)^\sigma dn$$
$$= [1/(2(\sigma+1)).(2n)^{\sigma+1} + C)]_1^\infty$$
$$= [\frac{1}{3}\{2n\}(2n)^{\frac{1}{2}} + C)]_1^\infty \text{ when } \sigma = \frac{1}{2} \quad (10)$$

For the equivalent R2 based on {2n-1} parameter,

$$\begin{aligned}
&\int_1^\infty (2n-1)^\sigma dn \\
&= [1/(2(\sigma+1)).(2n-1)^{\sigma+1}+C)]_1^\infty \\
&= [\frac{1}{3}\{2n-1\}(2n-1)^{\frac{1}{2}}+C)]_1^\infty \text{ when } \sigma = \frac{1}{2}
\end{aligned} \tag{11}$$

The Ratio R1 and Ratio R2 of Riemann-Dirichlet Ratio (for $\sigma = \frac{1}{2}$) is defined by the integral

$$\frac{[(\{2n\}.(e^{\frac{1}{t}\cdot\frac{3}{4}\pi}/2\sqrt{(t^2+1)}.e^{\frac{3}{4}\pi}).\sin(t.\log(2n)+\frac{3}{4}\pi-\arctan(t))]_1^\infty}{[(\{2n-1\}.e^{\frac{1}{t}\cdot\frac{3}{4}\pi}/2\sqrt{(t^2+1)}.e^{\frac{3}{4}\pi}).\sin(t.\log(2n-1)+\frac{3}{4}\pi-\arctan(t))]_1^\infty} = \frac{[\frac{1}{3}\{2n\}(2n)^{\frac{1}{2}}]_1^\infty}{[\frac{1}{3}\{2n-1\}(2n-1)^{\frac{1}{2}}]_1^\infty}$$

Canceling out the common parameter {2n} and {2n-1} terms,

$$\begin{aligned}
&\frac{[(e^{\frac{1}{t}\cdot\frac{3}{4}\pi}/2\sqrt{(t^2+1)}.e^{\frac{3}{4}\pi}).\sin(t.\log(2n)+\frac{3}{4}\pi-\arctan(t))]_1^\infty}{[(e^{\frac{1}{t}\cdot\frac{3}{4}\pi}/2\sqrt{(t^2+1)}.e^{\frac{3}{4}\pi}).\sin(t.\log(2n-1)+\frac{3}{4}\pi-\arctan(t))]_1^\infty} \leftarrow \textbf{this is R1} \\
&= \frac{[\frac{1}{3}(2n)^{\frac{1}{2}}]_1^\infty}{[\frac{1}{3}(2n-1)^{\frac{1}{2}}]_1^\infty} \leftarrow \textbf{this is R2}
\end{aligned} \tag{12}$$

The γ proportionality factor term in Riemann ζ function, viz. $\frac{1}{(1-2^{1-s})}$, can also be expressed with the aid of Euler formula as follows (with the formula for $\sigma = \frac{1}{2}$ substitution depicted last).

$$\begin{aligned}
&\frac{1}{(1-2^{1-s})} \\
&= \frac{(2^\sigma.2^{it})}{(2^\sigma.2^{it}-2)} \\
&= \frac{(2^\sigma.e^{t.\log(2\iota)})}{(2^\sigma.e^{t.\log(2\iota)}-2)} \\
&= \frac{(2^\sigma.(\cos(t.\log(2)+\iota.\sin(t.\log(2))}{(2^\sigma.(\cos(t.\log(2)+\iota.\sin(t.\log(2)-2)} \\
&= \frac{(2^{\frac{1}{2}}.(\cos(t.\log(2)+\iota.\sin(t.\log(2))}{(2^{\frac{1}{2}}.(\cos(t.\log(2)+\iota.\sin(t.\log(2)-2)}
\end{aligned} \tag{13}$$

The Dirichlet and Riemann σ-Power Laws are given by the exact formulae in Eqs. (14) to (17) below with ψ being the same proportionality constant valid for both power laws. We can now dispense with the constant of integration C. **Using Dimensional analysis (DA) approach we can easily conclude that the 'fundamental dimension' [Variable / Parameter / Number X to the power of Number Y] has to be represented by the particular 'unit of measure' [Variable / Parameter / Number X to the power of Number Y whereby Number Y needs to be of the specific value $\frac{1}{2}$] for DA homogeneity to occur. This *de novo* DA homogeneity equates to the location of the complete set of nontrivial zeros and is crucially a fundamental property present in all laws of Physics. The 'unknown' σ variable, now endowed with value of $\frac{1}{2}$, is treated as Number Y.**

Dirichlet σ-Power Law using the {2n} parameter:

$$\left[2^{\frac{1}{2}}.\{2n\}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n)+\frac{3}{4}\pi-\arctan(t))\right]_1^{\infty}=\left[\psi.\frac{1}{3}\{2n\}(2n)^{\frac{1}{2}})\right]_1^{\infty}$$

With common parameter {2n} canceling out on both sides, the equation reduces to

$$\left[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n)+\frac{3}{4}\pi-\arctan(t))-\psi.\frac{1}{3}(2n)^{\frac{1}{2}})\right]_1^{\infty}=0 \quad (14)$$

Similarly for the {2n-1} parameter, this equivalent equation is

$$\left[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n-1)+\frac{3}{4}\pi-\arctan(t))-\psi.\frac{1}{3}(2n-1)^{\frac{1}{2}})\right]_1^{\infty}=0 \quad (15)$$

Finally, the Riemann σ-Power Law is given by the exact formulae using {2n} and {2n-1} parameters with the $\gamma=(2^{\frac{1}{2}}.(\cos(t.\log(2)+\imath.\sin(t.\log(2))/(2^{\frac{1}{2}}.(\cos(t.\log(2)+\imath.\sin(t.\log(2)-2)$ substitution.

$$\left[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n)+\frac{3}{4}\pi-\arctan(t))-\psi.\gamma.\frac{1}{3}(2n)^{\frac{1}{2}})\right]_1^{\infty}=0 \quad (16)$$

$$\left[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n)+\tfrac{3}{4}\pi-\arctan(t))-\psi.\frac{2^{\frac{1}{2}}.(\cos(t.\log(2)+\imath.\sin(t.\log(2))))}{2^{\frac{1}{2}}.(\cos(t.\log(2)+\imath.\sin(t.\log(2)-2)))}.\tfrac{1}{3}(2n)^{\frac{1}{2}})\right]_1^{\infty}=0$$

$$\left[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n-1)+\frac{3}{4}\pi-\arctan(t))-\psi.\gamma.\frac{1}{3}(2n-1)^{\frac{1}{2}})\right]_1^{\infty}=0 \quad (17)$$

$$\left[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n-1)+\tfrac{3}{4}\pi-\arctan(t))-\psi.\frac{2^{\frac{1}{2}}.(\cos(t.\log(2)+\imath.\sin(t.\log(2))))}{2^{\frac{1}{2}}.(\cos(t.\log(2)+\imath.\sin(t.\log(2)-2)))}.\tfrac{1}{3}(2n-1)^{\frac{1}{2}})\right]_1^{\infty}=0$$

The proof is now complete for Proposition 5.2$\square$.

Proposition 5.3. Application of Dimensional analysis homogeneity to Sigma-Power Laws will always be associated with the one specific $\sigma=\frac{1}{2}$ value for Gram[x=0,y=0] points and this will enable the rigorous proof for Riemann hypothesis to mature.

Proof. We notice the γ proportionality factor given by Eq. (13) above when depicted with the $2^{\frac{1}{2}}$ constant numerical value (derived using $\sigma=\frac{1}{2}$ as conjectured in the original Riemann hypothesis) further allowing, and enabling, *de novo* Dimensional analysis homogeneity compliance in Riemann σ-Power Law in Eqs. (16) and (17) above. There is only one type of $\frac{1}{2}$ exponent present in Riemann σ-Power Law indicating Dimensional analysis homogeneity. *This two mathematical statements essentially complete the proof for Proposition 5.3 with complimentary demonstration below for the Dimensional analysis non-homogeneity case scenario*$\square$.

Corollary 5.4. Application of Dimensional analysis non-homogeneity to Sigma-Power Laws will never be associated with the one specific $\sigma=\frac{1}{2}$ value for Gram[x=0,y=0] points and this will enable the rigorous proof for Riemann hypothesis to mature.

Proof. We illustrate the Dimensional analysis non-homogeneity property for a $\sigma = \frac{1}{4}$ arbitrarily chosen value [clear-cut case with {2n}-parameter] of Riemann σ-Power Law lying on a non-critical line (with total absence of nontrivial zeros) in the following formula derived using Eqs. (13) and (16). **As Ratio R1 component of Riemann-Dirichlet Ratio is independent of σ variable, unlike the Ratio R2 component of Riemann-Dirichlet Ratio and the γ proportionality factor which are dependent on σ variable, we now note the mixture of $\frac{1}{4}$ and $\frac{1}{2}$ exponents subtly, but nonetheless, present in this formula indicating Dimensional analysis non-homogeneity.** Also the replacement of $\frac{1}{3}$ fraction with $\frac{2}{5}$ fraction [derived from substituting $\sigma = \frac{1}{4}$ into $\frac{1}{2(\sigma+1)}$] has occurred. Mathematically, this Dimensional analysis non-homogeneity property for any real number value of σ, when $\sigma \neq \frac{1}{2}$ and $0 < \sigma < 1$, will always be present.

$$[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n)+\frac{3}{4}\pi-\arctan(t))-\psi.\frac{2^{\frac{1}{4}}.(\cos(t.\log(2)+\imath.\sin(t.\log(2))))}{2^{\frac{1}{4}}.(\cos(t.\log(2)+\imath.\sin(t.\log(2)-2)))}.\frac{2}{5}(2n)^{\frac{1}{4}})]_1^{\infty} = 0 \quad (18)$$

The proof is now complete for Corrollary 5.4□.

6 {Mathematical-format-version} conjectures on Riemann zeta function

We now explore the corresponding Gram[x=0,y=0] (Riemann hypothesis), Gram[y=0], Gram[x=0], & Dirichlet-Gram-Riemann {mathematical-format-version} conjectures. The beautiful conjectures given in their geometrical-format-versions provide convincing but still insufficient evidence for their rigorous proofs. In this regard, mathematicians demand that in reference to the 'grand' Dirichlet-Gram-Riemann {mathematical-format-version} conjecture fully constituted by the three subsets (i) Riemann hypothesis or Gram[x=0,y=0], (ii) Gram[y=0], and (iii) Gram[x=0] {mathematical-format-version} conjectures; only when all are perfectly correct can they fulfill the absolute requirements for these rigorous proofs to be completely valid with [figuratively-speaking] 100% certainty. Fortunately, these mathematical-format-versions have conveniently been solved and proven not just beyond reasonable doubts, but beyond all doubts. The succinct summary on this point is appropriately expressed by the "Common Master Proof" outlined below for these three subsets using "Generic Gram conjecture" with their individualized "Generic Gram points". This Common Master Proof is centered on the four theorems (Virtual Container) below [with their complete proofs automatically resulting from mathematical materials in Section 5 above and Appendix A below]. Note that we have simplify all our mathematical arguments below but still maintain their total validity by not including the (Completely Predictable) very first negative Gram[y=0] point in being part of the "Generic Gram points".

Theorem {Riemann} I. The exact same {Generic Gram points}-Riemann-Dirichlet Ratio, directly derived from either the Riemann zeta or Dirichlet eta function, is an irrefutably accurate mathematical expression on the *de novo* criteria for the actual presence [but not the actual location] of the complete set of (identical) infinite Generic Gram points in both functions.

Proof. *This 'overall' proof for Theorem {Riemann} I is now complete as it literally contain the successful incorporation of the rigorous proofs for Lemmas 5.1 and A.1 which are associated with the complete set of the three types of Gram points [thus constituting the Generic Gram points]*□.

Theorem {Riemann} II. Both the near-identical (by proportionality factor-related) {Generic Gram points}-Riemann Sigma-Power Law and {Generic Gram points}-Dirichlet Sigma-Power Law with their derivations based on either the numerator or denominator of {Generic Gram points}-Riemann-Dirichlet Ratio have Dimensional analysis homogeneity only when their common and unknown σ variable has a value of $\frac{1}{2}$ as its solution.

Proof. *This 'overall' proof for Theorem {Riemann} II is now complete as it literally contain the successful incorporation of the rigorous proofs from Propositions 5.2 & A.2 on Sigma-Power Laws and Propositions 5.3 & A.3 on Dimensional analysis homogeneity*□. These are applicable to the complete set of the three types of Gram points [which constitute Generic Gram points].

Theorem {Riemann} III. The σ variable with value of $\frac{1}{2}$ derived using the {Generic Gram points}-Sigma-Power Laws [from Theorem {Riemann} II above] is the exact same σ variable in Generic Gram conjecture which proposed σ to also have the value of $\frac{1}{2}$ (representing the critical line with $\sigma = \frac{1}{2}$ in the critical strip with $0 < \sigma < 1$) for the location of all Generic Gram points of Riemann zeta function [and Dirichlet eta function by default], thus providing irrefutable evidence for this Generic Gram conjecture to be correct with further clarification supplied from Theorem {Riemann} IV below.

Proof. *This 'overall' proof for Theorem {Riemann} III is now complete as Theorem {Riemann} III simply reflect Theorem {Riemann} II with its {Generic Gram points}-Sigma-Power Laws having the exact same σ variable as that referred to by the Generic Gram conjecture with each case independently referring to (i) the Generic Gram points being endowed with the same value of $\frac{1}{2}$ for their σ variable and (ii) the critical line also simultaneously being endowed with the same value of $\frac{1}{2}$ for its σ variable*□. In relation to Dimensional analysis homogeneity in Sigma-Power Laws, the condition "If σ parameter is endowed with the $\frac{1}{2}$ value, then Dimensional analysis homogeneity will be satisfied" acts as the one (and only one) possible mathematical solution.

Theorem {Riemann} IV. Condition 1. Values of σ apart from the $\frac{1}{2}$ value, viz. $0 < \sigma < \frac{1}{2}$ and $\frac{1}{2} < \sigma < 1$, in the critical strip does not contain any Generic Gram points ["the DA-wise mathematical impossibility argument" with resulting *de novo* DA non-homogeneity], together with Condition 2. The one & only one value of $\frac{1}{2}$ for σ in the critical strip contains all Generic Gram points ["the DA-wise one & only one mathematical possibility argument" with resulting *de novo* DA homogeneity] from Theorem {Riemann} III, fully support the rather mute, but nonetheless the whole, point of study in this paper that Generic Gram conjecture is proven to be true when these two (mutually inclusive) conditions are met.

Proof. *This 'overall' proof for Theorem {Riemann} IV is now complete as Theorem {Riemann} IV simply reflect the rigorous proofs from Theorem {Riemann} III on Generic Gram points with the additional proofs from Corollaries 5.4 & A.4 on non-Generic Gram points being tightly incorporated into this mathematical framework*□.

Dimensions are properties which can be measured. With global consensus, Systeme International d'Unites (SI Units) is the standard elements we use to scientifically quantify dimensions. In Dimensional analysis (DA), we are only concerned with the nature of the dimension i.e. its quality (and not its quantity). The following abbreviations are commonly used for examples of various dimensions (expressed in their SI base or SI derived units): angle = θ in radian; length = L in meter; mass = M in kilogram; time = T in second; force = F in newton; temperature = Q in kelvin.

For instance, the traditional unit of measurement of angles is degree. Radian is the SI derived unit of measurement of angles equivalent to the angle subtended at the centre of a circle by an arc equal in length to the radius. One radian is equal to about 57.3 deg and

π radian is exactly 360 deg. Thus the term 'dimension' is traditionally used to refer to the units of measurement associated with various terms of an equation. However, we arbitrarily utilized 'dimension' to also refer to other mathematical properties such as the power or exponent associated with various terms of an equation. In other words, it is nothing more than a convenient language tool when using the term 'dimension' to directly refer to 'power' or 'exponent' in the sense that we could legitimately coin parallel terms such as Power analysis (PA) or Exponent analysis (EA) homogeneity [and non-homogeneity]. So what is this mysterious DA homogeneity (and non-homogeneity)? Any equation describing a physical situation will only be true (false) if both sides of the equation have the same (different) dimensions; that is, it must possess DA homogeneity (non-homogeneity). Examples: 2 kg + 3 kg = 5 kg is a valid equation because it possess DA homogeneity. 2 kg + 3 meter = 5 'something undefined nonsense unit' is meaningless and definitely not a valid equation because it possess DA non-homogeneity. It is a straight-forward exercise to arrive at this two verdicts.

The original equation $y^{\frac{1}{2}}$=$x^{\frac{1}{2}}$+3 which is equivalent to y=$x+6x^{\frac{1}{2}}$+9 possess DA or PA or EA homogeneity, has the same power or exponent $\frac{1}{2}$ [at least] in the original equation. But the original equation $y^{\frac{1}{2}}$=$x^{\frac{1}{3}}$+3 which is equivalent to y=$x^{\frac{2}{3}}+6x^{\frac{1}{3}}$+9 possess DA or PA or EA non-homogeneity, has different powers or exponents $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{2}{3}$ in both the original and equivalent equation. We make a brief comment here that determing the validity of the last two equations endowed with DA or PA or EA non-homogeneity is intuitively not such a straight-forward exercise in this setting. Alternatively stated, those last two equations looks like being physically invalid but may still be mathematically valid.

In this paper, it is to be explicitly elaborated here that a totally invalid comment such as "Dimensional analysis homogeneity can prove Riemann hypothesis" will contextually never be used by us to indicate a connotation that "Laws of Physics along with Scientific Principles even when all of them put together fully satisfying DA homogeneity *per se* can purportedly prove mathematical theorems". Rather, of utmost significance is this DA homogeneity (and non-homogeneity) being the secondary consequence [in a mathematical consistent albeit seemingly indirect manner] arising naturally out of our Virtual Container Research Method used to fully prove Theorems {Riemann} I - IV above. As shown in the above section and in Appendix A, the process to ultimately prove Theorems {Riemann} I - IV involves important mathematical tools such as Euler formula application, Ratio Study, Riemann integral, Calculus (Integration and Differentiation), Dimensional analysis, and concepts from the Hybrid method of Integer Sequence Classification.

Overall, the 6 mathematical steps (**'Riemann hypothesis mathematical foot-prints'**) in specific sequence required to prove Theorems {Riemann} I - IV are: *Step 1:* Riemann zeta or Dirichlet eta function [for the critical strip $0 < \sigma < 1$] $\rightarrow$ *Step 2:* Riemann zeta or Dirichlet eta function [with Euler formula application] $\rightarrow$ *Step 3:* Riemann zeta or Dirichlet eta function [simplified and identical version specifically indicating the criteria for the presence of the complete set of Generic Gram points without knowing their location] $\rightarrow$ *Step 4:* Riemann-Dirichlet Ratio [in discrete summation format] $\rightarrow$ *Step 5:* Riemann-Dirichlet Ratio [in continuous integral format] $\rightarrow$ *Step 6:* Riemann Sigma-power law and Dirichlet Sigma-power law [both with Dimensional analysis homogeneity].

In the process of deriving our rigorous proof, the seemingly small but utterly essential mathematical step in representing a 2-variable function with parameters {2n} or {2n-1} allows crucial moments where cancellation of the relevant "common" parameters in Riemann-Dirichlet Ratio and various Sigma-Power Laws can occur, further allowing the proper DA process to happen in the absolute correct way. These "common" parameters must be mathematically viewed as $(2n)^1$ or $(2n-1)^1$, viz. raised to a power (exponent) of 1 which will hamper proper DA if not serendipitously deleted. Once deleted, we now have the presence of parameters $(2n)^{\frac{1}{2}}$ or $(2n-1)^{\frac{1}{2}}$, viz. raised to the [same and uniform] power (exponent) of $\frac{1}{2}$, which will then enable proper DA (homogeneity) to proceed.

The key end-product equations arising out of, and fully conforming with, Theorems {Riemann} I - IV above then act as ultimate and final evidences for the complete proof of Riemann hypothesis, Gram[y=0] conjecture, and Gram[x=0] conjecture. These closely related equations, for the {2n}-parameter case with ψ being the proportionality constant, will **subtly manifest the necessary DA homogeneity for the single $\sigma = \frac{1}{2}$ value involving the $\frac{1}{2}$ exponents [and DA non-homogeneity for all other infinite σ values with the given example of $\sigma = \frac{1}{4}$ involving the mixture of $\frac{1}{4}$ and $\frac{1}{2}$ exponents]**. They are, respectively, regurgitated below from Section 5 and Appendix A for convenience.

Gram[x=0,y=0] or Riemann hypothesis

$$[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n)+\frac{3}{4}\pi-\arctan(t))-\psi.\frac{2^{\frac{1}{2}}.(\cos(t.\log(2)+\imath.\sin(t.\log(2))))}{2^{\frac{1}{2}}.(\cos(t.\log(2)+\imath.\sin(t.\log(2)-2)))}.\frac{1}{3}(2n)^{\frac{1}{2}})]_1^{\infty} = 0 \quad (19)$$

$$[2^{\frac{1}{2}}.\frac{e^{\frac{1}{t}\cdot\frac{3}{4}\pi}}{2(t^2+1)^{\frac{1}{2}}.e^{\frac{3}{4}\pi}}.\sin(t.\log(2n)+\frac{3}{4}\pi-\arctan(t))-\psi.\frac{2^{\frac{1}{4}}.(\cos(t.\log(2)+\imath.\sin(t.\log(2))))}{2^{\frac{1}{4}}.(\cos(t.\log(2)+\imath.\sin(t.\log(2)-2)))}.\frac{2}{5}(2n)^{\frac{1}{4}})]_1^{\infty} = 0 \quad (20)$$

Gram[y=0] conjecture

$$[\frac{e^{\frac{1}{t}}}{2(t^2+1)^{\frac{1}{2}}}.\sin(t.\log(2n)-\arctan(t))-\psi.\frac{2^{\frac{1}{2}}.(\cos(t.\log(2)+i.\sin(t.\log(2)))}{2^{\frac{1}{2}}.(\cos(t.\log(2)+i.\sin(t.\log(2))-2)}.\frac{1}{3}.(2n)^{\frac{1}{2}}]_1^{\infty} = 0 \quad (21)$$

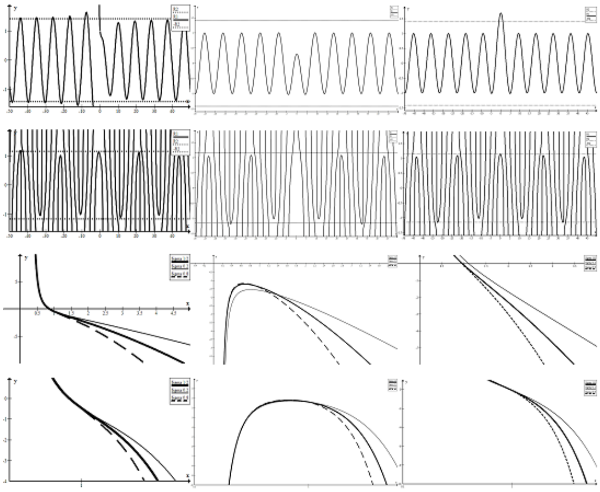
$$[\frac{e^{\frac{1}{t}}}{2(t^2+1)^{\frac{1}{2}}}.\sin(t.\log(2n)-\arctan(t))-\psi.\frac{2^{\frac{1}{4}}.(\cos(t.\log(2)+i.\sin(t.\log(2)))}{2^{\frac{1}{4}}.(\cos(t.\log(2)+i.\sin(t.\log(2))-2)}.\frac{2}{5}.(2n)^{\frac{1}{4}}]_1^{\infty} = 0 \quad (22)$$

Gram[x=0] conjecture

$$[\frac{e^{\frac{1}{t}}}{2(t^2+1)^{\frac{1}{2}}}.\sin(t.\log(2n)+\arctan(t))-\psi.\frac{2^{\frac{1}{2}}.(\cos(t.\log(2)+i.\sin(t.\log(2)))}{2^{\frac{1}{2}}.(\cos(t.\log(2)+i.\sin(t.\log(2))-2)}.\frac{1}{3}.(2n)^{\frac{1}{2}}]_1^{\infty} = 0 \quad (23)$$

$$[\frac{e^{\frac{1}{t}}}{2(t^2+1)^{\frac{1}{2}}}.\sin(t.\log(2n)+\arctan(t))-\psi.\frac{2^{\frac{1}{4}}.(\cos(t.\log(2)+i.\sin(t.\log(2)))}{2^{\frac{1}{4}}.(\cos(t.\log(2)+i.\sin(t.\log(2))-2)}.\frac{2}{5}.(2n)^{\frac{1}{4}}]_1^{\infty} = 0 \quad (24)$$

The near-identical "physical manifestations" depicted in Section 5, Fig. 2a & Fig. 2b, p. 17, Section 6, Fig. 3a & Fig. 3b, p. 18 from [8] and in Section 4, Fig. 2a & Fig. 2b, p. 7, Section 5, Fig. 3a & Fig. 3b, pp. 8-9, Section 6, Fig. 4a & Fig. 4b, p. 12, Fig. 5a & Fig.



Graphs.png

Fig. 6 Combined graphs giving us a snapshot on the "physical manifestations" of various Ratio and Law formulae derived out of Gram[x=0,y=0] (Riemann hypothesis), Gram[y=0] and Gram[x=0] conjectures.

5b, p. 13 from [9] on various Ratio and Law formulae which are obtained from our three conjectures demonstrating properties such as Mathematical Symmetry, Chaos (Chaos theory with Chaotic non-linear deterministic dynamics), and Fractals (Fractal geometry with Self-similarity) are portrayed in Figure 6. These miniaturized version graphs are grouped together from Left to Right as corresponding to Gram[x=0,y=0] (Riemann hypothesis), Gram[y=0] and Gram[x=0] conjectures; and depicted using $\sigma = \frac{1}{5}, \frac{1}{2}, \& \frac{4}{5}$ values.

7 Information-Complexity conservation

An Equation or Algorithm is simply a Black Box generating the necessary Output (with qualitative structural 'Complexity') when supplied with the given Input (with quantitative data 'Information'). In Set theory, the infinite sets are sets that are not finite sets, and they are further subdivided into two groups: "discrete" countable infinite sets and "continuous" uncountable infinite sets; with the later being conceptually larger than the former in magnitude [despite both being treated as objects endowed with the infinity property]. A set is countable if we can count its elements. If the set is finite, we can easily count its elements. If the set is infinite, being countable means that we are able to put the elements of the set in order (just like natural numbers are in order). The infinite sets of rational number and irrational number are, respectively, countable and uncountable. These two sets together give rise to the infinite set of real number which are uncountable.

The two-dimensional complex plane is typically specified by a one-dimensional real number line for horizontal or x-axis, and a one-dimensional imaginary number (i = $\sqrt{-1}$) line for vertical or y-axis. Complex numbers, each defined with a (pure) real number component and a (pure) imaginary number component, lie on this plane. Real numbers could alternatively be perceived as complex numbers with their imaginary number component being zero. In regards to the uncountable infinite set of real number line (with the understanding that every positive real number has its mirror image negative real number counterpart), this line is further seen to consist of both countable and uncountable infinite sets. The set and subsets of real numbers with some of their properties are comparatively illustrated below using the legends & abbreviations: =, $<$, $>$, $>>$, $\subset$, $\in$, CFS, CIS & UIS denoting (respectively) 'equal to', 'less than', 'greater than', 'much greater than', 'subset of', 'belongs to', 'countable finite set', 'countable infinite set' & 'uncountable infinite set'.

We can use the cardinality relation to describe the size of a set by comparing it with standard sets. Any set X with cardinality less than that for the set of natural numbers (set N), or $|X| < |N|$, is said to be a CFS. Any set X that has the same cardinality as set N, or $|X| = |N|$, is said to be a CIS endowed with "cardinality of the natural numbers". Any set X with cardinality greater than that for set N, or $|X| > |N|$ (for example, when set X = real numbers), is said to be a UIS endowed with "cardinality of the continuum". From smallest to biggest sets, natural numbers (CIS) $\subset$ integer numbers (CIS) $\subset$ rational numbers (CIS) $\subset$ algebraic numbers (CIS) $\subset$ real numbers (UIS). By definition, for relatively "smaller" set X (= even numbers or odd numbers or prime numbers or composite numbers) $\in$ set of natural numbers, we [counterintuitively] note that $|X|$ still = "cardinality of the natural numbers". The set of natural numbers has cardinality (of the natural numbers) that is strictly less than the set of real numbers having cardinality (of the continuum) as it can be shown that there does not exist a bijective function from natural numbers to real numbers using Cantor's diagonal argument or Cantor's first uncountability proof.

Irrational numbers (UIS) = (I) Transcendental numbers (UIS) + (II) Algebraic numbers (CIS) with (I) $>>$ (II). Almost all real and complex numbers are transcendental. All irrational numbers can imperatively be depicted as numbers with non-repeating decimal point digits of infinite length, with those decimal point digits being Incompletely Predictable. An algebraic number is any real or complex number that is a root of a non-zero polynomial in one variable with rational coefficients (or equivalently – by clearing denominators – with integer coefficients). All integers and rational numbers are algebraic, as are all roots of integers. Thus a transcendental number is a real or complex number that is not algebraic [associated with the criterion as just stated]; and it "transcends" the power of algebra to display it in its totality.

Real numbers (UIS) = (I) Irrational numbers (UIS) + (II) Rational numbers (CIS) with (I) $>$ (II). If real numbers are to be the union of two countable sets, they would have to be [incorrectly] countable; so the irrational numbers must be [correctly] uncountable by following this 'proof by contradiction' argument.

Rational numbers (CIS) = (I) Fractions (CIS) + (II) Integer numbers (CIS) with (I) $>$ (II). A rational number is any number that can be expressed as the quotient or fraction p/q of two integers, a numerator p and a non-zero denominator q. Since q may be equal to 1, every integer is a rational number. Fractions can imperatively be depicted as numbers with non-repeating decimal point digits of finite length type or repeating decimal point digits of infinite length type, with both sets of decimal point digits being Completely Predictable. Thus integers can specially be depicted either as the integer number itself followed by a (redundant) non-repeating decimal point digit '0' or as fractions with numerator given by the integer number itself and denominator given by the (redundant) number '1'.

Integers (CIS): -∞ ,..., -3, -2, -1, 0, 1, 2, 3,..., +∞. Whole numbers (CIS): 0, 1, 2, 3,..., ∞. Natural numbers (CIS): 1, 2, 3, 4,..., ∞. Let x be the set consisting of either one or two number(s) such that x $\in$ natural numbers, whole numbers or integers. Whenever relevant in this paper, we consider x for the relevant upper &/or lower boundary(ies) of interest in the study on a chosen set of numbers (such as even, odd, prime, and composite numbers).

Lemma 7.1. Natural numbers (CIS): 1, 2, 3, 4,..., ∞. The natural counting function Natural-$\pi(x)$, defined as the number of natural numbers $\leq$ x, is Completely Predictable to be simply = x.

Proof The formula for generating natural numbers with 100% certainty is N_i = i whereby N_i is the i^{th} natural number and i = 1, 2, 3,..., ∞. For a given N_i number, its i^{th} position is simply i. Natural gap (G_{Ni}) = N_{i+1} - N_i, with G_{Ni} always = 1. Thus there are x natural numbers $\leq$ x. The (coined) natural counting function, denoted here by Natural-$\pi(x)$, is defined as the number of natural numbers $\leq$ x – this is Completely Predictable to be simply = x. *The proof is now complete for Lemma 7.1*□.

Lemma 7.2. The even counting function Even-$\pi(x)$, defined as the number of even numbers $\leq$ x, is Completely Predictable to be simply = floor(x/2).

Proof. Even numbers (CIS): 2, 4, 6, 8,..., ∞. The formula for generating even numbers with 100% certainty is E_i = iX2 whereby E_i is the i^{th} even number and i = 1, 2, 3,..., ∞ abiding to the mathematical label "All natural numbers always ending with a digit 0, 2, 4, 6 or 8". For a given E_i number, its i^{th} position is calculated as i = E_i/2. Even gap (G_{Ei}) = E_{i+1} - E_i, with G_{Ei} always = 2. Thus there are $\lfloor \frac{x}{2} \rfloor$ even numbers $\leq$ x. The (coined) even counting function, denoted here by Even-$\pi(x)$, is defined as the number of even numbers $\leq$ x – this is Completely Predictable to be simply = floor(x/2). *The proof is now complete for Lemma 7.2*□.

Lemma 7.3. The odd counting function Odd-$\pi(x)$, defined as the number of odd numbers $\leq$ x, is Completely Predictable to be simply = ceiling(x/2).

Proof. Odd numbers (CIS): 1, 3, 5, 7,..., ∞. The formula for generating odd numbers with 100% certainty is O_i = (iX2)-1 whereby O_i is the i^{th} odd number and i = 1, 2, 3,..., ∞ abiding to the mathematical label "All natural numbers always ending with a digit 1, 3, 5, 7, or 9". For a given O_i number, its i^{th} position is calculated as i = (O_i + 1)/2. Odd gap (G_{Oi}) = O_{i+1} - O_i, with G_{Oi} always = 2. Thus there are $\lceil \frac{x}{2} \rceil$ odd numbers $\leq$ x. The (coined) odd counting function, denoted here by Odd-$\pi(x)$, is defined as the number of odd numbers $\leq$ x – this is Completely Predictable to be simply = ceiling(x/2). *The proof is now complete for Lemma 7.3*□.

Lemma 7.4. The prime counting function Prime-$\pi(x)$, defined as the number of prime numbers $\leq$ x, is Incompletely Predictable and always need to be calculated using the Sieve of Eratosthenes algorithm.

Proof. Prime numbers (CIS): 2, 3, 5, 7, 11, 13, 17,..., ∞. The algorithm for generating all prime numbers P_i whereby P_1 (= 2), P_2 (= 3), P_3 (= 5), P_4 (= 7),..., ∞ with 100% certainty is based on the Sieve of Eratosthenes abiding to the mathematical label "All natural numbers apart from 1 that are evenly divisible by itself and by 1". Suffice to state here that although we can check the primality of a given odd number [check whether a given odd number is a prime number or not] by trial division, we can never determine its position without knowing the positions of preceding prime numbers. All prime numbers must be odd numbers and the only even prime number is 2. Prime gap (G_{Pi}) = P_{i+1} - P_i, with G_{Pi} constituted by all even numbers except the 1^{st} G_{P1} = 3 - 2 = 1. The prime counting function, denoted here by Prime-$\pi(x)$ [which is traditionally denoted simply by $\pi(x)$], is defined as the number of prime numbers $\leq$ x – this is Incompletely Predictable and always need to be calculated via the mentioned algorithm. We notice that by the very definition of prime gap above, every prime number [represented here with the aid of 'n' notation instead the usual 'i' notation] can be written as $P_{n+1} = 2 + \sum_{i=1}^{n} G_{Pi}$ with '2' denoting P_1. Here i & n = 1, 2, 3, 4, 5, ..., ∞. *The proof is now complete for Lemma 7.4*□.

Lemma 7.5. The composite counting function Composite-$\pi(x)$, defined as the number of composite numbers $\leq$ x, is Incompletely Predictable and always need to be calculated indirectly as the set of natural numbers minus the set of prime numbers [obtained using the Sieve of Eratosthenes algorithm].

Proof. Composite numbers (CIS): 4, 6, 8, 9, 10, 12, 14,..., ∞. Composite numbers have the mathematical label "All natural numbers apart from 1 that are evenly divisible by numbers other than itself and 1". The algorithm for generating all composite numbers C_i whereby C_1 (= 4), C_2 (= 6), C_3 (= 8), C_4 (= 9),..., ∞ with 100% certainty is also based on the Sieve of Eratosthenes albeit in an indirect manner by simply selecting [the excluded] non-prime natural numbers to be composite numbers. We define the (coined) term Composite gap G_{Ci} as C_{i+1} - C_i with G_{Ci} constituted by 1 & 2. The (coined) composite counting function, denoted by Composite-$\pi(x)$, is defined as the number of composite numbers $\leq$ x – this is Incompletely Predictable and always need to be [indirectly] calculated via the mentioned algorithm. Applying similar ideas from prime numbers, we notice that by the very definition of composite gap above, every composite number [represented here with the aid of 'n' notation instead the usual 'i' notation] can be written as $C_{n+1} = 4 + \sum_{i=1}^{n} G_{Ci}$ with '4' denoting C_1. Here i & n = 1, 2, 3, 4, 5, ..., ∞. We crucially mention at this point that, in stark contrast to the equation "containing" but not identifying all prime numbers [outlined in the proof for Lemma 3.4 above] with prime gaps constituted by all even numbers [thus dealing with 'unfriendly' CIS property] except the very 1^{st} G_{P1} = 3 - 2 = 1; the equivalent equation "containing" but not identifying all composite numbers deals with 'friendly' CFS property for composite gaps which are constituted only by 1 & 2. Also we reinforce from the contents associated with Remark 1.1 above that we could conceptually and usefully visualize both the varying Incompletely Predictable prime gaps & composite gaps respectively as the varying Incompletely Predictable "prime gradients" & "composite gradients". *The proof is now complete for Lemma 7.5*□.

The following are useful mathematical relationships amongst the various groups of rational numbers: [Positive] Integers (CIS) = Whole numbers (CIS) = (I) Number '0' (CFS) + (II) Natural numbers (CIS) with (I) < (II). Natural numbers (CIS) = (I) Even numbers (CIS) + (II) Odd numbers (CIS) with (I) = (II). Natural numbers (CIS) = (I) Prime numbers (CIS) + (II) Composite numbers (CIS) + (III) Number '1' (CFS) with (I) < (II). Composite numbers (CIS) = (I) Even numbers (CIS) + (II) [Odd numbers (CIS) - Prime numbers (CIS) - Number '1' (CFS)] with (I) > (II) and Odd numbers > Prime numbers. Prime numbers (CIS) = (I) Natural numbers (CIS) - (II) Composite numbers (CIS) - (III) Number '1' (CFS) with (I) > (II). Prime numbers < composite numbers and they are (A) mutually exclusive to each other. In fact, (B) composite numbers are the exact complementary counterparts of prime numbers simply because the Incompletely Predictable composite numbers = Completely Predictable natural numbers - Incompletely Predictable prime numbers - Completely Predictable number '1'. Relationships (A) and (B) allow us to forge a useful mental picture of "monotonously, slowly and eternally increasing prime and composite numbers which are inseparable" with composite numbers doing so at a relatively faster rate than prime numbers. Although not carried out, better visualization of this picture could desirably be achieved by graphing various derived formulations of relevant counting functions for both elements.

The set of natural number can be visualized to consist of two 'mutually exclusive, complementary and inseparable' subset groupings of either (i) Completely Predictable even and odd numbers, or (ii) Incompletely Predictable prime and composite numbers plus the Completely Predictable number '1'. Denote 'A' to represent natural, even, odd, prime, and composite numbers. We define the relevant counting function A-$\pi(x)$ as the number of A $\leq$ x with x belonging to the set of natural number. As a prelude to outlining the all-important Information-Complexity conservation concept, we can easily define and compute below in a progressive manner the entity 'Grand-Total Gaps for A at x' (Grand-Total ΣA_x-Gaps) and their associated properties.

Proposition 7.6. For any given x value, the designated Information or Input is always validly represented by ΣNatural$_x$-Gaps = x - 1 for the (one) set of Completely Predictable natural numbers.

Proof. INPUT: Set of natural numbers (for x = 10): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Natural-$\pi(x)$ = 10. There are x - 1 = 9 Natural-Gaps each of '1' magnitude: 1, 1, 1, 1, 1, 1, 1, 1, 1. ΣNatural$_x$-Gaps = 9 X 1 = 9. This equates to "x - 1" by which we will regard as Information for Completely Predictable numbers. *The proof is now complete for Proposition 7.6*□.

Proposition 7.7. For all given x≥2 values, the designated Complexity or Output is continuously & validly represented by the solitary ΣEvenOdd$_x$-Gaps = 2x - N with N = 4 being maximal for the (two) sets of Completely Predictable even & odd numbers.

Proof. OUTPUT: Set of even and odd numbers (for x = 10): 2, 4, 6, 8, 10 and 1, 3, 5, 7, 9. Even-$\pi(x)$ = 5 and Odd-$\pi(x)$ = 5. There are predictably $\lfloor \frac{x}{2} \rfloor$ - 1 = 4 Even-Gaps each of '2' magnitude: 2, 2, 2, 2. ΣEven$_x$-Gaps = 4 X 2 = 8, and $\lceil \frac{x}{2} \rceil$ - 1 = 4 Odd-Gaps each of '2' magnitude: 2, 2, 2, 2. ΣOdd$_x$-Gaps = 4 X 2 = 8. Grand-Total ΣEvenOdd$_x$-Gaps = 8 + 8 = 16. As depicted by Table 7 and Figure 8 in Appendix B, this equates to 2x - N = "2x - 4" [with "N = 4 being maximal"] by which we will regard as Complexity for Completely Predictable numbers. *The proof is now complete for Proposition 7.7*□.

Proposition 7.8. For selective & relevant x≥4 values, the designated Complexity or Output is cyclically & validly represented by ΣPrimeComposite$_x$-Gaps = 2x - N with N = 7 being minimal for the (two) sets of Incompletely Predictable prime & composite numbers.

Proof. OUTPUT: Set of prime and composite numbers (for x = 12): 2, 3, 5, 7, 11 and 4, 6, 8, 9, 10, 12. Prime-$\pi(x)$ = 5 and Composite-$\pi(x)$ = 6. There are four Prime-Gaps of 1, 2, 2, 4 magnitude and five Composite-Gaps of 2, 2, 1, 1, 2 magnitude. ΣPrime$_x$-Gaps = 1 + 2 + 2 + 4 = 9. ΣComposite$_x$-Gaps = 2 + 2 + 1 + 1 + 2 = 8. Grand-Total ΣPrimeComposite$_x$-Gaps = 9 + 8 = 17. As depicted by Table 6 and Figure 7, this equates to 2x - N = "2x - 7" [with intermittent and cyclical appearances on a perpetual basis of this "N = 7 being minimal"] by which we will regard as Complexity for Incompletely Predictable numbers. *The proof is now complete for Proposition 7.8*□.

Incredibly, both the (defacto) baseline "2x - 4" Grand-Total Gaps for Completely Predictable Even-Odd number pairing output and the (defacto) baseline "2x - 7" Grand-Total Gaps for Incompletely Predictable Prime-Composite number pairing output will always be consistent entities. The consistency of this Grand-Total Gaps ingredient present in two outputs representing two vastly different groups of number is part of what we regard as fulfilling Information-Complexity conservation. Instead if we were to [incorrectly] let the number '1' be a composite number (with the addition of "extra" first Composite-Gap being 3), then the [incorrect] resulting (defacto) baseline "2x - 4" Grand-Total Gaps for Incompletely Predictable Prime-Composite number pairing output will now be identical to that for Completely Predictable Even-Odd number pairing output. This identical "2x - 4" Grand-Total Gaps occurrence in Prime-Composite number pairing is simply due to the fact that we are now [incorrectly] analyzing the exact same complete set of natural numbers as in Even-Odd number pairing – see Appendix C below depicting this artificial situation.

Let both x & N $\in$ 1, 2, 3,..., ∞. We utilize the word 'Dimension' here to contextually denote the relevant Dimension 2x - N whereby (i) the allocated [infinite] N integer values will result in Dimensions of the types 2x - 7, 2x - 8, 2x - 9, ..., 2x - ∞ for the Prime-Composite finite scale mathematical landscape below and (ii) the allocated [finite] N integer values for the Even-Odd finite scale mathematical landscape in Appendix B below will result in Dimensions of the type 2x - 4. For the Prime-Composite groupings, we note the initial one-off Dimensions 2x - 2, 2x - 4 and 2x - 5 (in consecutive order); and for the Even-Odd groupings, we note the very first one-off Dimension 2x - 2. [The term "mathematical

Table 6 Prime-Composite Prime-Composite finite scale mathematical (tabulated) landscape using data obtained for x = 64. The number '1' is neither a prime number nor a composite number. Legend: C = composite, P = prime, Y = Dimension 2x - 7 (for visual clarity), N/A = Not Applicable.

x	**P_i or C_i, Gaps**	**ΣPC_x-Gaps**	Dimension	**x**	**P_i or C_i, Gaps**	**ΣPC_x-Gaps**	Dimension
1	N/A	0	2x-2	33	C21, 1	58	2x-8
2	P1, 1	0	2x-4	34	C22, 1	59	2x-9
3	P2, 2	1	2x-5	35	C23, 1	60	2x-10
4	C1, 2	1	Y	36	C24, 2	61	2x-11
5	P3, 2	3	Y	37	P12, 4	67	Y
6	C2, 2	5	Y	38	C25, 1	69	Y
7	P4, 4	7	Y	39	C26, 1	70	2x-8
8	C3, 1	9	Y	40	C27, 1	71	2x-9
9	C4, 1	10	2x-8	41	P13, 2	75	Y
10	C5, 2	11	2x-9	42	C28, 2	77	Y
11	P5, 2	15	Y	43	P14, 4	79	Y
12	C6, 2	17	Y	44	C29, 1	81	Y
13	P6, 4	19	Y	45	C30, 1	82	2x-8
14	C7, 1	21	Y	46	C31, 2	83	2x-9
15	C8, 1	22	2x-8	47	P15, 6	87	Y
16	C9, 1	23	2x-9	48	C32, 1	89	Y
17	P7, 2	27	Y	49	C33, 1	90	2x-8
18	C10, 2	29	Y	50	C34, 1	91	2x-9
19	P8, 4	31	Y	51	C35, 1	92	2x-10
20	C11, 1	33	Y	52	C36, 1	93	2x-11
21	C12, 1	34	2x-8	53	P16, 6	99	Y
22	C13, 2	35	2x-9	54	C37, 1	101	Y
23	P9, 6	39	Y	55	C38, 1	102	2x-8
24	C14, 1	41	Y	56	C39, 1	103	2x-9
25	C15, 1	42	2x-8	57	C40, 1	104	2x-10
26	C16, 1	43	2x-9	58	C41, 1	105	2x-11
27	C17, 1	44	2x-10	59	P17, 2	111	Y
28	C18, 2	45	2x-11	60	C42, 2	113	Y
29	P10, 2	51	Y	61	P18, 6	115	Y
30	C19, 2	53	Y	62	C43, 1	117	Y
31	P11, 6	55	Y	63	C44, 1	118	2x-8
32	C20, 1	57	Y	64	C45, 1	119	2x-9

landscape" is self-explanatorily employed in this paper to denote tabulated and graphed data showing specific mathematical patterns and features.]

Using the relevant data, we have now painstakingly tabulate in Table 6 and graphically map in Figure 7 the all-important [Incompletely Predictable] Prime-Composite mathematical landscape for a relatively larger x = 64 as demonstrated below (and ditto for the [Completely Predictable] Even-Odd mathematical landscape as demonstrated in Appendix B at the end of this paper). Of utmost importance, we note this Prime-Composite mathematical landscape made up of the relevant Dimensions will intrinsically incorporate prime and composite numbers in an integrated manner; and that there will be infinite times whereby relevant Dimensions will deviate away from the 'baseline' Dimension 2x - 7 simply because prime [and composite] numbers in totality can always be rigorously and easily proven to be infinite in magnitude (as outlined in the commencement paragraph of subsection 1.3 above). For comparison, we have repeated this whole exercise for the [Completely Predictable] Even-Odd mathematical landscape in Appendix B and note the complete lack of deviation away from 'baseline' Dimension 2x - 4 apart from the one-off point of deviation as manifested by the initial Dimension 2x - 2.

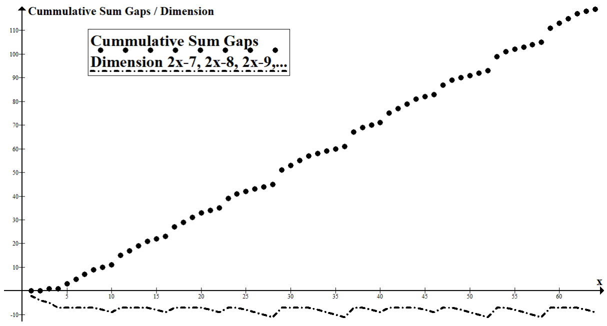


Fig. 7 Prime-Composite Prime-Composite finite scale mathematical (graphed) landscape using data obtained for x = 64. The bottom graph here on 'Dimensions when symbolically represented by ever larger negative integers' demonstrate the apparent manifestation of graphical "continuity".

In Figure 7, Dimensions 2x - 7, 2x - 8, 2x - 9, ..., 2x - ∞ are symbolically represented by -7, -8, -9, ..., ∞ with 2x - 7 displayed as 'baseline' Dimension whereby the Dimension trend (Cumulative Sum Gaps) must repeatedly reset itself onto this (Grand-Total Gaps) 'baseline' Dimension on a perpetual basis, thus manifesting Information-Complexity conservation and Dimensional analysis homogeneity. Graphical appearances of Dimensions symbolically represented by ever larger negative integers will correspond to prime numbers associated with ever larger prime gaps and this phenomenon will generally happen at ever larger x values. In other words, at ever larger x values, Prime-$\pi(x)$ will overall become larger but with a *decelerating* trend whereas Composite-$\pi(x)$ will overall become larger but with an *accelerating* trend. This highlights the inevitable mathematical event of ever larger prime gaps occurring at ever larger x values. We note that there is a complete presence of Chaos & Fractals phenomena being manifested in our graph.

The definitive derivation of the data in Table 6 is given next and this is illustrated by two examples given for position x = 31 & 32. For i & x ∈ 1, 2, 3,..., ∞; ΣPC_x-Gap = ΣPC_{x-1}-Gap + Gap value at P_{i-1} or Gap value at C_{i-1} whereby (i) P_i or C_i at position x is determined by whether the relevant x value belongs to a prime (P) or composite (C) number, and (ii) both ΣPC_1-Gap and ΣPC_2-Gap = 0. Example for position x = 31: 31 is a prime number (P11). Our desired Gap value at P10 = 2. Thus ΣPC_{31}-Gap (55) = ΣPC_{30}-Gap (53) + Gap value at P10 (2). Example for position x = 32: 32 is a composite number (C20). Our desired Gap value at C19 = 2. Thus ΣPC_{32}-Gap (57) = ΣPC_{31}-Gap (55) + Gap value at C20 (2).

Finally, we easily observe the 'overall magnitude of composite numbers to be always greater than that of prime numbers' criterion to hold true from x = 14 onwards. For instance, position x = 61 corresponds to prime number 61 which is the 18^{th} prime number, whereas [the one lower] position x = 60 corresponds to composite number 60 which is the [much higher] 42^{nd} composite number. For the sake of curiosity, we refer interested readers to Appendix C below whereby we have flawlessly and comparatively conducted near-identical calculations using the [incorrect] (defacto) baseline "2x - 4" Grand-Total Gaps for Incompletely Predictable Prime-Composite number pairing output when the number '1' is [incorrectly] assumed to be a composite number. The near-identical tabulating (Table 8) and graphing (Figure 9) of those calculations are also included in Appendix C.

8 Polignac's and Twin prime conjectures

The previous section alludes to the Prime-Composite finite scale mathematical landscape. This section mainly deals with the Prime-Composite infinite scale mathematical landscape. The "qualitative" aspect to the existence of the set of even number prime gaps and their associated sets of odd prime numbers, all as sets of infinite magnitude, will be shown to be correct by the Plus-Minus Composite Gap 2 Number Alternating Law and the Plus Composite Gap 2 Number Continuous Law. The 'Composite Gap 2 Number' occurrences are present on the finite scale mathematical landscape through direct observation (as depicted in Figure 7 and Table 6 above for x = 64) and on the infinite scale mathematical landscape through indirect logical deductions (as will be discussed below). These laws are Laws of Continuity based on the entity 'Composite Gap 2 Number' inferring underlying intrinsic driving mechanisms that enable the infinity magnitude association for both the set of even number prime gaps and their associated sets of odd prime numbers to co-exist. Thus these laws have the important implication that they will be perpetually applicable for all relevant even number prime gaps and their associated sets of odd prime numbers.

We have already established that prime and composite numbers are mutually exclusive, complementary, inseparable, and infinite in magnitude. With the letter 'Y' symbolizing (baseline) Dimension 2x - 7 and prime gap at $P_i = P_{i+1} - P_i$ with P_i & P_{i+1} respectively symbolizing consecutive "first" & "second" prime number in any P_i-P_{i+1} pairings, we can conveniently denote (i) Dimensions YY grouping [depicted by 2x - 7 initially appearing twice in (iii) below] as representing the signal for appearances of prime number pairings other than twin primes (with prime gap = 2) such as cousin primes (with prime gap = 4), sexy primes (with prime gap = 6), etc; (ii) Dimension YYYY grouping as representing the signal for appearances of prime number pairings as twin primes (with prime gap = 2); and (iii) Dimension (2x - $\geq$7)-Progressive-Grouping allocated to the 2x - 7, 2x - 7, 2x - 8, 2x - 9, 2x - 10, 2x - 11,..., 2x - ∞ as elements of the *precise* and *proportionate* countable finite set (CFS) Dimensions representation of an individual prime number P_i with its associated prime gap namely, Dimensions 2x - 7 & 2x - 7 pairing = twin prime (with both of its prime gap & CFS cardinality = 2); 2x - 7, 2x - 7, 2x - 8 & 2x - 9 pairing = cousin prime (with both of its prime gap & CFS cardinality = 4); 2x - 7, 2x - 7, 2x - 8, 2x - 9, 2x - 10 & 2x - 11 pairing = sexy prime (with both of its prime gap & CFS cardinality = 6); and so on. Then the higher order [which is traditionally defined as closest possible] prime groupings of three prime numbers as prime triplets, of four prime numbers as prime quadruplets, of five prime numbers as prime quintuplets, etc can each be mathematically deemed to consist of relevant serendipitous groupings required-by-law to always respect the following unwritten mathematical rule: With the exception of the three 'outlier' prime numbers 3, 5, & 7; groupings of any three prime numbers as the P, P+2, P+4 combination (viz. manifesting two consecutive twin primes with prime gap = 2) is a mathematical impossibility. The 'anomaly' that one of every three sequential odd numbers is a multiple of three, and hence this particular number cannot be prime, would clearly explain this mathematical impossibility. Then the closest possible prime grouping must be of either P, P+2, P+6 format or P, P+4, P+2 format.

Note that prime groupings not respecting the traditional closest-possible-prime groupings above are also the norm [occurring infinitely often], and they simply indicate the continual presence of prime gaps $\geq$ 6 [by which we tentatively propose here to the wider scientific community to arbitrarily represent 'large gaps']. As prime numbers become sparser at larger range; the perpetual presence of prime gaps $\geq$ 6 of progressive greater magnitude will, in a general and gentle manner, occur ever more frequently.

Based on not dissimilar rationale to above, we can deduce that as prime numbers become sparser at larger range; the permanent presence of prime gaps 2 & 4 [by which we tentatively propose here to the wider scientific community to arbitrarily represent 'small gaps'] will, in a general and gentle manner, occur ever less frequently. Thus nature seems to dictate that in order to comply with Information-Complexity conservation, the permanent requirement, at larger range, of intermittently resetting to baseline Dimension 2x - 7 occurring four times in a row as denoted by Dimension YYYY grouping [indicating the occurrence of twin primes] is inevitable.

We can now insightfully understand the Dimension YYYY unique signal of twin prime appearances in full details. The initial two CFS Dimensions YY components of YYYY fully represent the "first" prime number component of the twin prime number pairing. The last two Dimensions YY components of YYYY signifying the appearance of the "second" prime number component of the twin prime number pairing is also the initial first-two-element component of the full CFS Dimensions representation for the "first" prime number component of the following non-twin prime number pairing. The seemingly "bizarre" uniqueness of twin primes (with prime gap = 2) is that they are represented by repeating the *single* type Dimension 2x - 7 twice whereas in all other 'higher order' prime number pairings (with prime gaps $\geq$ 4), they will always require *multiple* types Dimension representation.

We next conveniently carry out the valid procedure of endowing all Dimensions with exponent / power / index of 1 for subsequent perusal in our on-going mathematical arguments. P_1 = 2 is represented by CFS as Dimension $(2x - 4)^1$ (with both of its prime gap & CFS cardinality = 1); P_2 = 3 is represented by CFS as Dimensions $(2x - 5)^1$ & $(2x - 4)^1$ (with both of its prime gap & CFS cardinality = 2); P_3 = 5 is represented by CFS Dimension $(2x - 7)^1$ & $(2x - 7)^1$ (with both of its prime gap & CFS cardinality = 2), etc.

Proposition 8.1. Let Case 1 be the Completely Predictable even & odd numbers pairing and let Case 2 be the Incompletely Predictable prime & composite numbers & the Completely Predictable number '1' pairing. Furthermore, let Case 1 and Case 2 be totally independent of each other. Then for any given x value, there exist the grand total number of Dimensions [Complexity] such that it exactly equal to either the two combined subtotal number of Dimensions [Complexity] to precisely represent each of the Completely Predictable even & odd numbers in Case 1, or the combined subtotal number of Dimensions [Complexity] to precisely represent each of the Incompletely Predictable prime & composite numbers & the Completely Predictable number '1' in Case 2.

Proof. Natural numbers can directly be constituted from either the combined even & odd numbers in Case 1 or the combined prime & composite numbers & the number '1' in Case 2. The correctly designated infinitely many CFS of Dimensions that are used to precisely represent both the combined even & odd numbers in Case 1 and the combined prime & composite numbers & the number '1' in Case 2 must also directly and proportionately be representative of the relevant natural numbers arising from the combined subtotal of even & odd numbers in Case 1 and from the combined subtotal of prime & composite numbers & the number '1' in Case 2. *The proof is now complete for Proposition 8.1*□.

Proposition 8.2. Let Case 1 be the Completely Predictable even & odd numbers pairing and let Case 2 be the Incompletely Predictable prime & composite numbers & the Completely Predictable number '1' pairing. Furthermore, let Case 1 and Case 2 be totally independent of each other. Part I: Then for any given x value apart from the x = 1 value in Case 1 and the x = 1, 2, and 3 values in Case 2; the Dimension $(2x - N)^1$ [Complexity] representations of all Completely Predictable even & odd numbers in Case 1 and all Incompletely Predictable prime & composite numbers & the Completely Predictable number '1' in Case 2 are such that they are respectively given by N = 4 in Case 1 and by N $\geq$ 7 in Case 2. Part

II: In addition, prime numbers will obey 'Plus-Minus Composite Gap 2 Number Alternating Law' for prime gaps $\geq$ 4 and 'Plus Composite Gap 2 Number Continuous Law' for prime gap = 2.

Proof. Apart from the very first Dimension $(2x - 2)^1$ representation in groupings of even & odd numbers in Case 1 and the first three Dimension $(2x - 2)^1$, Dimension $(2x - 4)^1$ and Dimension $(2x - 5)^1$ representations in groupings of prime & composite numbers & the number '1' in Case 2; the possible N value in Dimension $(2x - N)^1$ representation for Case 1 has been shown to be (maximal) 4 and for Case 2 has been shown to be (minimal) 7. These nominated Dimensions simply represent the possible (defacto) baseline "2x - 4" Grand-Total Gaps as per Proposition 7.7 for Case 1 & "2x - 7" Grand-Total Gaps as per Proposition 7.8 for Case 2, and they intrinsically comply in full with Information-Complexity [Input-Output] conservation. Note that all the CFS of Dimensions that can be used to precisely represent the combined even & odd numbers in Case 1 will persistently consist of the same [solitary] Dimension $(2x - 4)^1$ after the very first Dimension $(2x - 2)^1$. The perpetual repeated deviation of N values away from the N = 7 (minimum) in Case 2 is simply representing the infinite magnitude of prime & composite numbers. *The proof is now complete for Part I of Proposition 8.2*□.

The Information-Complexity of each prime number is conserved in that it must always remain constant. This is explained using prime number '61'. At Position x = 61 equating to P_{18} = 61, it is exactly represented by CFS Dimensions $(2x - 7)^1$, $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$ & $(2x - 11)^1$ (with both its prime gap & CFS cardinality = 6). This Virtual Container CFS Dimensions style of representation at that particular Position x = 61 seems to indicate an "unknown but correct" prime number with prime gap = 6 [without revealing its associated full information consisting of '61' = 31^{st} odd number = 18^{st} prime number at Position x = 61 with prime gap = 6] if the Position x = 61's associated full information [which can only be completely determined by calculating all preceding CFS Dimensions/prime gaps prior to this particular CFS Dimensions/prime gap] is with-held from us. Put in a different manner, we can always confirm that '61' is prime by primality tests such as trial division but we will not glean the prime gap of 6 information associated with '61' unless it is displayed in the unique CFS Dimensions representation at Position x = 61 whereby we have now seemingly gained the extra "prime gap of 6 information". However on closer inspection, in order to ultimately arrive at this unique CFS Dimensions representation containing the extra "prime gap of 6 information" in prime number '61' at Position x = 61, will still require prior computing of all preceding CFS Dimensions/prime gaps – this is simply manifesting the hallmark property of Incompletely Predictable entities such as prime numbers (or their equivalent CFS Dimensions/prime gaps representation).

By invoking certain broad principles such as expressed through the Universality of Physical & Mathematical Laws, Pigeonhole principle and Proof by contradiction technique, we can categorically make the following valid statements using sound mathematical judgment. The total number of individual CFS Dimensions required to represent each and every known prime numbers will have to be infinite in magnitude simply because prime numbers are [overall] infinite in magnitude. This is equivalent to the exact mathematical statement that the standalone Dimensions YY groupings [representing the signals for "higher order" non-twin primes appearances] &/or as the front Dimensions YY (sub)groupings [which by itself is fully representative of twin primes] from the Dimensions YYYY appearances, must always recur on an indefinite basis. Common sense alone would suggest that twin primes and the "higher order" cousin primes, sexy primes, etc should aesthetically all be infinite in magnitude simply because they should regularly and universally arise as part of the components in Dimensions YY and Dimensions YYYY appearances. We provide the proof for this statement in the following paragraphs.

An isolated prime is defined as a prime number P such that neither P - 2 nor P + 2 is prime. In other words, P is not part of a twin prime pair. For example, 23 is an isolated prime, since 21 and 25 are both composite. We note that the repeated inevitable presence of Dimension YY grouping is nothing more than indicating the repeated occurrences of isolated prime. This constitutes yet another view on Dimension YY.

Prime gaps = 2, 4, 6,... are CIS and composite gaps = 1 & 2 are CFS. These are general principles which are fully applicable except right at the beginning of prime number integer sequence. Composite numbers with composite gap = 1 are the ”defacto” basic numbers needing to eternally recur simply because they are present in any two consecutive natural numbers [which themselves are also fundamentally and eternally present in the Prime-Composite infinite scale mathematical landscape representing the 'composite gap = 1 signatures' to signify the actual prime gaps *per se* for non-twin prime numbers]. Composite numbers with composite gap = 2 can then be considered as the ”default” basic numbers needing to eternally recur simply because they must be present as 'composite gap = 2 signatures' to signify the appearances of any prime numbers *per se*.

An alternative & advantageous view on prime numbers is from the perspective of the ”manageable” CFS composite gaps [instead of the ”unmanageable” CIS prime gaps] with various observable clear-cut *intrinsic* patterns involving ALTERNATING PRESENCE and ABSENCE of composite numbers with composite gap = 2 in association with every CFS Dimensions representations of prime numbers with prime gaps ≥ 4. This important observation in the context of our Prime-Composite infinite scale mathematical landscape can be deemed a mathematical law needing to be abided by all non-twin prime numbers. Twin primes with CFS Dimensions YY representations are always associated with a composite number with composite gap = 2, and are thus exempted from this law [now designated with the conveniently shortened name 'Plus-Minus Composite Gap 2 Number Alternating Law']. We coined the law for the prime gap = 2 situation as 'Plus Composite Gap 2 Number Continuous Law'. Two illustrative examples for both laws: A twin prime (with prime gap = 2) in its unique CFS Dimensions format is always followed by a composite number with composite gap = 2 [constant] pattern. A cousin prime (with prime gap = 4) in its unique CFS Dimensions format is always followed by two composite numbers with composite gap = 1 & then one composite number with composite gap = 2 [combined] pattern ALTERNATING with three consecutive composite numbers with composite gap = 1 [non-combined] pattern. From this simple observation alone, one can rigorously deduce that we can already/always generate an infinite magnitude of composite numbers from each of the composite gaps of 1 & 2 [automatically endowed with the same composite gaps of 1 & 2 respectively]. We can see that this composite gap = 2 ALTERNATING pattern behavior in cousin primes will not hold true unless twin primes & all other non-cousin primes are infinite in magnitude and integratedly supplying essential ”driving mechanism” to eternally sustain this composite gap = 2 ALTERNATING pattern behavior in cousin primes. Thus we have already discussed and established that twin primes and cousin primes in their CFS Dimensions formats are CIS closely intertwined together when depicted using composite numbers with composite gaps = 1 & 2 with each supplying their own peculiar (infinite) share of associated composite numbers with composite gap = 2 [thus contributing to the overall pool of composite numbers with composite gap = 2].

A inevitable mathematical statement in relation to ”composite gap = 2 pool contribution” based on mathematical reasoning above is that, at the bare minimum, *either* twin prime numbers *or* at least one of the non-twin prime numbers must be infinite in magnitude. A beautiful natural question that follows is: Why then should all the generated sets of prime numbers from 'small gaps' [of 2 & 4] and 'large gaps' [of $\geq$ 6] alike not all belong to CIS thus allowing true uniformity in prime number distribution? Again we can see in Table 6 above depicting the Prime-Composite data for x = 64 that, for instance, prime numbers with prime gap = 6 must also persistently have this 'last-place' composite numbers endowed with composite gap = 2 intermittently appearing in certain rhythmic ALTERNATING patterns, thus complying with the Plus-Minus Composite Gap 2 Number Alternating Law. This CFS Dimensions representation for prime numbers with prime gaps = 6 will again generate their infinite share to the pool of associated composite numbers with composite gap = 2. The presence of this last-place composite numbers with composite gap = 2 in various alternating pattern in their appearances & non-appearances must SELF-GENERATINGLY be similarly extended in a mathematically consistent fashion *ad infinitum* to all remaining infinite number of prime gaps [which were not discussed in details above]. *The proof is now complete for Part II of Proposition 8.2*□.

The preceding few paragraphs above then provide the rigorous proofs for Polignac's and Twin prime conjectures in that we have mathematically shown in a self-consistent manner that prime gaps are [necessarily] infinite (and arbitrarily large) in magnitude with each individual prime gap [necessarily] generating prime numbers which are again infinite in magnitude. The Plus-Minus Composite Gap 2 Number Alternating Law, only clearly seen when prime & composite numbers are depicted in their CFS Dimensions formats giving rise to their respective prime & composite number gaps, is crucial in regards to achieving those rigorous proofs. To comply with this Information-Complexity conservation, which is literally the appropriate recurrences of Dimensions $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, ..., $(2x - \infty)^1$ [all endowed with the same exponent] on an eternal basis, is what we dubbed "Dimensional analysis homogeneity" for prime [and composite] numbers whereby this 'same exponent' has to be consistently 1. **[However, we must perspectively contrast our desired [conjure up] exponent '1' here in the prime-composite number setting (for Polignac's and Twin prime conjectures) with the totally different desired [natural occurring] exponent '$\frac{1}{2}$' in the Riemann zeta function setting (for Riemann hypothesis).]**

Incorrect/incomplete recurrences in any of those mentioned Dimensions or in their exponents [e.g. using exponents $\frac{2}{5}$ or $\frac{3}{5}$ instead of exponent 1] would have the dire consequence of "Dimensional analysis non-homogeneity" resulting in drastically incorrect or incomplete representation of all known prime numbers. Each of the [fixed] finite scale mathematical landscape "page" as part of the [fixed] infinite scale mathematical landscape "pages" for prime numbers will have to permanently display Chaos [sensitivity to initial conditions viz. positions of subsequent prime numbers are "sensitive" to positions of initial prime numbers] and Fractals [manifesting fractal dimensions with self-similarity viz. those aforementioned Dimensions for prime numbers must always be present, albeit in a non-identical manner, for all ranges of x]. Advocated in another manner, the Chaos and Fractals phenomena of those Dimensions for prime numbers above must always be correctly present signifying the accurate composition of prime and composite numbers in different (predetermined) finite scale mathematical landscape "pages" for prime numbers that are self-similar but never identical – and there are, of course, an infinite number of those finite scale mathematical landscape "pages". As previously mentioned in this paper, we regard a 'conjecture' to become a 'hypothesis' when that particular conjecture has been rigorously proven to be true. Abiding to this notational use for those terms, we should now call Polignac's & Twin prime conjectures as Polignac's & Twin prime hypotheses.

9 Polignac's and Twin prime hypotheses

The lemmas and propositions from the preceding section above should now supply all necessary evidences to support the following Theorem {Polignac-Twin prime} I to IV (Virtual Container) which will be seen to further contribute towards fully strengthening the rigorous proofs for Polignac's and Twin prime conjectures in a succinct manner. Only after successfully procuring those rigorous proofs are we finally permitted to term Polignac's and Twin prime conjectures more appropriately as Polignac's and Twin prime hypotheses.

The complete set of even number prime gaps are traditionally & conveniently divided into 'small gaps' and 'large gaps'. In this paper, we arbitrarily denote prime numbers with 'small gaps' as having the finite number of prime gaps = 2 & 4 and prime numbers with 'large gaps' as having the infinite number of prime gaps ≥ 6. We have already established in the previous section above that (i) composite numbers with composite gap = 1 are involved in representing the infinite magnitude of all possible prime gaps except that for twin primes, viz. for prime gaps = 4, 6, 8,... [whereby twin primes whose prime gap = 2 will always involve composite numbers with composite gap = 2 representations] and (ii) composite numbers with composite gap = 2 represent the appearances of relevant prime numbers which must compulsorily be present on an indefinite basis. Furthermore, we ingeniously establish through the Plus-Minus Composite Gap 2 Number Alternating Law that composite numbers with composite gaps = 2 present in each of the prime numbers with prime gaps $\geq$ 4 situation must be observed to appear as some sort of rhythmic patterns of alternating presence and absence for relevant composite numbers with composite gap = 2. The prime gap = 2 situation obeys the Plus Composite Gap 2 Number Continuous Law. These are the dominant underlying driving mechanisms for the infinite magnitude of prime numbers generated by each of the prime gaps ≥ 4 and prime gap = 2 scenarios in a mathematically consistent manner. The case for twin primes with prime gap = 2 scenario can best be understood as the special situation of "rhythmic patterns with CONTINUAL presence" for relevant composite numbers with composite gap = 2. All these prime number INTERLINKED driving mechanisms must be perpetually present ("self-generating") in every single prime gap in order to contribute towards generating the [complete] infinite size pool of composite numbers with composite gap = 2.

Alphonse de Polignac (1826 - 1863) was a French mathematician. In 1849, the year he was admitted to Polytechnique, he made what is known as Polignac's conjecture which relates the complete set of prime numbers to all prime gaps = 2, 4, 6, ..., ∞ [viz. all the even numbers]. We reiterate here again that Twin prime conjecture, which relates twin prime numbers to prime gap = 2, is nothing more than a subset of Polignac's conjecture.

Theorem {Polignac-Twin prime} I. The set of prime numbers P_n = 2, 3, 5, 7, 11, ..., ∞ or the *proxy* set of composite numbers C_n = 4, 6, 8, 9, 10, ..., ∞ is infinite in magnitude with each and every conceivable prime or composite number [but not its actual identity] irrefutably, accurately and completely represented by the following formula involving prime gaps G_{Pi} viz. $P_{n+1} = 2 + \sum_{i=1}^{n} G_{Pi}$ or involving composite gaps viz. $C_{n+1} = 4 + \sum_{i=1}^{n} G_{Ci}$ whereby prime and composite numbers are symbolically represented here with the aid of 'n' notation instead of the usual 'i' notation used in this research paper; and i & n = 1, 2, 3, 4, 5, ..., ∞. The number '2' in the first formula represent P_1, the very first (and only even) prime number. All even numbers except the number '2' which is the only even prime number with odd prime gap = 1, and all odd numbers except all (remaining) odd prime numbers with even number prime gaps and the number '1', are composite numbers. Then the number '4' in the second formula represent C_1, the very first (and even) composite number. Note that the natural numbers required to represent prime gaps must be infinite in magnitude whilst that required to represent composite gaps must be finite in magnitude.

Proof. We treat and closely analyze the above formulae as unique mathematical objects looking for key intrinsic properties and behaviors. By definition, each prime or composite number is assigned a unique prime or composite gap. The absolute number of prime or composite numbers and (thus) prime or composite gaps are known to be infinite in magnitude. As original true formulae containing all possible prime or composite numbers by themselves (viz. without computationally supplying prime or composite gaps as "input information" to generate the necessary prime or composite numbers as "output complexity"), these formulae will intrinsically incorporate the actual presence [but not the actual locations] of the complete set of prime or composite numbers. See Proposition 9.1 below, based on the language using cardinality and pigeonhole principle, for further supporting materials. *The proof is now complete for Theorem {Polignac-Twin prime} I□.*

Theorem {Polignac-Twin prime} II. The set of prime gaps G_{Pi} = 2, 4, 6, 8, 10, ..., ∞ is infinite (& arbitrarily large) in magnitude with each & every conceivable prime gap irrefutably, accurately and completely represented by Dimensions $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, ..., $(2x - \infty)^1$ which must satisfy Information-Complexity conservation in a self-consistent manner. Furthermore, this nominated method of prime gap representation using these Dimensions is purportedly the only (solitary) way to achieve the required conservation.

Proof. The relevant Part I of Proposition 8.2 stated that all prime numbers can be represented by the Dimension $(2x - N)^1$ with $N \geq 7$ for any given x value (except for the x = 1, 2 and 3 values). If each prime number is endowed with a specific prime gap value, then each such prime gap must [via logical mathematical deduction] be able to be represented by the mentioned Dimension $(2x - N)^1$. The preceding mathematical statement is absolutely correct as there is, by definition, a unique prime gap value associated with each prime number. Proposition 9.1 below predominantly based on cardinality language provides further supporting materials that prime gaps are infinite (and arbitrarily large) in magnitude. *The proof is now complete for Theorem {Polignac-Twin prime} II□.*

Theorem {Polignac-Twin prime} III. To maintain Dimensional analysis (DA) homogeneity, those aforementioned [endowed with exponent 1] Dimensions $(2x - N)^1$ from Theorem {Polignac-Twin prime} II must repeat themselves indefinitely in the following specific combinations – (i) Dimension $(2x - 7)^1$ only appearing as twin [two-times-in-a-row] and quadruplet [four-times-in-a-row] sequences, and (ii) Dimensions $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$,..., $(2x - \infty)^1$ appearing as progressive groupings of [even numbers] 2, 4, 6, 8, 10,..., ∞. To accommodate the (only) even prime number '2', exceptions to this DA homogeneity compliance will expectedly occur right at the beginning of prime number sequence – (i) the one-off appearance of Dimensions $(2x - 2)^1$, $(2x - 4)^1$ and $(2x - 5)^1$ and (ii) the one-off appearance of Dimension $(2x - 7)^1$ as a quintuplet [five-times-in-a-row] sequence which is equivalent to (eternal) non-appearance of Dimension $(2x - 6)^1$ at x = 4. Theorem {Polignac-Twin prime} III can be more succinctly stated as the eternal repetitions of well-ordered sets constituted by Dimensions $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$, ..., $(2x - \infty)^1$. These sequentially arranged sets consist of countable finite sets (CFS) whereby from x = 11 onwards, each set will always commence initially as 'baseline' Dimension $(2x - 7)^1$ at x = odd number values and will always end with its last Dimension at x = even number values. Each set will also have varying cardinality with the value derived from 2, 4, 6, 8, 10, 12,..., ∞; and the correctly combined sets will always intrinsically generate the two infinite sets of prime and, by default, composite numbers in an integrated manner whereby at ever larger x values, Prime-$\pi(x)$ will overall become larger but with a decelerating trend and Composite-$\pi(x)$ will overall become larger but with an accelerating trend.

Proof. Theorem {Polignac-Twin prime} III simply represent a mathematical summary of all the expressed characteristics of Dimension $(2x - N)^1$ when used to represent prime numbers with intrinsic display of Dimensional analysis homogeneity. This summary was rigorously derived in Section 7 & 8 above. See Proposition 9.2 below for supporting details on the Dimensional analysis aspect. *The proof is now complete for Theorem {Polignac-Twin prime} III*□.

Theorem {Polignac-Twin prime} IV. Aspect 1. The "quantitive" aspect to the existence of both prime gaps and their associated prime numbers as sets of infinite magnitude will be shown to be correct by simultaneously utilizing concepts derived from Set theory, and incorporating arguments based on 'pigeonhole principle'. Aspect 2. The "qualitative" aspect to the existence of both prime gaps and their associated prime numbers as sets of infinite magnitude will be shown to be correct by the 'Plus-Minus Composite Gap 2 Number Alternating Law' and the 'Plus Composite Gap 2 Number Continuous Law'. Aspect 1 and Aspect 2 are mutually inclusive of each other.

Proof. The required concepts derived from Set theory will mainly involve cardinality of a set together with its 'well-ordering principle' application. Supporting materials for these concepts and for arguments based on 'pigeonhole principle' in relation to Aspect 1 are found in Proposition 9.1 below. The 'Plus-Minus Composite Gap 2 Number Alternating Law' is applicable to all even number prime gaps [apart from the Completely Predictable special case of first even number prime gap = 2 for twin primes]. The prime gap = 2 situation will obey 'Plus Composite Gap 2 Number Continuous Law'. These laws are essentially Laws of Continuity inferring underlying intrinsic driving mechanisms that enables infinity magnitude association for both prime gaps & prime numbers to co-exist. By the same token, these laws have the important implication that they must be applicable to those relevant prime gaps on an perpetual time scale. Supporting materials in relation to Aspect 2 are found in Proposition 8.2 above. *The proof is now complete for Theorem {Polignac-Twin prime} IV*□.

We now generate parallel insightful arguments to that obtained in the process of proving Riemann hypothesis. These powerful arguments, based similarly on utilizing Dimensional analysis (DA), are equally valid despite the desired [conjure up] exponent '1' in prime-composite number setting here for Polignac's & Twin prime conjectures being totally different to the desired [natural occurring] exponent '$\frac{1}{2}$' in Riemann zeta function setting for Riemann hypothesis. We obtain two mutually inclusive conditions from these arguments: Condition 1. The presence of all Dimensions that do repeat themselves on an indefinite basis and with exponent of 1 will give rise to the complete set of prime numbers ["the DA-wise one & only one mathematical possibility argument" associated with inevitable *de novo* DA homogeneity], together with Condition 2. The presence of any Dimension(s) that do not repeat itself (themselves) on an indefinite basis or with exponent other than 1 will give rise to the incomplete set of prime numbers or incorrect set of non-prime numbers ["the DA-wise mathematical impossibility argument" associated with inevitable *de novo* DA non-homogeneity], fully support the rather mute but whole point of this study in that the CFS Dimensions format Virtual Container representations of prime & composite numbers [and their respective prime and composite gaps] are proven to be totally accurate when these two (mutually inclusive) conditions are met. We see that Condition 1 simply reflect the proof from Theorem {Polignac-Twin prime} III above in that all prime numbers will be associated with DA homogeneity when all their Dimensions are endowed with exponent of 1. In addition, Condition 2 invoke the corollary on the inevitable appearance of incomplete prime numbers or non-prime numbers [which will always be associated with DA non-homogeneity] being tightly incorporated into this mathematical framework. See Propositions 9.1 & 9.2, and Corollary 9.3 below for more supporting materials.

Ignoring the glitch caused by the (only) even prime number '2' at the commencement of prime number sequence, we can further analyze the two components prime numbers and composite numbers in terms of (i) measurements based on cardinality of countable infinite set (CIS) and (ii) the pigeonhole principle which states that if n items are put into m containers, with $n>m$, then at least one container must contain more than one item. The composite gaps can only be "finitely" constituted by the numerical values 1 & 2 and the prime gaps can only be "infinitely" constituted by the numerical values 2, 4, 6, ..., ∞ (except the first prime gap = 1 situation). We note that the ordinality of all infinite prime (and infinite composite) numbers is "fixed" implying that each one of the infinite well-ordered Dimension sets conforming to the countable finite set (CFS) type as constituted by Dimensions $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$, ..., $(2x - \infty)^1$ on the respective gaps for prime (and composite) numbers, must also be "fixed". Only by this method alone can we then accommodate each and every one of the infinite prime (and composite) numbers.

Proposition 9.1. Even number prime gaps are infinite (and arbitrarily large) in magnitude with each individual even number prime gap generating odd prime numbers which are again infinite in magnitude.

Proof. In this proof, we note that the two sets of prime and composite numbers are mutually exclusive sets. We can hereby validly ignore the Completely Predictable first & only known even prime number '2' associated with the first & only known Completely Predictable odd number prime gap = 1 and assume the complete set of all prime numbers to only consist of all known odd prime numbers. Let the cardinality of (i) all prime numbers (CIS) derived from all prime gaps 2, 4, 6,..., ∞ sets (CIS) = T, (ii) all prime numbers (CIS) derived from prime gap 2 set (CIS) = T_2, all prime numbers (CIS) derived from prime gap 4 set (CIS) = T_4, all prime numbers (CIS) derived from prime gap 6 set (CIS) = T_6, etc. Paradoxically $T = T_2 + T_4 + T_6 + ... + T_\infty$ is mathematically valid despite $T = T_2 = T_4 = T_6 = ... = T_\infty$ (when defined in terms of the 'well-ordering principle' applied to the cardinality of each set). But if prime numbers derived from one or more prime gap(s) are finite in magnitude of the CFS variety, this will breach the CIS 'uniformity' property resulting in (i) DA non-homogeneity and (ii) the inequality $T > T_2 + T_4 + T_6 + ... + T_\infty$. In the language of pigeonhole principle, residual prime numbers (still CIS in magnitude) not accounted for by the CFS-type prime gap(s) will have to be [incorrectly] contained in one (or more) of the composite gap(s). Ditto for composite numbers with a similar but less complicated argument able to be conjured up for the same case scenario. With the T notations now referring to all composite numbers and their associated composite gaps 1 & 2 in totality; $T = T_1 + T_2$ is mathematically valid despite $T = T_1 = T_2$. If composite numbers derived from one of the composite gaps are finite in magnitude of the CFS variety, this will breach the CIS 'uniformity' property resulting in (i) DA non-homogeneity and (ii) the inequality $T > T_1 + T_2$. Again in the language of pigeonhole principle, a breach in the infinity magnitude for this *vice versa* situation will inevitably lead to those residual composite numbers being [incorrectly] assigned as prime numbers to one (or more) of the prime gap(s).

Finally, the Plus-Minus Composite Gap 2 Number Alternating Law has an underlying built-in intrinsic mechanism to automatically apply to [& further generate] all prime gaps $\geq$ 4 in a mathematically consistent *ad infinitum* manner. Twin primes with prime gap = 2 obeys the Plus Composite Gap 2 Number Continuous Law which is endowed with its own unique underlying built-in intrinsic mechanism to automatically apply to [& further generate] prime gap = 2 appearances in a mathematically consistent *ad infinitum* manner. The above arguments using the cardinality of number property [with "quantitative" features/patterns] thus constitute the rigorous proof for Proposition 9.1 as it has been shown that (i) prime gaps and (ii) prime numbers generated from each of the prime gaps, must all consist of CIS. *The proof on the "quantitative" aspect is now complete for Proposition 9.1*□.

Proposition 9.1 fully encompass Polignac's and Twin prime conjectures. When Proposition 9.1 is rigorously proven to be correct, it will be the overall mathematical "quantitative" statement to fully describe the complete set of prime numbers as generated by the Sieve of Erastosthenes algorithm. This complete set of prime numbers derived from all even number prime gaps can be fully represented by the Dimensions $(2x - N)^1$ concept as rigorously stated in Theorem {Polignac-Twin prime} I - IV above. As these four Theorems are not falsifiable, they must act as valid Virtual Container for all prime & composite numbers, and their respective gaps. Table 6 and Figure 7 on Prime-Composite finite scale mathematical landscape clearly depict perpetual "qualitative" features/patterns by which overall mathematical "qualitative" statements can be made supporting (i) the Plus-Minus Composite Gap 2 Number Alternating Law (which literally can be stated as composite numbers with composite gaps = 2 present in each of the prime numbers with prime gaps ≥ 4 situation must be observed to appear as some sort of rhythmic patterns of alternating presence and absence for relevant composite numbers with composite gap = 2), and (ii) the Plus Composite Gap 2 Number Continuous Law (which literally can be stated as composite numbers with composite gaps = 2 continual appearances in each of the (twin) prime numbers with prime gap = 2 situation). *The proof on the "qualitative" aspect is now complete for Proposition 9.1*□.

From all the above tedious mathematical reasoning, Polignac's and Twin prime conjectures have now been proven to be true thus becoming Polignac's and Twin prime hypotheses with the overall implication that all odd prime numbers generated from each of the infinite (and arbitrarily large) in magnitude prime gaps 2, 4, 6, ..., ∞ are again infinite in magnitude. The four mathematical steps (**'Polignac's and Twin prime conjectures mathematical footprints'**) in specific sequence required to prove Theorem {Polignac-Twin prime} I - IV can be outlined next as:

Step 1: Use the 2-variable formula with 'prime number' variable & 'prime gap' variable to "contain" all prime numbers without revealing their true identities. *Step 2:* Use Dimensions $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, ..., $(2x - \infty)^1$ to "contain" all [even number] prime gaps without knowing their true identities (in a virtual manner). *Step 3:* Define DA homogeneity as the perpetual recurrences of specific groupings of those Dimensions with exponent 1 for all ranges of x. *Step 4:* Supporting mathematical arguments for the "quantitative aspects" will represent all prime numbers in a complete manner, and that for the "qualitative aspects" will also represent all prime numbers in a complete manner.

Proposition 9.2. Only the defined Dimensional analysis homogeneity will always result in the correct & complete set of prime numbers.

Proof. The DA definition is completely dependent on these Dimensions. As all prime (& composite) numbers are "fixed", we can deduce from Figure 7 and Table 6 above that there is one (& only one) way to represent Information-Complexity conservation using our defined Dimensions. Thus, there is one (& only one) way to depict all prime numbers using these Dimensions in a self-consistent manner and this can only be achieved with the one (& only one) DA homogeneity possibility. *The proof is now complete for Proposition 9.2*□.

Corollary 9.3. The defined Dimensional analysis non-homogeneity will always result in the incorrect &/or incomplete set of prime numbers.

Proof. Proposition 9.2 equates DA homogeneity with the correct & complete set of prime numbers with full mathematical consistency. There are "more than one" DA non-homogeneity possibilities. For instance, if a particular $(2x - 7)^1$ Dimension derived from $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$,..., $(2x - \infty)^1$ terminates prematurely and does not perpetually repeat [resulting in loss of continuity and thus depicting one DA non-homogeneity possibility];

then there are intuitively two 'broad' DA possibilities here; namely, (one) DA homogeneity possibility and "all others" endowed with DA non-homogeneity possibilities. This meant that the mathematical consistency of Dimensions $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$,..., $(2x - \infty)^1$ appearing as progressive groupings of [even numbers] 2, 4, 6, 8, 10,..., ∞ will be halted without justification. For optimal clarity, we have treated all those Dimensions using exponents and depict them as $(2x - 7)^1$, $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$,..., $(2x - \infty)^1$. Then a particular Dimension, using the $(2x - 7)^1$ example (endowed with exponent 1), that stop recurring at some point in the prime number sequence would have DA non-homogeneity and be depicted against-all-trends as $(2x - 7)^0$ when endowed with a totally different exponent – which is arbitrarily set as '0' in this case. Thus a Dimension that stop recurring will result in the well-ordered CFS sets from the progressive groupings of [even numbers] 2, 4, 6, 8, 10,..., ∞ for Dimensions $(2x - 8)^1$, $(2x - 9)^1$, $(2x - 10)^1$, $(2x - 11)^1$,..., $(2x - \infty)^1$ to stop existing (and ultimately for sequential prime numbers to stop appearing) at that point using this grouping method – with the likely ensuing outcome that prime & composite numbers are overall [incorrectly] finite in magnitude. Finally, a Dimension with fractional exponent values (other than 1) will always result in non-prime & non-composite (fractional) numbers. *The proof is now complete for Corollary 9.3*□.

Thus the seemingly small but utterly essential sequential mathematical steps in (i) representing all prime numbers using a '2-variable function' (made up of prime number variable and prime gap variable) and (ii) then further representing all prime gaps with the defined Dimensions, will crucially allow proper DA process to happen in the absolute correct way. The 'strong' principle argument mathematical end-result is that DA homogeneity will equate to the complete set of prime numbers whereas DA non-homogeneity will not equate to the complete set of prime numbers. One could additionally advocate for a 'weak' principle argument supporting DA homogeneity for prime numbers in that nature should not "favor" any particular Dimension(s) to terminate and therefore DA non-homogeneity does not, and cannot, exist for prime numbers.

10 Conclusions

In this research paper, we have provided readers with our version of relatively elementary but nevertheless rigorous proofs for the three open problems of Riemann hypothesis, Polignac's and Twin prime conjectures by predominantly utilizing our invented Virtual Container Research Method (VCRM). The way we achieve these proofs is succinctly outlined by our 'Executive Summary' in the Introduction section above. One could envision the VCRM employed in this paper to anticipatedly be accepted as [futuristic] applied mathematics especially for solving the 'Special-Class-of-Mathematical-Problems with Solitary-Proof-Solution' (or simply the 'Incompletely Predictable problems') containing Incompletely Predictable entities of infinite magnitude.

The following overall property has been harnessed to obtain the successful proof on Riemann hypothesis. The countable finite set (CFS) of exactly three types of axes intercepts occurring in Riemann zeta function when $\sigma = \frac{1}{2}$ as opposed to the CFS of exactly two types of axes intercepts occurring in Riemann zeta function when $\sigma \neq \frac{1}{2}$. The three types of axes intercepts in this function occurring only when $\sigma = \frac{1}{2}$ are completely constituted by the three complementary countable infinite sets (CIS) of Gram[x=0,y=0] points (or nontrivial zeros), Gram[y=0] points ('usual' Gram points), and Gram[x=0] points, and the one CFS of negative Gram[y=0] point.

The following overall property has been harnessed to obtain the successful proofs on Polignac's and Twin prime conjectures. The CIS of [Completely Predictable] natural numbers 1, 2, 3, 4, 5, 6, 7,... with their associated CIS of [Completely Predictable] natural gap consisting of the numbers 1, 1, 1, 1, 1, 1,... are completely constituted by two complementary sets of numbers, namely (i) the CIS of [Incompletely Predictable] odd prime numbers 3, 5, 7, 11, 13, 17,... with their associated CIS of [Incompletely Predictable] prime gaps consisting of the numbers 2, 2, 4, 2, 4,... plus the CFS of solitary [Completely Predictable] even prime number 2 with its associated CFS of [Completely Predictable] prime gap consisting of the number 1 (ii) the CIS of [Incompletely Predictable] even & odd composite numbers 4, 6, 8, 9, 10, 12,... with their associated CIS of [Incompletely Predictable] composite gaps consisting of the numbers 2, 2, 1, 1, 2, 2,.... and (iii) the CFS of [Completely Predictable] solitary number '1'. The following are useful expressions: All the CIS of prime numbers must be odd numbers except the very first prime number '2' which is an even number and the majority of odd numbers are not prime numbers and belong to the CIS of composite numbers. All even numbers must be composite numbers except the first even number '2' which is a prime number. This last expression can reciprocally be written as: the even number '2' is not a composite number despite the caveat "all even numbers must in general be composite numbers".

The commonly used public key encryption system RSA (Rivest-Shamir-Adleman) was first described in 1977 by Ron Rivest, Adi Shamir and Leonard Adleman from Massachusetts Institute of Technology. It gets its security from the difficulty of factoring large integers that are the product of two large prime numbers. Multiplying these two numbers is easy but determining the original prime numbers from the total (viz, factoring) is considered infeasible due to the time it would take to achieve this task even using powerful supercomputers. The public and private key-generation algorithm is the most complex part of RSA cryptography. Two large prime numbers, p and q, are generated using the Rabin-Miller primality test algorithm. A modulus n is calculated by multiplying p and q. This number is used by both the public and private keys and provides the link between them. Its length, usually expressed in bits, is called the key length. The public key consists of the modulus n, and a public exponent, e, which is normally set at 65537 as this is a prime number that is not too large. The e figure does not have to be a secretly selected prime number as the public key is shared with everyone. The private key consists of the modulus n and the private exponent d which is calculated using the Extended Euclidean algorithm to find the multiplicative inverse with respect to the totient of n.

Internet transactions in e-commerce depends on the integrity of humongous [non-prime] numbers to be anonymously or secretly constituted from its basic prime numbers such as that utilized by the public key encryption system RSA. It is often thought that breaching this integrity by being able to easily identify prime numbers constituents of relevant humongous numbers after successfully solving Riemann hypothesis would have massive implication in that it will now brought the whole of e-commerce to its knees overnight.

The truthfulness of the preceding narrative paragraph can now be beautifully refuted by us here as follows. Having solved Riemann hypothesis, Polignac's and Twin prime conjectures is simply irrelevant because the CIS of nontrivial zeros and prime numbers must be treated as Incompletely Predictable entities abiding by Complex Elementary Fundamental Laws that involve Incompletely Predictable Laws [and not Simple Elementary Fundamental Laws that involve Completely Predictable Laws]. Thus, in principle, we have dispelled the doom-and-gloom prophecy that financial disaster might follow when successful proof of Riemann hypothesis occur.

However, in practice, there may be a twist to this sentiment. Building ever more powerful supercomputers, which are classical computers based on classical physics, could more easily crack crptic codes but this issue can progressively be negated by employing ever larger prime numbers in cryptic codes. The world's smallest transitor made from a single atom was created in 2012. This should hypothetically assist in the future building of the most powerful supercomputers. But the infinitely more powerful quantum computer, based on quantum mechanics phenomena such as superposition and entanglement, could solve problems in minutes which would otherwise take thousands of years due to the theoretical ability of quantum computers to do a huge range of calculations simultaneously rather than sequentially as in classical computers. This will revolutionize research into areas like artificial intelligence, self-driving cars and drug design in a positive manner but will likely impact the desired role of many cryptic codes in a negative manner by easily cracking them ("cryptocalypse"). Quantum computers could easily crack many crytic codes such as that used by RSA in polynomial time by using Shor's algorithm to find the prime number factors of large integers. To circumvent this problem, the use of alternative crytic codes such as the lattice-based cryptosystems which are known not to be broken by quantum computers will be desirable. Finally, 'quantum cryptography' (as opposed to 'classical cryptography') could potentially fulfill some of the functions of public key cryptography.

Named after Danish mathematician Jørgen Pedersen Gram (June 27, 1850 – April 29, 1916), the 'usual' Gram points are the other conjugate pairs values on the critical line defined by $Im\{\zeta(\frac{1}{2} \pm \imath t)\} = 0$ whereby they obey Gram's Rule and Rosser's Rule with many other interesting characteristics. As a bonus, we additionally provide detailed explanations below on this Gram points (or Gram[y=0] points) and its closely related Gram[x=0] points.

The Z function is a function used for studying the Riemann zeta function along the critical line. It is also called the Riemann-Siegel Z function, the Riemann-Siegel zeta function, the Hardy function, the Hardy Z function, and the Hardy zeta function. It can be defined in terms of the Riemann-Siegel theta function and the Riemann zeta function by $Z(t) = e^{\imath\theta(t)}\zeta(\frac{1}{2} + \imath t)$ whereby $\theta(t) = \arg(\Gamma(\frac{(2\imath t+1)}{4})) - \frac{\log\pi}{2}t$. In the next paragraph, we will only outline a brief exposition of some of the useful properties of Gram points.

The algorithm used to compute Z(t) is called the Riemann-Siegel formula. The zeta function on the critical line, $\zeta(\frac{1}{2} + \imath t)$, will be real when $\sin(\theta(t)) = 0$. Positive real values of t where this occurs are called Gram points and can also be described as the points where $\frac{\theta(t)}{\pi}$ is an integer. The real part of zeta function on the critical line tends to be positive, while the imaginary part alternates more regularly between positive and negative values. That means that the sign of Z(t) must be opposite to that of the sine function most of the time, so one would expect the nontrivial zeros of Z(t) to alternate with zeros of the sine term, i.e. when θ takes on integer multiples of π. This turns out to hold most of the time and is known as Gram's Rule (Law) – a law which is violated infinitely often though. Thus Gram's Law is the statement that nontrivial zeros of Z(t) alternate with Gram points. Gram points which satisfy Gram's Law are called 'good', while those that do not are called 'bad'. A Gram block is an interval such that its very first and last points are good Gram points and all Gram points inside this interval are bad. The exercise of counting nontrivial zeros then reduces to that of counting all Gram points where Gram's Law is satisfied, and adding to that the count of nontrivial zeros inside each Gram block. With this process we do not have to locate nontrivial zeros exactly, and we just have to compute Z(t) accurately enough to show that it changes sign. Up to now, a crucial obervation to note from the above is that Riemann zeta function (and its *proxy* Dirichlet eta function) will always generate an infinite number of relevant spirals/loops on which will be located all nontrivial zeros and 'usual'

Gram points (and our Gram [x=0] points) in a fixed relationship manner. Then the aforementioned Gram's Law and its violation, Gram block, etc will be Completely Predicted to periodically occur an infinite number of times, albeit in an Incompletely Predictable manner.

Hadamard product:

$$\zeta(s)$$
$$= \frac{e^{(\log(2\pi)-1-\frac{\gamma}{2}).s)}}{2(s-1).\Gamma(1+\frac{s}{2})}.\Pi_\rho\left(1-\frac{s}{\rho}\right).e^{\frac{s}{\rho}}$$
$$= \pi^{\frac{s}{2}}.\frac{\Pi_\rho\left(1-\frac{s}{\rho}\right)}{2(s-1).\Gamma\left(1+\frac{s}{2}\right)}$$

Euler product formula:

$$\sum_{n=1}^{\infty}\frac{1}{n^s} \qquad \{\text{which is } \zeta(s)\}$$
$$= \Pi_{p\ prime}\frac{1}{(1-p^{-s})}$$
$$= \frac{1}{(1-2^{-s})}.\frac{1}{(1-3^{-s})}.\frac{1}{(1-5^{-s})}.\frac{1}{(1-7^{-s})}.\frac{1}{(1-11^{-s})}\cdots\frac{1}{(1-p^{-s})}\cdots$$

The beautiful Hadamard product above is the infinite product expansion of Riemann zeta function, $\zeta(s)$, based on Weierstrass's factorization theorem – this product simultaneously contains both trivial and nontrivial zeros. The beautiful Euler product formula above connects Riemann zeta function and prime numbers and was discovered by Euler – this identity has, by definition, the left hand side being $\zeta(s)$ and the infinite product on the right hand side extends over all prime numbers p. The form of the Hadamard product clearly displays the simple pole at s = 1, the trivial zeros at all even negative integers due to the gamma function term in the denominator, and the nontrivial zeros at $s = \rho$; with the letter γ in the expansion here specifically denoting the Euler-Mascheroni constant. Note that with the second simpler infinite product expansion formula of Hadamard, to ensure convergence, the product should be taken over "matching pairs" of zeroes, i.e. the factors for a pair of zeroes of the form ρ and $1-\rho$ should be combined.

Resulting crucial primary and secondary beneficiary by-products arising out of proving Riemann hypothesis promise to be aplenty. The usual primary by-products arising out of the rigorous proof for Riemann hypothesis are often stated as "With this one solution, we have proven five hundred theorems or more at once". This apply to many important theorems in number theory (mostly about prime numbers) that rely on properties of Riemann zeta or Dirichlet eta functions such as where trivial & nontrivial zeros are, and are not, located. A classical example of this primary by-product is the resulting absolute and full delineation of prime number theorem, which relates to prime counting function. This function, usually denoted by $\pi(x)$, is defined as the number of prime numbers less than or equal to x. In mathematics, the logarithmic integral function or integral logarithm li(x) is a special function. It is relevant in problems of physics and has number theoretic significance, occurring in prime number theorem as an estimate of the number of prime numbers less than a given value. In prime number theorem, the form of this function is defined so that li(2)=0; viz. li(x) $\equiv \int_2^x \frac{du}{\ln u}$ = li(x)-li(2). The symbol 'ln' here denotes natural logarithm. Thus the rigorous proof for Riemann hypothesis on nontrivial zeros location at $\sigma=\frac{1}{2}$, together with the negative even number locations for trivial zeros, is instrumental in proving the efficacy of techniques that estimate $\pi(x)$ efficiently and reasonably well. In particular, our rigorous proof of Riemann hypothesis will now confirm the "best possible" bound for the error (the "smallest possible" error) of prime number theorem.

We mention here that there are other less accurate ways of estimating $\pi(x)$ such as that conjectured by Gauss and Legendre at the end of the 18th century. This is approximately x/ln$x$ in the sense $\lim_{x\to\infty}\frac{\pi(x)}{x/\ln x} = 1$. For prime counting function, other functions more convenient to work with can also be utilized and they open up a whole new world of marvelous mathematical relationships. An example is Riemann prime counting function (aka prime power counting function), commonly denoted by *J*(x). This non-infinite series function has jumps of 1/n for prime powers p^n, and with it taking a value halfway between the two sides at discontinuities. Amazingly, the prime counting function $\pi(x)$ is related to *J*(x) by the Mobius transform. More amazingly still, *J*(x) is related to Riemann zeta function through the Mellin transform (which is an integral transform).

In number theory, Skewes' number is any of several extremely large numbers used by South African mathematician Stanley Skewes as upper bounds for the smallest natural number x for which li(x)$<\pi(x)$. These bounds have since been improved by others: there is a crossing near $e^{727.95133}$ but it is not known whether this is the smallest. John Edensor Littlewood, who was Skewes' research supervisor, proved in 1914 [10] that there is such a number (and so, a first such number); and indeed found that the sign of the difference $\pi(x)$ - li(x) changes infinitely often. This then refute all prior numerical evidence available that seem to suggest li(x) was always more than $\pi(x)$. The key point here is that the [100% accurate] $\pi(x)$ mathematical tool being "wrapped around" by the [less-than-100% accurate] approximate mathematical tool li(x) infinitely often via this 'sign of difference' changes meant that li(x) must be the most efficient approximate mathematical tool. Contrast this with the "crude" x/ln$x$ approximate mathematical tool where values obtained diverge away from $\pi(x)$ at increasingly greater rate when larger range of prime numbers are being studied. Finally, we point out to readers interested in "miscellanous materials" which are useful but not required for actual proofs on Riemann hypothesis, Polignac's & Twin prime conjectures that we have assign these materials (often endowed with philosophical overtones) to Appendix D below.

Appendix A: Prerequisite lemma, corollary and propositions for Gram[x=0] and Gram[y=0] conjectures

For the mathematical treatment of our two cases on Gram[x=0] and Gram[y=0] points here, we will follow a similar procedure carried out for the above case on nontrivial zeros (Gram[x=0,y=0] points).

Lemma A.1. The {Modified-for-Gram[x=0] & [y=0] points}-Riemann-Dirichlet Ratio is derived from Riemann zeta or Dirichlet eta function and thus has the resultant capability to incorporate the actual presence [but not the actual locations] of the complete sets of Gram[x=0] points and Gram[y=0] points, and the solitary negative Gram[y=0] point.

Proof. We hereby depict the case on Gram[y=0] points (which is the usual 'Gram points') and the solitary negative Gram[y=0] point to obtain the relevant [simply named] {Modified-for-Gram points}-Riemann-Dirichlet Ratio. Apply n^2(Euler) to Eq. (3), we have

$\zeta(s) = \gamma.\eta(s) = \gamma.[Re\{\eta(s)\} + i.Im\{\eta(s)\}]$ whereby

$$Re\{\eta(s)\} = \sum_{n=1}^{\infty}((2n-1)^{-\sigma}.\cos(t.\log(2n-1)) - (2n)^{-\sigma}.\cos(t.\log(2n))) \text{ and}$$

$$Im\{\eta(s)\} = i.\sum_{n=1}^{\infty}((2n)^{-\sigma}.\sin(t.\log(2n)) - (2n-1)^{-\sigma}.\sin(t.\log(2n))) \qquad (25)$$

Here γ is the proportionality factor $\frac{1}{(1-2^{1-s})}$.

As Gram[y=0] points and the solitary negative Gram[y=0] point based on $\zeta(s)$ is identical to that based on its proxy $\eta(s)$, then Gram[y=0] conjecture is satisfied when

$$\sum ReIm\{\eta(s)\} = Re\{\eta(s)\} + 0, \text{ or simply } Im\{\eta(s)\} = 0 \qquad (26)$$

Applying Eq. (26) to Eq. (25), this equation can be simplified and be reduced to

$$\sum_{n=1}^{\infty}((2n)^{-\sigma}.\sin(t.\log(2n)) - (2n-1)^{-\sigma}.\sin(t.\log(2n))) = 0$$

$$\sum_{n=1}^{\infty}(2n)^{\sigma}.\sin(t.\log(2n)) = \sum_{n=1}^{\infty}(2n-1)^{-\sigma}.\sin(t.\log(2n)) \qquad (27)$$

We note from the above sequential mathematical derivation of Eq. (27) that this equation will completely and intrinsically fulfills the 'presence of the complete set of Gram[y=0] points and the solitary negative Gram[y=0] point without knowing their actual location' criteria.

$$\frac{\sum_{n=1}^{\infty}\sin(t.\log(2n))}{\sum_{n=1}^{\infty}\sin(t.\log(2n-1))} = \frac{\sum_{n=1}^{\infty}(2n)^{\sigma}}{\sum_{n=1}^{\infty}(2n-1)^{\sigma}} \qquad (28)$$

Eq. (28) will also abide to this specified criteria as it is simply the result of rearranging the terms in Eq. (27) giving rise to our {Modified-for-Gram points}-Riemann-Dirichlet Ratio. *This proof is now complete for Lemma A.1*□.

Denote the left hand side ratio as Ratio R1 (of a 'cyclical' nature) and the right hand side ratio as Ratio R2 (of a 'non-cyclical' nature). The {Modified-for-Gram points}-Riemann-Dirichlet Ratio calculations, valid for all continuous real number values of t, would theoretically result in infinitely many non-Hybrid integer sequences [here arbitrarily] for the $0 < \sigma < 1$ critical strip region of interest with $n = 1, 2, 3, \ldots, \infty$ being discrete integer number values, or n being continuous real numbers from 1 to ∞ with Riemann integral applied in the interval from 1 to ∞. This infinitely many integer sequences can geometrically be interpreted to representatively cover the entire plane of the critical strip bounded by σ values of 0 and 1, thus (at least) allowing our proposed proof on Gram[y=0] conjecture to be of a 'complete' nature.

Proposition A.2. The equivalent Sigma-Power Laws can be rigorously derived from {Modified-for-Gram[x=0] & [y=0] points}-Riemann-Dirichlet Ratio.

Proof. We hereby depict the case on Gram[y=0] points (which is the usual 'Gram points') and the solitary negative Gram[y=0] point to obtain its equivalent Sigma-Power Laws. We apply Riemann integral to the four continuous functions of Ratio R1 and Ratio R2 in Eq. (28) thus depicting the {Modified-for-Gram points}-Riemann-Dirichlet Ratio in the integral forms – see the subsequent Eq. (33) below.

Thereafter, step-by-step we derive the closely related {Modified-for-Gram points}-Dirichlet σ-Power Law [expressed in real numbers] and the {Modified-for-Gram points}-Riemann σ-Power Law [expressed in real and complex numbers] – these two laws are further elaborated below. The {Modified-for-Gram points}-Sigma-Power Law has its Dirichlet and Riemann versions directly related to each other via Dirichlet $\eta(s)$ being the equivalence of Riemann $\zeta(s)$ but without the $\frac{1}{(1-2^{1-s})}$ proportionality factor. We stress that it is the main underlying mathematically-consistent properties of *symmetry* and *constraints* arising from this power law that also allowed our most direct, basic and elementary proof for the Gram[y=0] conjecture to mature. An important characteristic to note of {Modified-for-Gram points}- σ-Power Law is that its exact formula expression in the usual mathematical language [$y = f(x_1, x_2)$ format description for a 2-variable function] consists of $y = \{2n\}$ or $\{2n-1\} = f(t, \sigma)$ with $n = 1, 2, 3, \ldots, \infty$ or n = 1 to ∞ with Riemann integral application; -∞ < t < +∞; and σ being of real number values $0 < \sigma < 1$ corresponding to the [arbitrarily defined] critical strip of interest in this particular case scenario.

For the, initially, {2n} parameter integration of R1, $\int_1^\infty \sin(t.\log(2n)).dn$

Use integration by u-substitution technique to obtain $u = t.\log(2n)$, $n = \frac{1}{2}e^{\frac{1}{t}(u)}$, $\frac{du}{dn} = \frac{2t}{2n} = \frac{t}{n}$, $du = t.\frac{dn}{n}$, $dn = 2n.\frac{du}{2t} = n.\frac{du}{t}$
$\int_1^\infty \sin(u).\frac{n}{t}.du = \int_1^\infty \sin(u).\frac{1}{t}.\frac{1}{2}.e^{\frac{1}{t}(u)}.du = \frac{1}{2t}\int_1^\infty \sin(u).e^{\frac{1}{t}u}.du$

Use the Products of functions proportional to their second derivatives, namely the indefinite integral $\int \sin(a.u).e^{b.u}du = \frac{e^{bu}}{a^2+b^2}(b.\sin(a.u) - a.\cos(a.u)) + C$ (Comparatively, we observe that $\int \cos(a.u).e^{b.u}du = \frac{e^{bu}}{a^2+b^2}(b.\cos(a.u) + a.\sin(a.u)) + C$). Then a = 1, b = $\frac{1}{t}$, and temporarily ignore the $\frac{1}{2t}$ term, we have

$$\int_1^\infty \sin(u).e^{\frac{1}{t}u}.du$$
$$= [(e^{\frac{1}{t}u})/(1 + \frac{1}{t^2}).(\frac{1}{t}.\sin(u) - \cos(u)) + C]_1^\infty$$
$$= [(t^2.e^{\frac{1}{t}u})/(t^2+1).(\frac{1}{t}.\sin(u) - \cos(u)) + C]_1^\infty$$

Now apply the non-linear combination of sine and cosine functions identity, namely $a.\sin(u) + b.\cos(u) = c.\sin(u+\varphi)$ where $c = \sqrt{(a^2+b^2)}$ and $\varphi = a\tan 2(b,a)$. Here $a = \frac{1}{t}$, b = -1, $c = \sqrt{(\frac{1}{t})2 + 1)} = \frac{\sqrt{(t^2+1)}}{t}$. Then we have

$$\int_1^\infty \sin(u).e^{\frac{1}{t}u}.du$$
$$= [(t^2.e^{\frac{1}{t}u})/(t^2+1).\frac{\sqrt{(t^2+1)}}{t}.\sin(u + atan2(b,a)) + C]_1^\infty$$
$$= [(t.e^{\frac{1}{t}u})/(\sqrt{(t^2+1)}).\sin(u + \arctan(-t)) + C]_1^\infty$$

But there was a $\frac{1}{2t}$ term in front of this integral as can be seen above. Then after substituting this term and simplifying, the integral

$$\int_1^\infty \sin(u).e^{\frac{1}{t}u}.du$$
$$= [(e^{\frac{1}{t}u})/2\sqrt{(t^2+1)}.\sin(u - \arctan(t)) + C]_1^\infty$$

But $u = t.\log(2n)$. Reverting back to the n variable, the equation for the {2n} parameter finally becomes

$$\int_1^\infty \sin(t.\log(2n)).dn$$
$$= [(\{2n\}.e^{\frac{1}{t}})/(2\sqrt{(t^2+1)}.\sin(t.\log(2n) - \arctan(t)) + C)]_1^\infty \tag{29}$$

In a similar manner integration for the {2n-1} parameter, this equation becomes

$$[(\{2n-1\}.e^{\frac{1}{t}})/(2\sqrt{(t^2+1)}.\sin(t.\log(2n-1) - \arctan(t)) + C)]_1^\infty \tag{30}$$

In R2 using {2n} parameter,

$$\int_1^\infty (2n)^\sigma dn$$
$$= [1/(2(\sigma+1)).(2n)^{\sigma+1} + C)]_1^\infty$$
$$= [\frac{1}{3}\{2n\}(2n)^{\frac{1}{2}} + C)]_1^\infty \text{ when } \sigma = \frac{1}{2} \tag{31}$$

For the equivalent R2 based on 2n-1 parameter,

$$\int_1^\infty (2n-1)^\sigma dn$$
$$= [1/(2(\sigma+1)).(2n-1)^{\sigma+1} + C)]_1^\infty$$
$$= [\frac{1}{3}\{2n-1\}(2n-1)^{\frac{1}{2}} + C)]_1^\infty \text{ when } \sigma = \frac{1}{2} \tag{32}$$

The Ratio R1 and Ratio R2 of {Modified-for-Gram points}-Riemann-Dirichlet Ratio (for $\sigma = \frac{1}{2}$) is defined by the integral

$$\frac{[(\{2n\}.(e^{\frac{1}{t}}\cdot/2\sqrt{(t^2+1)}).\sin(t.\log(2n) - \arctan(t))]_1^\infty}{[(\{2n-1\}.e^{\frac{1}{t}}/2\sqrt{(t^2+1)}).\sin(t.\log(2n-1) - \arctan(t))]_1^\infty} = \frac{[\frac{1}{3}\{2n\}(2n)^{\frac{1}{2}}]_1^\infty}{[\frac{1}{3}\{2n-1\}(2n-1)^{\frac{1}{2}}]_1^\infty}$$

Canceling out the common parameter {2n} and {2n-1} terms,

$$\frac{[(e^{\frac{1}{t}}/2\sqrt{(t^2+1)}).\sin(t.\log(2n) - \arctan(t))]_1^\infty}{[(e^{\frac{1}{t}}/2\sqrt{(t^2+1)}).\sin(t.\log(2n-1) - \arctan(t))]_1^\infty} \leftarrow \textbf{this is R1}$$
$$= \frac{[\frac{1}{3}(2n)^{\frac{1}{2}}]_1^\infty}{[\frac{1}{3}(2n-1)^{\frac{1}{2}}]_1^\infty} \leftarrow \textbf{this is R2} \tag{33}$$

The {Modified-for-Gram points}-Dirichlet and {Modified-for-Gram points}-Riemann σ-Power Laws are given by exact formulae in Eqs. (34) to (37) below with ψ being the same proportionality constant valid for both power laws. We can now dispense with the constant of integration C. **Using Dimensional analysis (DA) approach we easily conclude that the 'fundamental dimension' [Variable / Parameter / Number X to the power of Number Y] has to be represented by the particular 'unit of measure' [Variable / Parameter / Number X to the power of Number Y whereby Number Y needs to be of specific value $\frac{1}{2}$] for DA homogeneity to occur. This *de novo* DA homogeneity equates to the**

location of complete set of Gram[y=0] points and solitary negative Gram[y=0] point, and is crucially a fundamental property present in all laws of Physics. The 'unknown' σ variable, now endowed with value of $\frac{1}{2}$, is treated as Number Y.

{Modified-for-Gram points}-Dirichlet σ-Power Law using the {2n} parameter:

$$[\{2n\}.\frac{e^{\frac{1}{t}}}{2(t^2+1)^{\frac{1}{2}}}.\sin(t.\log(2n)-\arctan(t))]_1^\infty = \psi.\frac{1}{3}\{2n\}(2n)^{\frac{1}{2}}]_1^\infty$$

With common parameter {2n} canceling out on both sides, the equation reduces to

$$[\frac{e^{\frac{1}{t}}}{2(t^2+1)^{\frac{1}{2}}}.\sin(t.\log(2n)-\arctan(t))-\psi.\frac{1}{3}(2n)^{\frac{1}{2}}]_1^\infty = 0 \tag{34}$$

Similarly for the {2n-1} parameter, this equivalent equation is

$$[\frac{e^{\frac{1}{t}}}{2(t^2+1)^{\frac{1}{2}}}.\sin(t.\log(2n-1)-\arctan(t))-\psi.\frac{1}{3}(2n-1)^{\frac{1}{2}}]_1^\infty = 0 \tag{35}$$

Finally, the {Modified-for-Gram points}-Riemann σ-Power Law is given by the exact formulae using {2n} and {2n-1} parameters with the $\gamma = \frac{2^{\frac{1}{2}}.(\cos(t.\log(2)+i.\sin(t.\log(2))}{(2^{\frac{1}{2}}.(\cos(t.\log(2)+i.\sin(t.\log(2)-2)}$ substitution.

$$\left[\frac{e^{\frac{1}{t}}}{2(t^2+1)^{\frac{1}{2}}}.\sin(t.\log(2n)-\arctan(t))-\psi.\gamma.\frac{1}{3}.(2n)^{\frac{1}{2}}\right]_1^\infty = 0$$
$$\left[\frac{e^{\frac{1}{t}}}{2(t^2+1)^{\frac{1}{2}}}.\sin(t.\log(2n)-\arctan(t))-\psi.\frac{2^{\frac{1}{2}}.(\cos(t.\log(2)+i.\sin(t.\log(2)))}{2^{\frac{1}{2}}.(\cos(t.\log(2)+i.\sin(t.\log(2))-2)}.\frac{1}{3}.(2n)^{\frac{1}{2}}\right]_1^\infty = 0 \tag{36}$$

$$\left[\frac{e^{\frac{1}{t}}}{2(t^2+1)^{\frac{1}{2}}}.\sin(t.\log(2n-1)-\arctan(t))-\psi.\gamma.\frac{1}{3}.(2n-1)^{\frac{1}{2}}\right]_1^\infty = 0$$
$$\left[\frac{e^{\frac{1}{t}}}{2(t^2+1)^{\frac{1}{2}}}.\sin(t.\log(2n-1)-\arctan(t))-\psi.\frac{2^{\frac{1}{2}}.(\cos(t.\log(2)+i.\sin(t.\log(2)))}{2^{\frac{1}{2}}.(\cos(t.\log(2)+i.\sin(t.\log(2))-2)}.\frac{1}{3}.(2n-1)^{\frac{1}{2}}\right]_1^\infty = 0 \tag{37}$$

The proof is now complete for Proposition A.2□.

Proposition A.3. Application of Dimensional analysis homogeneity to the equivalent Sigma-Power Laws will always be associated with the one specific $\sigma = \frac{1}{2}$ value for Gram[x=0] & Gram[y=0] points & the solitary negative Gram[y=0] point, and this will enable the rigorous proofs for Gram[x=0] & Gram[y=0] conjectures to mature.

Proof. We again depict the case on Gram[y=0] points and the solitary negative Gram[y=0] point here. We note the γ proportionality factor given by Eq. (13) above when depicted with the $2^{\frac{1}{2}}$ constant numerical value (derived using $\sigma = \frac{1}{2}$ as proposed in the original Gram[y=0] conjecture) further allowing, and enabling, *de novo* Dimensional analysis homogeneity compliance in the {Modified-for-Gram points}-Riemann σ-Power Law in Eqs. (36) and (37) above. There is only one type of $\frac{1}{2}$ exponent present in {Modified-for-Gram points}-Riemann σ-Power Law indicating Dimensional analysis homogeneity. *This two mathematical statements essentially complete the proof for Proposition A.3 with complimentary demonstration below for the Dimensional analysis non-homogeneity case scenario*□.

Corollary A.4. Application of Dimensional analysis non-homogeneity to Sigma-Power Laws will never be associated with the one specific $\sigma = \frac{1}{2}$ value for Gram[x=0] & Gram[y=0] points & the solitary negative Gram[y=0] point, and this will enable the rigorous proofs for Gram[x=0] & Gram[y=0] conjectures to mature.

Proof. We again depict the case on Gram[y=0] points and the solitary negative Gram[y=0] point here. We illustrate the Dimensional analysis non-homogeneity property for a $\sigma = \frac{1}{4}$ arbitrarily chosen value [clear-cut case with {2n}-parameter] of {Modified-for-Gram points}-Riemann σ-Power Law lying on a non-critical line (with total absence of Gram[y=0] points and the solitary negative Gram[y=0] point) in the following formula derived using Eqs. (13) and (36). **As Ratio R1 component of {Modified-for-Gram points}-Riemann-Dirichlet Ratio is independent of σ variable, unlike the Ratio R2 component of {Modified-for-Gram points}-Riemann-Dirichlet Ratio and the γ proportionality factor which are dependent on σ variable, we now note the mixture of $\frac{1}{4}$ and $\frac{1}{2}$ exponents subtly, but nevertheless, present in this formula confirming Dimensional analysis non-homogeneity.** Also the replacement of $\frac{1}{3}$ fraction with $\frac{2}{5}$ fraction [derived from substituting $\sigma = \frac{1}{4}$ into $\frac{1}{2(\sigma+1)}$] has occurred. Mathematically, this Dimensional analysis non-homogeneity property for any real number value of σ, when $\sigma \neq \frac{1}{2}$ and $0 < \sigma < 1$, will always be present indicative of the full presence of {Non-critical lines}-Gram[y=0] points and the solitary negative {Non-critical lines}-Gram[y=0] point, or by the same token, indicative of total absence of Gram[y=0] points and the solitary negative Gram[y=0] point.

$$\left[\frac{e^{\frac{1}{t}}}{2(t^2+1)^{\frac{1}{2}}}.\sin(t.\log(2n)-\arctan(t))-\psi.\frac{2^{\frac{1}{4}}.(\cos(t.\log(2)+i.\sin(t.\log(2)))}{2^{\frac{1}{4}}.(\cos(t.\log(2)+i.\sin(t.\log(2))-2)}.\frac{2}{5}.(2n)^{\frac{1}{4}}\right]_1^{\infty}=0 \tag{38}$$

The proof is now complete for Corollary A.4□.

The $\frac{1}{2}$ exponent in Eq. (38) only occur once in the denominator of the first term. The subtlety of Dimensional analysis non-homogeneity for {Non-critical lines}-Gram[y=0] points and the solitary negative {Non-critical lines}-Gram[y=0] point is even more pronounced when compared to its closely related cousin Eq. (18) above for Riemann σ-Power Law [with easy clarification and confirmation of the $\frac{1}{2}$ exponent occurring twice in the first term].

For Gram[x=0] points, Gram[x=0] conjecture is satisfied by Eqs. (39) to (41) below, whereby Eq. (39) is the equivalent of Eq. (26) above.

$$\sum ReIm\{\eta(s)\} = 0 + Im\{\eta(s)\}, \text{ or simply } Re\{\eta(s)\} = 0 \tag{39}$$

Not unexpectedly with only minor subtraction (-) operator to addition (+) operator sign change required, the equivalent to Eq. (36) and Eq. (38) above using {2n} parameter for Gram[x=0] points can easily be derived to (respectively) be:

$$\left[\frac{e^{\frac{1}{t}}}{2(t^2+1)^{\frac{1}{2}}}.\sin(t.\log(2n)+\arctan(t))-\psi.\frac{2^{\frac{1}{2}}.(\cos(t.\log(2)+i.\sin(t.\log(2)))}{2^{\frac{1}{2}}.(\cos(t.\log(2)+i.\sin(t.\log(2))-2)}.\frac{1}{3}.(2n)^{\frac{1}{2}}\right]_1^{\infty}=0 \tag{40}$$

$$\left[\frac{e^{\frac{1}{t}}}{2(t^2+1)^{\frac{1}{2}}}.\sin(t.\log(2n)+\arctan(t))-\psi.\frac{2^{\frac{1}{4}}.(\cos(t.\log(2)+i.\sin(t.\log(2)))}{2^{\frac{1}{4}}.(\cos(t.\log(2)+i.\sin(t.\log(2))-2)}.\frac{2}{5}.(2n)^{\frac{1}{4}}\right]_1^{\infty}=0 \tag{41}$$

Table 7 Even-Odd mathematical (tabulated) landscape using data obtained for x = 64. Legend: E = even, O = odd, Y = Dimension 2x - 4 (for visual clarity).

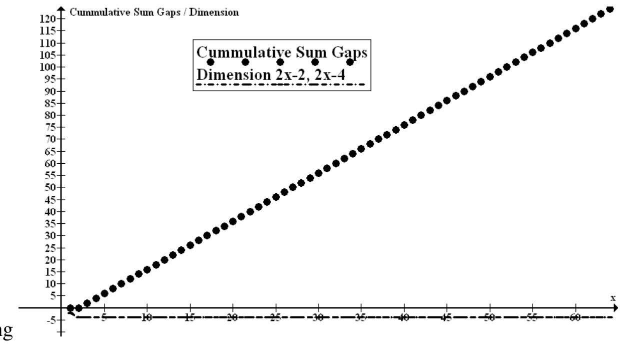
x	**E_i or O_i, Gaps**	**ΣEO_x-Gaps**	Dimension	**x**	**E_i or O_i, Gaps**	**ΣEO_x-Gaps**	Dimension
1	O1, 2	0	2x-2	33	O17, 2	62	Y
2	E1, 2	0	Y	34	O17, 2	64	Y
3	O2, 2	2	Y	35	O17, 2	66	Y
4	E2, 2	4	Y	36	O17, 2	68	Y
5	O3, 2	6	Y	37	O17, 2	70	Y
6	E3, 2	8	Y	38	O17, 2	72	Y
7	O4, 2	10	Y	39	O17, 2	74	Y
8	E4, 2	12	Y	40	O17, 2	76	Y
9	O5, 2	14	Y	41	O17, 2	78	Y
10	E5, 2	16	Y	42	O17, 2	80	Y
11	O6, 2	18	Y	43	O17, 2	82	Y
12	E6, 2	20	Y	44	O17, 2	84	Y
13	O7, 2	22	Y	45	O17, 2	86	Y
14	E7, 2	24	Y	46	O17, 2	88	Y
15	O8, 2	26	Y	47	O17, 2	90	Y
16	E8, 2	28	Y	48	O17, 2	92	Y
17	O9, 2	30	Y	49	O17, 2	94	Y
18	E9, 2	32	Y	50	O17, 2	96	Y
19	O10, 2	34	Y	51	O17, 2	98	Y
20	E10, 2	36	Y	52	O17, 2	100	Y
21	O11, 2	38	Y	53	O17, 2	102	Y
22	E11, 2	40	Y	54	O17, 2	104	Y
23	O12, 2	42	Y	55	O17, 2	106	Y
24	E12, 2	44	Y	56	O17, 2	108	Y
25	O13, 2	46	Y	57	O17, 2	110	Y
26	E13, 2	48	Y	58	O17, 2	112	Y
27	O14, 2	50	Y	59	O17, 2	114	Y
28	E14, 2	52	Y	60	O17, 2	116	Y
29	O15, 2	54	Y	61	O17, 2	118	Y
30	E15, 2	56	Y	62	O17, 2	120	Y
31	O16, 2	58	Y	63	O17, 2	122	Y
32	E16, 2	60	Y	64	O17, 2	124	Y

Dimensional analysis homogeneity and non-homogeneity are demonstrated once again by Eq. (40) and Eq. (41) respectively for the Gram[x=0] points case scenario.

Appendix B: Tabulated and graphical depictions on Even-Odd mathematical landscape for x = 64

In Figure 8, Dimensions 2x - 2 & 2x - 4 are symbolically represented by -2 & -4 with 2x - 4 displayed as 'baseline' Dimension whereby the Dimension trend (Cumulative Sum Gaps) must reset itself onto this (Grand-Total Gaps) 'baseline' Dimension after the initial Dimension 2x - 2 on a permanent basis, thus manifesting Information-Complexity conservation and Dimensional analysis homogeneity. Graphical appearances of Dimensions symbolically represented by the two negative integers are Completely Predictable with both Even-$\pi(x)$ and Odd-$\pi(x)$ becoming larger at a constant rate. We note that there is a complete absence of Chaos & Fractals phenomena being manifested in our graph.

The definitive derivation of the data in Table 7 is given next and this is clearly illustrated by two examples given for position x = 31 & 32. For i & x ∈ 1, 2, 3, ..., ∞; ΣEO_x-Gap = ΣEO_{x-1}-Gap + Gap value at E_{i-1} or Gap value at O_{i-1} whereby (i) E_i or O_i at position x is



Even Odd.png

Fig. 8 Even-Odd mathematical (graphed) landscape using data obtained for x = 64.

determined by whether the relevant x value belongs to an even (E) or odd (O) number, and (ii) both ΣEO_1-Gap and ΣEO_2-Gap = 0. Example for position x = 31: 31 is an odd number (O16). Our desired Gap value at O15 = 2. Thus ΣEO_{31}-Gap (58) = ΣEO_{30}-Gap (56) + Gap value at O15 (2). Example 2 for position x = 32: 32 is an even number (E16). Our desired Gap value at E15 = 2. Thus ΣEO_{32}-Gap (60) = ΣEO_{31}-Gap (58) + Gap value at E15 (2).

Using the relevant data above, we have now painstakingly tabulate (in Table 7) and graphically map (in Figure 8) the [Completely Predictable] Even-Odd mathematical landscape for x = 64. Involved Dimensions are 2x - 2 & 2x - 4 with Y denoting Dimension 2x - 4 for visual clarity. This Even-Odd mathematical landscape, made up of Dimension 2x - 4 (except for the very first and only Dimension 2x - 2), will intrinsically incorporate even and odd numbers in an integrated manner. Except for the very first odd number, we note that all Completely Predictable even and odd numbers, and all their Completely Predictable identical gaps, can be represented by the countable finite set of [single] Dimension 2x - 4.

Appendix C: Tabulated and graphical depictions on Prime-Composite mathematical landscape for x = 64 with the number '1' incorrectly treated as a composite number

'1' incorrectly treated as Composite number.png

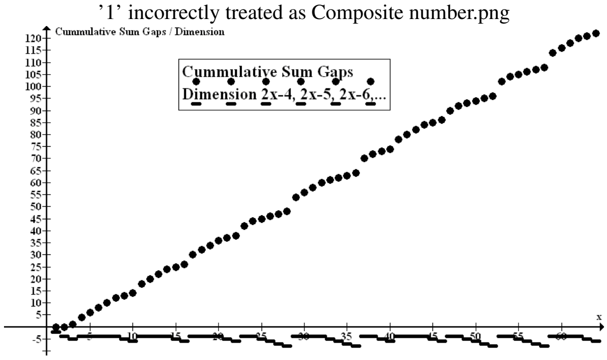


Fig. 9 Prime-Composite Prime-Composite finite scale mathematical (graphed) landscape using data obtained for x = 64 with the number '1' incorrectly treated as a composite number. The bottom graph here on 'Dimensions when symbolically represented by ever larger negative integers' demonstrate the apparent manifestation of graphical "discontinuity" when compared to that shown in Figure 7 whereby the number '1' is correctly treated as neither prime nor composite.

Table 8 Prime-Composite Prime-Composite finite scale mathematical (tabulated) landscape using data obtained for x = 64 with the number '1' incorrectly treated as a composite number. Legend: C = composite, P = prime, Y = Dimension 2x - 4 (for visual clarity), N/A = Not Applicable.

x	**P_i or C_i, Gaps**	**ΣPC_x-Gaps**	Dimension	**x**	**P_i or C_i, Gaps**	**ΣPC_x-Gaps**	Dimension
1	C1, 3	0	2x-2	33	C22, 1	61	2x-5
2	P1, 1	0	Y	34	C23, 1	62	2x-6
3	P2, 2	1	2x-5	35	C24, 1	63	2x-7
4	C2, 2	4	Y	36	C25, 2	64	2x-8
5	P3, 2	6	Y	37	P12, 4	70	Y
6	C3, 2	8	Y	38	C26, 1	72	Y
7	P4, 4	10	Y	39	C27, 1	73	2x-5
8	C4, 1	12	Y	40	C28, 1	74	2x-6
9	C5, 1	13	2x-5	41	P13, 2	78	Y
10	C6, 2	14	2x-6	42	C29, 2	80	Y
11	P5, 2	18	Y	43	P14, 4	82	Y
12	C7, 2	20	Y	44	C30, 1	84	Y
13	P6, 4	22	Y	45	C31, 1	85	2x-5
14	C8, 1	24	Y	46	C32, 2	86	2x-6
15	C9, 1	25	2x-5	47	P15, 6	90	Y
16	C10, 1	26	2x-6	48	C33, 1	92	Y
17	P7, 2	30	Y	49	C34, 1	93	2x-5
18	C11, 2	32	Y	50	C35, 1	94	2x-6
19	P8, 4	34	Y	51	C36, 1	95	2x-7
20	C12, 1	36	Y	52	C37, 1	96	2x-8
21	C13, 1	37	2x-5	53	P16, 6	102	Y
22	C14, 2	38	2x-6	54	C38, 1	104	Y
23	P9, 6	42	Y	55	C39, 1	105	2x-5
24	C15, 1	44	Y	56	C40, 1	106	2x-6
25	C16, 1	45	2x-5	57	C41, 1	107	2x-7
26	C17, 1	46	2x-6	58	C42, 1	108	2x-8
27	C18, 1	47	2x-7	59	P17, 2	114	Y
28	C19, 2	48	2x-8	60	C43, 2	116	Y
29	P10, 2	54	Y	61	P18, 6	118	Y
30	C20, 2	56	Y	62	C44, 1	120	Y
31	P11, 6	58	Y	63	C45, 1	121	2x-5
32	C21, 1	60	Y	64	C46, 1	122	2x-6

We now painstakingly tabulate and graphically map the [Incompletely Predictable] Prime-Composite mathematical landscape for x=64 when the number '1' is incorrectly treated as a composite number. In Figure 9, the Dimensions 2x - 4, 2x - 5, 2x - 6, ..., 2x - ∞ are symbolically represented by -4, -5, -6, ..., ∞ with 2x - 4 displayed as the [incorrect] 'baseline' Dimension [because the number '1' is incorrectly treated as a composite number] whereby the Dimension trend (Cumulative Sum Gaps) must repeatedly reset itself onto this (Grand-Total Gaps) 'baseline' Dimension on a perpetual basis, thus manifesting Information-Complexity conservation and Dimensional analysis homogeneity. Graphical appearances of Dimensions symbolically represented by ever larger negative integers will correspond to prime numbers associated with ever larger prime gaps and this phenomenon will generally happen at ever larger x values. In other words, at ever larger x values, Prime-$\pi(x)$ will overall become larger but with a decelerating trend whereas Composite-$\pi(x)$ will overall become larger but with an accelerating trend. This highlights the inevitable mathematical event of ever larger prime gaps occurring at ever larger x values. We note that there is a complete presence of Chaos & Fractals phenomena being manifested in our graph.

The definitive derivation of the data in Table 8 is given next and this is illustrated by two examples given for position x = 31 & 32. For i & x ∈ 1, 2, 3,..., ∞; ΣPC_x-Gap = ΣPC_{x-1}-Gap + Gap value at P_{i-1} or Gap value at C_{i-1} whereby (i) P_i or C_i at position x is determined by whether the relevant x value belongs to a prime (P) or composite (C) number, and (ii) both ΣPC_1-Gap and ΣPC_2-Gap = 0. Example for position x = 31: 31 is a prime number (P11). Our desired Gap value at P10 = 2. Thus ΣPC_{31}-Gap (58) = ΣPC_{30}-Gap (56) + Gap value at P10 (2). Example for position x = 32: 32 is a composite number (C21). Our desired Gap value at C20 = 2. Thus ΣPC_{32}-Gap (60) = ΣPC_{31}-Gap (58) + Gap value at C20 (2).

Finally, we easily observe the 'overall magnitude of composite numbers to be always greater than that of prime numbers' criterion to hold true from x = 8 onwards [when the number '1' is incorrectly treated as a composite number]. For instance, position x = 61 corresponds to prime number 61 which is the 18^{th} prime number, whereas [the one lower] position x = 60 corresponds to composite number 60 which is the much higher [incorrect] 43^{rd} composite number [when '1' is incorrectly assumed to be a composite number].

Appendix D: Miscellanous materials: L-function, LMFDB, Moonshine theory, GUT, TOE and Fundamental Laws

There are many deep-seated connections between prime numbers, prime counting function, Sieve of Eratosthenes, Polignac's & Twin prime conjectures, Riemann zeta (or its *proxy* Dirichlet eta) function, and Riemann hypothesis. With the visual aid of Figure 3, our (now-successful) geometrical proof version of Riemann hypothesis can be exploited to explicitly state that "All Gram[x=0,y=0] intercepts [which are all the nontrivial zeros] of Riemann zeta function only occur when $\sigma = \frac{1}{2}$". It is commonly advocated that the rigorous proof for Riemann hypothesis would be instrumental in proving efficacy of techniques that estimate prime counting function (traditionally denoted by $\pi(x)$ but in this paper by Prime-$\pi(x)$) efficiently & reasonably well.

The sets of nontrivial zeros, Gram[y=0] points and Gram[x=0] points in Riemann zeta function are complementary to each other. This can only happen when all three sets are directly stipulated by the same function. The sets of prime and composite number are complementary to each other because they are both, respectively, stipulated directly and indirectly by the same Sieve of Eratosthenes. The prime counting function is the function counting the number of prime numbers less than or equal to some real number x. One can easily deduce that inventing a complementary and equivalent 'composite counting function' is absolutely a valid mathematical exercise. This *proxy* composite counting function, denoted in this paper by Composite-$\pi(x)$, can then be categorically stated as the function counting the number of composite numbers less than or equal to some real number x. It must also enjoy the exact same mathematical privileges that are extended to prime counting function.

By an L-function, we generally refer to a Dirichlet series with a functional equation and an Euler product. Contextually, the simplest example of an L-function is Riemann zeta function on which the 1859 Riemann hypothesis is based upon. L-functions are ubiquitous in number theory and hence have applications to mathematical physics and cryptography. They arise from and encode information about a number of mathematical objects and it is necessary to exhibit these objects along with the L-functions themselves since typically we need these objects to compute L-functions. For examples, L-functions can come from modular forms, elliptic curves, number fields, and Dirichlet characters, as well as more generally from automorphic forms, algebraic varieties, and Artin representations. Broadly based on these examples, the mammoth 'L-functions and Modular Forms Database' (LMFDB) creation was conducted with massive team-effort collaboration from an international group of more than 80 researchers from 12 countries which included prominent mathematicians such as from American Institute of Mathematics in United States, University of Bristol in United Kingdom, and Dartmouth College in United States. The LMFDB idea was first conceived at an American Institute of Mathematics workshop in 2007. Six years after commencing the LMFDB project [website address http://www.lmfdb.org/], its launching was celebrated on May 10, 2016. In effect, LMFDB can be considered an uncharted mathematical terrain providing a detailed atlas of mathematical objects that highlights deep relationships and serves as a guide to latest research happening in physics, computer science and mathematics. Elliptic curves arise naturally in many parts of mathematics and can be described by a simple cubic equation. They also form the basis of cryptographic protocols used by most of the major internet companies including Google, Facebook and Amazon. Modular forms are more mysterious objects constituted by complex functions with an almost unbelievable degree of symmetry. The two mathematical worlds of elliptic curves and modular forms are remarkably connected via their L-functions. It is this deep connection that was in essence required in the late 20th century by famous British number theorist Andrew John Wiles to successfully achieve his proof of Fermat's Last Theorem. To put into perspective the importance of LMFDB in relation to active research areas such as involving Monstrous moonshine (Moonshine theory), Mathieu moonshine, and Umbral moonshine with their conjectured roles in Quantum gravity and String theory; we think that most physicists would have a positive opinion or consensus on the potential role of these research areas in successfully merging gravity with Grand Unified Theory (GUT) – consisting of the unification of electromagnetism, weak nuclear force, and strong nuclear force – thus giving rise to the holy grail Theory of Everything (TOE).

We briefly divert here to mention that the name 'Standard Model of particle physics', commenced in the 1970s, denotes the theory describing three of the four known fundamental forces in the universe (viz. the electromagnetic, weak, and strong interactions of GUT), as well as classifying all known elementary particles. Despite all its predictive power, it is not "perfect" in that it can't explain gravity, dark matter or dark energy.

String theories assume that fundamental building blocks of the universe are strings instead of point particles. String duality is a class of symmetries in physics that link different String theories, with K3 surfaces appearing almost ubiquitously in string duality. A K3 surface is a complex or algebraic smooth minimal complete surface that is regular and has trivial canonical bundle. Not least because of this difficulty of multiple String theories (and hence multiple possibilities), an alternative view is that all four fundamental forces of nature will always exist as the current *status quo* with gravity obeying laws [perhaps endowed with certain "continuous" Completely Predictable properties] derived from Einstein's Theory of General Relativity and the three forces of GUT obeying laws [perhaps endowed with certain "discrete" Incompletely Predictable properties] based on Quantum mechanics. Alternatively stated, nature will intrinsically never allow the mathematical merging together of those two totally incompatible situations; namely, the "continuous" property on the one hand and "discrete" property on the other hand. Despite this issue, LMFDB with one of its crucial features acting as "intricate catalog of mathematical objects" will, metaphorically speaking, be the source supplying the required mathematical objects in those mentioned research areas.

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