Electromagnetic-Power-based Modal Classification, Modal Expansion, and Modal Decomposition for Perfect Electric Conductors

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***Abstract*—Traditionally, all working modes of a perfect electric conductor are classified into resonant modes, inductive modes, and capacitive modes, and the resonant modes are further classified into internal resonant modes and external resonant modes. In this paper, the resonant modes are alternatively classified into intrinsically resonant modes and non-intrinsically resonant modes, and the intrinsically resonant modes are further classified into non-radiative intrinsically resonant modes and radiative intrinsically resonant modes. Based on the modal expansion corresponding to this new modal classification, an alternative modal decomposition method is proposed. In addition, it is also proved that: all intrinsically resonant modes and all non-radiative intrinsically resonant modes constitute linear spaces respectively, but all resonant modes and all radiative intrinsically resonant modes cannot constitute spaces respectively.**

***Index Terms*—Characteristic mode (CM), modal classification, modal decomposition, modal expansion, radiation, resonance.**

I. Introduction

RESONANCE is an important concept in electromagnetics. Based on whether the resonant modes radiate, they are classified into internal resonant modes and external resonant modes [1], and these two kinds of resonant modes are widely applied in electromagnetic (EM) cavities [2] and EM antennas [3] respectively.

The most commonly used mathematical method for researching internal resonant modes is normal eigen-mode theory (EMT) [2], [4], and the normal EMT can construct a basis of internal resonance space (which is constituted by all internal resonant modes [5]), and the basis are called as normal eigen-modes. The most commonly used mathematical methods for researching external resonant modes are singular EMT [6] and characteristic mode theory (CMT) [7], and the "basis" constructed by singular EMT and CMT are respectively called as singular eigen-modes and radiative characteristic modes (CMs), where the quotation mark on "basis" will be explained in Sec. IV. Recently, paper [8] generalized the traditional CMT to internal resonance problem, and proved that: all non-radiative modes constitute a linear space called as non-radiation space, and this space is the same as internal resonance space; all non-radiative CMs constitute a basis of non-radiation space and internal resonance space, and then they are equivalent to the normal eigen-modes from the aspect of modal expansion.

Based on above observations, the "basis" used to expand resonant modes can be classified into four categories internal resonant normal eigen-modes, external resonant singular eigen-modes, radiative CMs, and non-radiative CMs, and the relationships and differences among the first three of these "basis" are analyzed in paper [1]. This paper alternatively classifies all resonant modes into three categories non-radiative intrinsically resonant modes, radiative intrinsically resonant modes, and non-intrinsically resonant modes, and discusses the relationships and differences among them. By employing the modal expansion corresponding to this new modal classification, an alternative modal decomposition method is proposed in this paper, and at the same time some further conclusions are obtained.

II. Modal Classification

When a field $\vec{F}$ incidents on a perfect electric conductor (PEC), a current $\vec{J}$ will be induced on the PEC. All possible working modes $\vec{J}$ constitute a linear space called as *modal space* [4]-[8]. If the $\vec{J}$ is expanded in terms of independent and complete basis functions, there exists a one-to-one correspondence between the $\vec{J}$ and its expansion vector $\bar{a}$ [5], [7], [8], and the linear space constituted by all possible $\bar{a}$ is called as *expansion vector space* (where the $\bar{a}$ is the vector constituted by all expansion coefficients). The following parts of this paper are discussed in expansion vector space and frequency domain.

In expansion vector space, the complex power P done by $\vec{E}$ on $\vec{J}$ has the matrix form $P = \bar{a}^H \cdot \bar{\bar{P}} \cdot \bar{a}$, and then the radiated power $P^{rad} = \text{Re}\{P\}$ and the reactively stored power $P^{sto} = \text{Im}\{P\}$ can be correspondingly expressed as the matrix forms $P^{rad} = \bar{a}^H \cdot \bar{\bar{P}}^{rad} \cdot \bar{a}$ and $P^{sto} = \bar{a}^H \cdot \bar{\bar{P}}^{sto} \cdot \bar{a}$ [8]. Here, the superscript "H" represents the transpose conjugate of a matrix or vector, and the method to obtain the matrix $\bar{\bar{P}}$ can be found in papers [7] and [8], and $\bar{\bar{P}}^{rad} = (\bar{\bar{P}} + \bar{\bar{P}}^H)/2$ and $\bar{\bar{P}}^{sto} = (\bar{\bar{P}} - \bar{\bar{P}}^H)/2j$ [8].

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A. Traditional modal classification

Matrix $\overline{\overline{P}}^{rad}$ is positive semi-definite [8], so $\bar{a}^H \cdot \overline{\overline{P}}^{rad} \cdot \bar{a} \geq 0$ for any $\bar{a}$, and the modes corresponding to $\bar{a}^H \cdot \overline{\overline{P}}^{rad} \cdot \bar{a} > 0$ and $\bar{a}^H \cdot \overline{\overline{P}}^{rad} \cdot \bar{a} = 0$ are called as *radiative modes* and *non-radiative modes* respectively. In addition, the semi-definiteness of matrix $\overline{\overline{P}}^{rad}$ implies that: $\bar{a}^H \cdot \overline{\overline{P}}^{rad} \cdot \bar{a} = 0$ if and only if $\overline{\overline{P}}^{rad} \cdot \bar{a} = 0$ [9], i.e.,

$$\overline{\overline{P}}^{rad} \cdot \bar{a} = 0 \quad \Leftrightarrow \quad \bar{a}^H \cdot \overline{\overline{P}}^{rad} \cdot \bar{a} = 0 \quad \Leftrightarrow \quad \text{Mode } \bar{a} \text{ is non-radiative} \tag{1}$$

Thus, all non-radiative modes $\bar{a}_{non-rad}$ constitute a linear space (i.e. the null space of $\overline{\overline{P}}^{rad}$ [9]) called as *non-radiation space* (which is identical to the *internal resonance space* [8]), and any $\bar{a}_{non-rad}$ satisfies the following orthogonality:

$$\bar{a}^H \cdot \overline{\overline{P}}^{rad} \cdot \bar{a}_{non-rad} \;=\; 0 \;=\; (\bar{a}_{non-rad})^H \cdot \overline{\overline{P}}^{rad} \cdot \bar{a} \tag{2}$$

for any working mode $\bar{a}$.

The matrix $\overline{\overline{P}}^{sto}$ is indefinite [7], [8], so $\bar{a}^H \cdot \overline{\overline{P}}^{sto} \cdot \bar{a}$ can be zero or positive or negative, and the modes corresponding to $\bar{a}^H \cdot \overline{\overline{P}}^{sto} \cdot \bar{a} = 0$, $\bar{a}^H \cdot \overline{\overline{P}}^{sto} \cdot \bar{a} > 0$, and $\bar{a}^H \cdot \overline{\overline{P}}^{sto} \cdot \bar{a} < 0$ are called as *resonant modes*, *inductive modes*, and *capacitive modes* respectively [3], [4], [7], [8]. According to whether the resonant modes radiate, the resonant modes are further classified into *internal resonant modes* (which don't radiate, so this paper calls them as *non-radiative resonant modes*) and *external resonant modes* (which radiate, so this paper calls them as *radiative resonant modes*) [1], [5], [8]. As demonstrated in [8], the non-radiative modes must be resonant, so all inductive and capacitive modes must be radiative, and then this paper calls them as *radiative inductive modes* and *radiative capacitive modes* respectively.

B. New modal classification (An alternative classification for resonant modes)

Besides classifying all modes into resonant modes (including non-radiative resonant modes and radiative resonant modes), radiative inductive modes, and radiative capacitive modes traditionally, an alternative classification for the resonant modes is proposed in this sub-section.

Matrix $\overline{\overline{P}}^{sto}$ is indefinite, so $\bar{a}^H \cdot \overline{\overline{P}}^{sto} \cdot \bar{a} = 0$ doesn't imply that $\overline{\overline{P}}^{sto} \cdot \bar{a} = 0$ [9], though $\overline{\overline{P}}^{sto} \cdot \bar{a} = 0$ always implies that $\bar{a}^H \cdot \overline{\overline{P}}^{sto} \cdot \bar{a} = 0$. This is equivalent to saying that

$$\overline{\overline{P}}^{sto} \cdot \bar{a} = 0 \quad \underset{\not\Leftarrow}{\Rightarrow} \quad \bar{a}^H \cdot \overline{\overline{P}}^{sto} \cdot \bar{a} = 0 \quad \Leftrightarrow \quad \text{Mode } \bar{a} \text{ is resonant} \tag{3}$$

i.e., the condition $\overline{\overline{P}}^{sto} \cdot \bar{a} = 0$ is a stronger condition than the condition $\bar{a}^H \cdot \overline{\overline{P}}^{sto} \cdot \bar{a} = 0$ to guarantee resonance. Based on this, the $\overline{\overline{P}}^{sto} \cdot \bar{a} = 0$ can be particularly called as *intrinsic resonance condition*, if the $\bar{a}^H \cdot \overline{\overline{P}}^{sto} \cdot \bar{a} = 0$ is viewed as *resonance condition*.

Correspondingly, the modes satisfying $\overline{\overline{P}}^{sto} \cdot \bar{a} = 0$ are called as *intrinsically resonant modes*, and the resonant modes not satisfying $\overline{\overline{P}}^{sto} \cdot \bar{a} = 0$ are called as *non-intrinsically resonant modes*. Obviously, all intrinsically resonant modes constitute a linear space, i.e. the null space of $\overline{\overline{P}}^{sto}$, and this space is called as *intrinsic resonance space*. Similarly to (2), any intrinsically resonant mode $\bar{a}^{int\ res}$ satisfies the following (4) for any $\bar{a}$:

$$\bar{a}^H \cdot \overline{\overline{P}}^{sto} \cdot \bar{a}^{int\ res} \;=\; 0 \;=\; \left(\bar{a}^{int\ res}\right)^H \cdot \overline{\overline{P}}^{sto} \cdot \bar{a} \tag{4}$$

When intrinsically resonant mode $\bar{a}^{int\ res}$ satisfies condition $(\bar{a}^{int\ res})^H \cdot \overline{\overline{P}}^{rad} \cdot \bar{a}^{int\ res} = 0$, it is called as *non-radiative intrinsically resonant mode*, and correspondingly denoted as $\bar{a}^{int\ res}_{non-rad}$. When intrinsically resonant mode $\bar{a}^{int\ res}$ satisfies condition $(\bar{a}^{int\ res})^H \cdot \overline{\overline{P}}^{rad} \cdot \bar{a}^{int\ res} > 0$, it is called as *radiative intrinsically resonant mode*, and correspondingly denoted as $\bar{a}^{int\ res}_{rad}$. As demonstrated in paper [8], $\overline{\overline{P}}^{sto} \cdot \bar{a} = 0$, if $\overline{\overline{P}}^{rad} \cdot \bar{a} = 0$. This implies that the intrinsic resonance space contains the whole non-radiation space. Then, the set constituted by all $\bar{a}^{int\ res}_{non-rad}$ must be a linear space, and this space is just the non-radiation space; all non-intrinsically resonant modes $\bar{a}^{non-int\ res}$ are radiative, and they are particularly denoted as $\bar{a}^{non-int\ res}_{rad}$; for any mode $\bar{a}$, the $\bar{a}^{int\ res}_{non-rad}$ satisfies the following orthogonality:

$$\bar{a}^H \cdot \overline{\overline{P}}^{rad} \cdot \bar{a}^{int\ res}_{non-rad} \;=\; 0 \;=\; \left(\bar{a}^{int\ res}_{non-rad}\right)^H \cdot \overline{\overline{P}}^{rad} \cdot \bar{a} \tag{5.1}$$

$$\bar{a}^H \cdot \overline{\overline{P}}^{sto} \cdot \bar{a}^{int\ res}_{non-rad} \;=\; 0 \;=\; \left(\bar{a}^{int\ res}_{non-rad}\right)^H \cdot \overline{\overline{P}}^{sto} \cdot \bar{a} \tag{5.2}$$

In summary, by introducing the concepts of intrinsic resonance and non-intrinsic resonance, this sub-section alternatively classifies all resonant modes into non-radiative intrinsically resonant modes $\bar{a}^{int\ res}_{non-rad}$, radiative intrinsically resonant modes $\bar{a}^{int\ res}_{rad}$, and radiative non-intrinsically resonant modes $\bar{a}^{non-int\ res}_{rad}$. Because the non-radiative intrinsically resonant modes $\bar{a}^{int\ res}_{non-rad}$ are just the traditional internal resonant modes, the introduction of radiative intrinsically resonant modes $\bar{a}^{int\ res}_{rad}$ and radiative non-intrinsically resonant modes $\bar{a}^{non-int\ res}_{rad}$ is essentially a subdivision for the traditional external resonant modes.

C. Classification for characteristic modes

Because both above traditional and new modal classifications are suitable for whole modal space, they are also valid for CM set $\{\bar{\alpha}_\xi\}$. Here, the symbol "$\bar{\alpha}_\xi$" is used to represent the expansion vector of CM $\vec{J}_\xi$ in order to be distinguished from the expansion vector $\bar{a}$ of general mode $\vec{J}$.

Traditional classification for CMs

Traditionally, CM set $\{\bar{\alpha}_\xi\}$ are divided into four sub-sets [1], [7], [8]: non-radiative resonant CM set $\{\bar{\alpha}^{res}_{non-rad;\xi}\}$, radiative resonant CM set $\{\bar{\alpha}^{res}_{rad;\xi}\}$, radiative inductive CM set $\{\bar{\alpha}^{ind}_{rad;\xi}\}$, and radiative capacitive CM set $\{\bar{\alpha}^{cap}_{rad;\xi}\}$. For the convenience of the following parts of this sub-section, the non-radiative and radiative resonant CMs are collectively referred to as resonant CMs, and the union of sets $\{\bar{\alpha}^{res}_{non-rad;\xi}\}$ and $\{\bar{\alpha}^{res}_{rad;\xi}\}$ is correspondingly denoted as $\{\bar{\alpha}^{res}_{\xi}\}$, i.e., $\{\bar{\alpha}^{res}_{\xi}\} = \{\bar{\alpha}^{res}_{non-rad;\xi}\} \cup \{\bar{\alpha}^{res}_{rad;\xi}\}$.

An alternative classification for resonant CMs

As illustrated in papers [1], [7], and [8], all $\bar{\alpha}^{res}_{\xi}$ satisfy the characteristic equation $\overline{\overline{P}}^{sto} \cdot \bar{\alpha}^{res}_{\xi} = 0$. In fact, this equation is just the intrinsic resonance condition introduced in Sec. II-B, so all the $\bar{\alpha}^{res}_{\xi}$ are intrinsically resonant, and then they are particularly denoted as $\bar{\alpha}^{int\ res}_{\xi}$. Correspondingly, the $\bar{\alpha}^{res}_{non-rad;\xi}$ and $\bar{\alpha}^{res}_{rad;\xi}$ are particularly denoted as $\bar{\alpha}^{int\ res}_{non-rad;\xi}$ and $\bar{\alpha}^{int\ res}_{rad;\xi}$

respectively.

All $\bar{\alpha}_{\xi}^{int\,res}$ are independent of each other [7], and the rank of set $\{\bar{\alpha}_{\xi}^{int\,res}\}$ equals to the rank of the null space of $\bar{\bar{P}}^{sto}$, so they constitute a basis of intrinsic resonance space [9], i.e., any intrinsically resonant mode $\bar{a}^{int\,res}$ can be uniquely expanded in terms of $\{\bar{\alpha}_{\xi}^{int\,res}\}$. In addition, the $\{\bar{\alpha}_{non-rad;\xi}^{int\,res}\}$ constitute a basis of non-radiation space [8], i.e., any non-radiative mode $\bar{a}_{non-rad}$ can be uniquely expanded in terms of $\{\bar{\alpha}_{non-rad;\xi}^{int\,res}\}$.

III. Modal Expansion

In this section, a further discussion on the CM-based modal expansions for various modes is provided, based on the new modal classification proposed in above Sec. II.

A. Modal expansion for general modes

Based on the independence property and completeness of CM set $\{\bar{\alpha}_{\xi}\}$ [7], [8], any mode $\bar{a}$ can be uniquely expanded in terms of some non-radiative resonant CMs $\bar{\alpha}_{non-rad;\xi}^{int\,res}$, some radiative resonant CMs $\bar{\alpha}_{rad;\xi}^{int\,res}$, some radiative inductive CMs $\bar{\alpha}_{rad;\xi}^{ind}$, and some radiative capacitive CMs $\bar{\alpha}_{rad;\xi}^{cap}$ as follows:

$$\bar{a} \sim \sum c_{non-rad;\xi}^{int\,res}\bar{\alpha}_{non-rad;\xi}^{int\,res} + \sum c_{rad;\xi}^{int\,res}\bar{\alpha}_{rad;\xi}^{int\,res} + \sum c_{rad;\xi}^{ind}\bar{\alpha}_{rad;\xi}^{ind} + \sum c_{rad;\xi}^{cap}\bar{\alpha}_{rad;\xi}^{cap} \tag{6}$$

where the reason to use "$\sim$" instead of "$=$" will be explained in Sec. IV. Based on expansion (6), some valuable conclusions shown in Fig. 1 can be derived, and they are proved as below.

- The proof for "$1\Downarrow$" is obvious.
- The proof for "$2\Downarrow$": It is obvious that $\bar{\bar{P}}^{sto}\cdot 0 = 0$, so mode 0 is intrinsically resonant. Thus, if $\sum c_{rad;\xi}^{ind}\bar{\alpha}_{rad;\xi}^{ind} + \sum c_{rad;\xi}^{cap}\bar{\alpha}_{rad;\xi}^{cap} = 0$, then $\sum c_{rad;\xi}^{ind}\bar{\alpha}_{rad;\xi}^{ind} + \sum c_{rad;\xi}^{cap}\bar{\alpha}_{rad;\xi}^{cap}$ is intrinsically resonant.
- The proofs for "$3\Downarrow$" and "$\Uparrow 7$": It is obvious that the term $\sum c_{non-rad;\xi}^{int\,res}\bar{\alpha}_{non-rad;\xi}^{int\,res} + \sum c_{rad;\xi}^{int\,res}\bar{\alpha}_{rad;\xi}^{int\,res}$ is intrinsically resonant. Thus, the $\bar{a}$ is intrinsically resonant, if and only if the term $\sum c_{rad;\xi}^{ind}\bar{\alpha}_{rad;\xi}^{ind} + \sum c_{rad;\xi}^{cap}\bar{\alpha}_{rad;\xi}^{cap}$ is intrinsically resonant, based on the intrinsic resonance condition introduced in Sec. II-B.
- The proof for "$4\Downarrow$" is obvious, because of (3).
- The proofs for "$5\Downarrow$" and "$\Uparrow 6$": Because the term $\sum c_{non-rad;\xi}^{int\,res}\bar{\alpha}_{non-rad;\xi}^{int\,res} + \sum c_{rad;\xi}^{int\,res}\bar{\alpha}_{rad;\xi}^{int\,res}$ is intrinsically resonant, the reactively stored power of $\bar{a}$ equals to the reactively stored power of term $\sum c_{rad;\xi}^{ind}\bar{\alpha}_{rad;\xi}^{ind} + \sum c_{rad;\xi}^{cap}\bar{\alpha}_{rad;\xi}^{cap}$ due to the orthogonality (4). Thus, both the "$5\Downarrow$" and "$\Uparrow 6$" hold.
- The proof for "$\Uparrow 8$": If $\sum c_{rad;\xi}^{ind}\bar{\alpha}_{rad;\xi}^{ind} + \sum c_{rad;\xi}^{cap}\bar{\alpha}_{rad;\xi}^{cap}$ is intrinsically resonant, then the $\bar{a}$ is intrinsically resonant due to "$3\Downarrow$". This implies that the $\bar{a}$ can be expanded in terms of $\{\bar{\alpha}_{rad;\xi}^{int\,res}\}\cup\{\bar{\alpha}_{non-rad;\xi}^{int\,res}\}$ as concluded in Sec. II-C. Because of the uniqueness of the CM-based modal expansion for a certain $\bar{a}$, the coefficients $\{c_{rad;\xi}^{ind}\}$ and $\{c_{rad;\xi}^{cap}\}$ in (6) must be zeros, and then both the terms $\sum c_{rad;\xi}^{ind}\bar{\alpha}_{rad;\xi}^{ind}$ and $\sum c_{rad;\xi}^{cap}\bar{\alpha}_{rad;\xi}^{cap}$ must be zeros.
- The proof for "$\Uparrow 9$" is obvious, because of "$\Uparrow 8$" and "$1\Downarrow$".

B. Modal expansion for general resonant modes

Obviously, any resonant mode $\bar{a}^{res}$ can be expanded as follows:

$$\begin{array}{c}
\sum c_{rad;\xi}^{ind}\bar{\alpha}_{rad;\xi}^{ind}\ ,\ \sum c_{rad;\xi}^{cap}\bar{\alpha}_{rad;\xi}^{cap} = 0 \Leftarrow 8 \\
1\Downarrow \\
\sum c_{rad;\xi}^{ind}\bar{\alpha}_{rad;\xi}^{ind} + \sum c_{rad;\xi}^{cap}\bar{\alpha}_{rad;\xi}^{cap} = 0 \\
2\Downarrow\ \Uparrow 9 \\
\sum c_{rad;\xi}^{ind}\bar{\alpha}_{rad;\xi}^{ind} + \sum c_{rad;\xi}^{cap}\bar{\alpha}_{rad;\xi}^{cap} \text{ is intrinsically resonant} \\
3\Downarrow\ \Uparrow 7 \\
\text{Mode } \bar{a} \text{ is intrinsically resonant} \\
4\Downarrow \\
\text{Mode } \bar{a} \text{ is resonant} \\
5\Downarrow\ \Uparrow 6 \\
\sum c_{rad;\xi}^{ind}\bar{\alpha}_{rad;\xi}^{ind} + \sum c_{rad;\xi}^{cap}\bar{\alpha}_{rad;\xi}^{cap} \text{ is resonant}
\end{array}$$

Fig. 1. Some "equivalence" relationships related to resonance.

$$\bar{a}^{res} \simeq \sum c_{non-rad;\xi}^{int\,res}\bar{\alpha}_{non-rad;\xi}^{int\,res} + \sum c_{rad;\xi}^{int\,res}\bar{\alpha}_{rad;\xi}^{int\,res} + \sum c_{rad;\xi}^{ind}\bar{\alpha}_{rad;\xi}^{ind} + \sum c_{rad;\xi}^{cap}\bar{\alpha}_{rad;\xi}^{cap} \tag{7}$$

where the reason to use "$\simeq$" instead of "$=$" will be explained in Sec. IV.

C. Modal expansion for intrinsically resonant modes

The conclusions given in Sec. II-C and Fig. 1 imply that any intrinsically resonant mode $\bar{a}^{int\,res}$ can be expanded as follows:

$$\bar{a}^{int\,res} \sim \sum c_{non-rad;\xi}^{int\,res}\bar{\alpha}_{non-rad;\xi}^{int\,res} + \sum c_{rad;\xi}^{int\,res}\bar{\alpha}_{rad;\xi}^{int\,res} \tag{8}$$

i.e., there doesn't exist any inductive CMs $\bar{\alpha}_{rad;\xi}^{ind}$ and capacitive CMs $\bar{\alpha}_{rad;\xi}^{cap}$ in the CM-based modal expansion formulation for an intrinsically resonant mode $\bar{a}^{int\,res}$.

As pointed out in Sec. II-C and paper [8], any non-radiative intrinsically resonant mode $\bar{a}_{non-rad}^{int\,res}$ can be expanded as follows:

$$\bar{a}_{non-rad}^{int\,res} \sim \sum c_{non-rad;\xi}^{int\,res}\bar{\alpha}_{non-rad;\xi}^{int\,res} \tag{9}$$

However, it cannot be guaranteed that the non-radiative term $\sum c_{non-rad;\xi}^{int\,res}\bar{\alpha}_{non-rad;\xi}^{int\,res}$ in the modal expansion of radiative intrinsically resonant mode $\bar{a}_{rad}^{int\,res}$ is zero because of the (5), i.e.,

$$\bar{a}_{rad}^{int\,res} \cong \sum c_{non-rad;\xi}^{int\,res}\bar{\alpha}_{non-rad;\xi}^{int\,res} + \sum c_{rad;\xi}^{int\,res}\bar{\alpha}_{rad;\xi}^{int\,res} \tag{10}$$

where the reason to use "$\cong$" instead of "$=$" will be explained in Sec. IV.

D. Modal expansion for non-intrinsically resonant modes

If a non-intrinsically resonant mode $\bar{a}_{rad}^{non-int\,res}$ is expanded as follows:

$$\bar{a}_{rad}^{non-int\,res} \cong \sum c_{non-rad;\xi}^{int\,res}\bar{\alpha}_{non-rad;\xi}^{int\,res} + \sum c_{rad;\xi}^{int\,res}\bar{\alpha}_{rad;\xi}^{int\,res} + \sum c_{rad;\xi}^{ind}\bar{\alpha}_{rad;\xi}^{ind} + \sum c_{rad;\xi}^{cap}\bar{\alpha}_{rad;\xi}^{cap} \tag{11}$$

it can be concluded that

$$\sum c_{rad;\xi}^{ind}\bar{\alpha}_{rad;\xi}^{ind} + \sum c_{rad;\xi}^{cap}\bar{\alpha}_{rad;\xi}^{cap} \neq 0 \tag{12}$$

based on Fig. 1. In fact, it can be further concluded that

$$\sum c^{ind}_{rad;\xi}\bar{\alpha}^{ind}_{rad;\xi} \;\; , \;\; \sum c^{cap}_{rad;\xi}\bar{\alpha}^{cap}_{rad;\xi} \;\neq\; 0 \tag{13}$$

because: if the inductive/capacitive term is zero and the capacitive/inductive term is non-zero, then the reactively stored power of $\bar{a}^{non-int\ res}_{rad}$ is less/larger than zero due to the orthogonality (4), and this leads to a contradiction; if both the inductive term and capacitive term are zeros, then the term $\sum c^{ind}_{rad;\xi}\bar{\alpha}^{ind}_{rad;\xi} + \sum c^{cap}_{rad;\xi}\bar{\alpha}^{cap}_{rad;\xi}$ must be zero, and this leads to a contradiction with (12).

IV. Modal Decomposition

If the terms $\sum c^{int\ res}_{non-rad;\xi}\bar{\alpha}^{int\ res}_{non-rad;\xi}$, $\sum c^{int\ res}_{rad;\xi}\bar{\alpha}^{int\ res}_{rad;\xi}$, $\sum c^{ind}_{rad;\xi}\bar{\alpha}^{ind}_{rad;\xi}$, and $\sum c^{cap}_{rad;\xi}\bar{\alpha}^{cap}_{rad;\xi}$ in above-mentioned modal expansion formulations are denoted as $\bar{\beta}^{int\ res}_{non-rad}$, $\bar{\beta}^{int\ res}_{rad}$, $\bar{\beta}^{ind}_{rad}$, and $\bar{\beta}^{cap}_{rad}$ respectively, then the CM-based modal expansions (6)-(11) can be alternatively written as follows:

$$\bar{a} \quad \sim \quad \bar{\beta}^{int\ res}_{non-rad} + \bar{\beta}^{int\ res}_{rad} + \bar{\beta}^{ind}_{rad} + \bar{\beta}^{cap}_{rad} \tag{6'}$$

and

$$\bar{a}^{res} \quad \simeq \quad \bar{\beta}^{int\ res}_{non-rad} + \bar{\beta}^{int\ res}_{rad} + \underline{\bar{\beta}^{ind}_{rad}} + \underline{\bar{\beta}^{cap}_{rad}} \tag{7'}$$

$$\bar{a}^{int\ res} \quad \sim \quad \bar{\beta}^{int\ res}_{non-rad} + \bar{\beta}^{int\ res}_{rad} \tag{8'}$$

$$\bar{a}^{int\ res}_{non-rad} \quad \sim \quad \bar{\beta}^{int\ res}_{non-rad} \tag{9'}$$

$$\bar{a}^{int\ res}_{rad} \quad \cong \quad \bar{\beta}^{int\ res}_{non-rad} + \underline{\underline{\bar{\beta}^{int\ res}_{rad}}} \tag{10'}$$

$$\bar{a}^{non-int\ res}_{rad} \quad \cong \quad \bar{\beta}^{int\ res}_{non-rad} + \bar{\beta}^{int\ res}_{rad} + \underline{\underline{\bar{\beta}^{ind}_{rad}}} + \underline{\underline{\bar{\beta}^{cap}_{rad}}} \tag{11'}$$

where to utilize symbol "$\bar{\beta}$" is to emphasize that these terms are the building block terms in CM-based modal expansions. The (6') and (7')-(11') are respectively called as the electromagnetic-power-based (EMP-based) modal decompositions for general modes and various resonant modes. In fact, the EMP-based modal decompositions for any radiative inductive mode $\bar{a}^{ind}_{rad}$ and any radiative capacitive mode $\bar{a}^{cap}_{rad}$ can be similarly expressed as follows:

$$\bar{a}^{ind}_{rad} \;\cong\; \bar{\beta}^{int\ res}_{non-rad} + \bar{\beta}^{int\ res}_{rad} + \underline{\underline{\bar{\beta}^{ind}_{rad}}} + \bar{\beta}^{cap}_{rad} \tag{14}$$

$$\bar{a}^{cap}_{rad} \;\cong\; \bar{\beta}^{int\ res}_{non-rad} + \bar{\beta}^{int\ res}_{rad} + \bar{\beta}^{ind}_{rad} + \underline{\underline{\bar{\beta}^{cap}_{rad}}} \tag{15}$$

As the continuation of the conclusions given in Secs. II and III, the following further conclusions can be derived based on above EMP-based modal decompositions.

▪ In (6'), (8'), and (9'), all the terms in the right-hand sides of these expansions can be zero or non-zero. In (7'), the $\bar{\beta}^{int\ res}_{non-rad}$ and $\bar{\beta}^{int\ res}_{rad}$ can be zero or non-zero, and the $\bar{\beta}^{ind}_{rad}$ and $\bar{\beta}^{cap}_{rad}$ marked by single underlines can be simultaneously zero or simultaneously non-zero. In (11'), the $\bar{\beta}^{int\ res}_{non-rad}$ and $\bar{\beta}^{int\ res}_{rad}$ can be zero or non-zero, and the $\bar{\beta}^{ind}_{rad}$ and $\bar{\beta}^{cap}_{rad}$ marked by double underlines must be simultaneously non-zero. In (10'), (14), and (15), the terms marked by double underlines must be non-zero. These above are just the reasons to use "$\sim$", "$\simeq$", and "$\cong$" in (7)-(11), (7')-(11'), (14), and (15).

▪ Because the term $\bar{\beta}^{int\ res}_{non-rad}$ in (10') can be non-zero, then the set constituted by all radiative intrinsically resonant modes is not closed for addition, so all radiative intrinsically resonant modes cannot constitute a linear space [9]. Obviously, similar conclusions hold for the sets constituted by all non-intrinsically resonant modes, radiative inductive modes, and radiative capacitive modes, because of (11'), (14), and (15). In addition, all resonant modes also cannot constitute a linear space. For example: If the reactively stored powers of CMs $\bar{\alpha}^{ind}_{rad;1}$ and $\bar{\alpha}^{cap}_{rad;1}$ are normalized to 1 and -1, then the modes $\bar{a} = A\bar{\alpha}^{ind}_{rad;1} + A\bar{\alpha}^{cap}_{rad;1}$ and $\bar{a}' = A\bar{\alpha}^{ind}_{rad;1} + Ae^{j\varphi}\bar{\alpha}^{cap}_{rad;1}$ must be resonant for any $A, \varphi \in \mathbb{R}^{+}$, due to the orthogonality of CMs [7]. However, the mode $\bar{a} + \bar{a}'$ might be non-resonant, because of the arbitrariness of φ. This implies that the set constituted by all resonant modes is not closed for addition. These are just the reasons to use some quotation marks on the "basis" in Sec. I.

▪ The (7') implies that the $\bar{a}^{res}$ might contain the $\bar{\beta}^{int\ res}_{non-rad}$, $\bar{\beta}^{ind}_{rad}$, and $\bar{\beta}^{cap}_{rad}$ terms; the (8') and (10') imply that the $\bar{a}^{int\ res}$ and $\bar{a}^{int\ res}_{rad}$ might contain the $\bar{\beta}^{int\ res}_{non-rad}$ term; the (11') implies that the $\bar{a}^{non-int\ res}_{rad}$ must contain the $\bar{\beta}^{ind}_{rad}$ and $\bar{\beta}^{cap}_{rad}$ terms. In fact, these are just the reasons to call the $\bar{\bar{P}}^{sto} \cdot \bar{a} = 0$ as intrinsic resonance condition and to call the modes satisfying $\bar{\bar{P}}^{sto} \cdot \bar{a} = 0$ as intrinsically resonant modes.

The relationships of various modes are illustrated in Fig. 2, where the modal classes in boxes are linear spaces.

V. Conclusions

This paper alternatively proposes an EMP-based modal classification. Based on the new modal classification and corresponding CM-based modal expansion, an alternative modal decomposition method is obtained, i.e., any mode can be expressed as the superposition of a non-radiative intrinsically resonant mode, a radiative intrinsically resonant mode, a radiative inductive mode, and a radiative capacitive mode. In addition, some conclusions are obtained, for example: all intrinsically resonant modes and all non-radiative modes constitute linear spaces respectively, but other kinds of resonant modes cannot constitute linear spaces respectively.

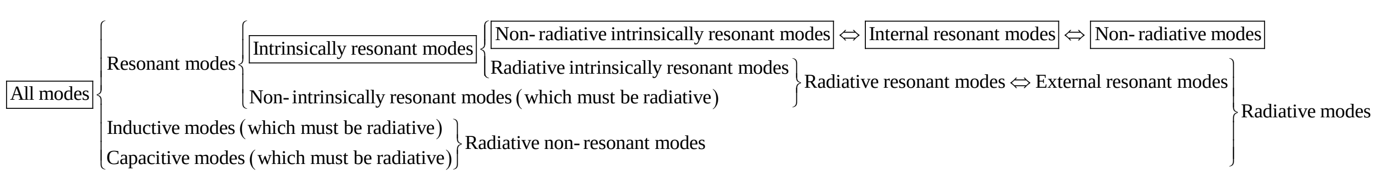

Fig. 2. The EMP-based modal classification for all working modes and the relationships among various modal classes.

REFERENCES

[1] T. Sarkar, E. Mokole, and M. Salazar-Palma, “An expose on internal resonance, external resonance, and characteristic modes,” *IEEE Trans. Antennas Propag.*, vol. 64, no. 11, pp. 4695–4702, Nov. 2016.

[2] R. Collin, *Field Theory of Guided Waves*. New York: Wiley, 1960.

[3] C. Balanis, *Antenna Theory: Analysis and Design*. NY: Wiley, 1982.

[4] J. Jackson, *Classical Electrodynamics, 3rd*. New York: Wiley, 1999.

[5] A. Peterson, S. Ray, and R. Mittra, *Computational Methods for Electromagnetics*. New York: IEEE Press, 1998.

[6] C. Baum, “On the singularity expansion method for the solution of electromagnetic interaction problems,” *AFWL Inter. Notes 88*, Dec. 1971.

[7] R. Harrington and J. Mautz, “Theory of characteristic modes for conducting bodies,” *IEEE Trans. Antennas Propag.*, vol. AP-19, no. 5, pp. 622-628, May 1971.

[8] R. Lian (Oct. 2016), “Electromagnetic-power-based characteristic mode theory for perfect electric conductors,” [Online]. Available: https://arxiv.org/abs/1610.05099.

[9] R. Horn and C. Johnson, *Matrix Analysis, 2nd*. New York: Cambridge University Press, 2013.