

## Refutation of Tarski's undefinability of truth theorem

© Copyright 2018 by Colin James III All rights reserved.

From: Salehji, S. Theorems of Tarski's undefinability and Gödel's second incompleteness--computationally". 2017. [arxiv.org/pdf/1509.00164.pdf](https://arxiv.org/pdf/1509.00164.pdf)

"Gödel's first incompleteness theorem is usually stated as "*no sound and R[ecursively] E[numerable] extension of P[eano's] A[rithmetic] can be complete*"; in notation  $PA \subseteq T \ \& \ T \in \Sigma_1 \ \& \ T \subseteq Th(N) \Rightarrow T \neq Th(N)$ ." (2.2.1)

"So, Tarski's theorem states that for any  $n$ ,  $Th(N) \notin \Sigma_n$ . For the sake of unifying it with Gödel's theorem let us present this theorem as  $(*)_n \ PA \subseteq T \ \& \ T \in \Sigma_n \ \& \ T \subseteq Th(N) \Rightarrow Th(N) \not\subseteq T$  stating that "no definable and sound extension of PA can be complete". (2.2.2)

We rewrite Eq. 2.2.2 because "for any  $n$ ,  $Th(N) \notin \Sigma_n$ " is not expressed correctly.

$PA \subseteq T \ \& \ T \in \Sigma_n \ \& \ T \subseteq Th(N) \Rightarrow Th(N) \not\subseteq \forall n \Sigma_n$ . (2.2.3)

We assume the apparatus and method of Meth8/VL4 to evaluate Eq. 2.2.3.

The designated proof value is  $\mathbb{T}$  for tautology;  $\mathbb{F}$  is for contradiction.  
The 16-valued truth table is presented row-major and horizontally.

LET:  $\sim$  Not;  $\&$  And;  $+$  Or;  $>$  Imply, greater than;  $<$  Not Imply, less than;  
# necessity, all; % possibility, some;  
pqrs: "PA"; T;  $n$ ;  $N$   
#r  $\forall n \Sigma_n$ ; (%s>#s) non-contingency truth value for  $Th(N)$ ;  $\sim(q<p)$  ( $p \subseteq q$ ).

$(\sim(q<p) \ \& \ ((q<r) \ \& \ \sim((\%s>\#s) <q))) \ > \ (\#r < (\%s > \#s))$  ; T T T F T T T T T T T F T T T T (2.2.4)

Eqs. 2.2.4 as rendered for Eq. 2.2.3 is *not* tautologous.