A Simple Newtonian Quantum Gravity Theory That Predicts the Same Light Bending as GR And a New Gravitational Prediction that Can Be Tested!

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Abstract

In this paper we propose a new and simple theory of quantum gravity, inspired by Newton, that gives the same prediction of light bending as Einstein's theory of general relativity. This new quantum gravity theory also predicts that non-light beams, that is to say beams of particles with rest-mass such as electron and proton beams, will only have half the bending of light as GR. In other words, this theory is testable. Based on this theory, we will suggest that it is a property of light that makes it bend twice as much as the amount that is predicted by Newton's theory. This quantum gravity theory also seems to predict that for masses below the Planck mass, we are dealing with quantum probabilities and gravity force expectations. This may explain the difference between the strong and weak force – the difference is simply related to a probability factor at the Planck time scale.

We are also suggesting a minor adjustment to the Newtonian gravitational acceleration field, which renders that field equal to the Planck acceleration at the Schwarzschild radius, and gives the same results as predicted by Newton when we are dealing with weak gravitational fields. This stands in contrast to standard Newtonian theory, which predicts a very weak gravitational acceleration field at the Schwarzschild radius for super-massive objects.

Key words: Quantum gravity, Newton's gravitational constant, bending of light, bending of non-light, strong force, gravitational acceleration field.

1 Introduction to Big G as a Composite Constant and Planck Quantization of Newton

The role of Newton's gravitational constant is to calibrate data generated by empirical observations in order to get Newton's theory of gravity to work. In addition, the inverse is also possible: Newton's theory can be combined with gravitational observations to find the gravitational constant. In 1798, Cavendish was the first to indirectly measure the gravitational constant; see [1].

One hundred years later, in 1899, Max Planck [2] first described his natural units, which he thought represented something fundamental and deep. He derived the Planck length, the Planck second, the Planck mass, and the Planck temperature (energy) from what he assumed were the most fundamental constants, namely the speed of light, Newton's gravitational constant, and the Planck constant; see also [3]. The Planck length was given by Planck himself as

$$l_p = \sqrt{\frac{G\hbar}{c^3}} \tag{1}$$

Based on this, it has always been assumed that the Planck length is a derived constant and the gravitational constant is a more fundamental constant. However, from the formula above we can see that this can be rewritten so that the gravitational constant is a function of the Planck length, the speed of light, and the Planck constant

$$G = \frac{l_p^2 c^3}{\hbar} \tag{2}$$

This is Haug's suggested way to look at the gravitational constant as a composite constant; see [4, 5, 6, 7]. This can also be derived from dimensional analysis, and in fact, it is not so strange that one can derive the gravitational constant from dimensional analysis assuming the Planck length, the speed of light, and the Planck constant are the fundamental constants, even if that not should be given too much weight. It is basically just flipping the Planck coin around, so to speak. In 2013, McCulloch [8] derived basically the same gravitational constant based on Heisenberg's uncertainty principle, see also [9]. Further, Haug [10] has also derived the same composite gravitational constant using Heisenberg's uncertainty principle in combination with his newly-introduced maximum velocity for matter.

Some physicists will likely protest here. Because we can measure the gravitational constant, and one can mistakenly assume that the Planck length must be derived from it. In this view, claiming the gravitational constant is a composite constant seems to introduce a circular problem, at least until recently. Haug [6] has shown that the Planck length can be found through a Cavendish-style experiment, without any knowledge of big G. The Planck length, as a logical idea that we can relate to, is simply a length, and likely the shortest length we can measure, even hypothetically. Studying this length in greater depth, Haug has also recently shown that for all observable gravity phenomena, only one of the Planck lengths in the gravitational composite constant cancels out in the corresponding formulas. In other words, the Planck length is always there, at least in gravity calculations above the subatomic scale. Thus, all observable gravitational phenomena seem to be dependent on the Planck length. Continuing along the track of measurements that are intuitive, the speed of light is also something we can easily relate to: it represents how far the light has traveled during a given time interval. Of these fundamental constants, only the Planck constant is hard to conceptualize and therefore leads us to a different story, which is discussed in [6].

Another strong indication that the gravitational constant is a composite constant may be seen in its units, which are $m^3 \cdot kg^{-1} \cdot s^{-2}$. It would be very strange if something concerning the fundamental nature of reality would be meters cubed, divided by kg and seconds squared. What kind of exotic animal is that? If it quacks like a composite, it most likely is a composite.

There is yet another argument that strengthens our hypothesis that Newton's gravitational constant is a composite constant. In 2014, McCulloch published an interesting paper where he derives Newton's gravitational formulation based on Heisenberg's uncertainty principle utilizing the Planck mass [8]. We will argue this might only be possible if the Planck length (and therefore the Planck mass) plays a special role, even for macroscopic gravitational phenomena.

As previously shown by Haug [4], all of the Planck units are much simpler to understand from a logical point of view when we replace big G with its composite structure. In the standard Planck units, one can find c^7, c^8, and even c^9. What is the intuition of the light speed powered to the seven, or even to the nine? When we replace big G with its composite structure, no Planck unit has more than c^2 in its formula. Suddenly things start to make logical sense. And if we have two theories that give the same result, shouldn't we give preference to the simpler theory? We can leave such philosophical questions for another day. However, we will show that using the composite structure of Newton's gravitational constant can take us one step further; we will obtain a gravity prediction, different than the one that is predicted by GR and by Newton, that actually can be tested.

2 Mass and Energy under Atomism

We suggest that the key to developing a consistent theory of quantum gravity is to understand that Newton's gravitational constant is a composite constant and also to grasp how it should be adjusted to hold for things like light as well. To do this properly, it is helpful to understand something about the recent rise of atomism.

In recent years, Haug has published a new theory rooted in ancient atomism, which holds that at the deepest level of nature, there only exists one indivisible particle and empty space (void) that makes up all matter and energy. This particle always moves at a constant speed, which must be the speed of light, except when it is colliding with another indivisible particle. The collision lasts for only one Planck second. This particle has no rest-mass and therefore no mass when it is moving; it only has mass when it is colliding. The collision points between indivisible particles are what modern physicists calls mass. Viewed in the light of atomism, this leads to an invariant Planck mass particle, an invariant Planck second, and an invariant Planck length.

In his book [11] and a series of papers, Haug has shown that, based on such postulates, one must get the same mathematical end results as Einstein's special relativity theory as long as one uses Einstein-Poincaré synchronized clocks. In addition, he gets an upper limit on the maximum velocity matter can take, which leads to a series of maximum limits on kinetic energy, proper velocity, maximum acceleration, and more. Under modern physics, even an electron can basically attain any level of kinetic energy as long as it is below infinity. However, we ask: Is a limit that almost reaches infinity, really that different than on that truly does touch infinity? Bear in mind that we know the latter is impossible, as it would require an infinite amount of energy; see [12] for interesting examples. Modern physics has no clear mechanism to explain why an electron cannot achieve a relativistic mass equal to one kg, or even equal to the rest-mass of the Moon or the Earth.

There is a solid mathematical and logical framework behind this renewed atomism theory. The view of matter and energy in atomism plays a central role in producing a simple Newtonian-type theory of quantum gravity that predicts the observed bending of light, something classical Newtonian gravity does not do.

3 Back to the Planck Mass

While Max Planck was the first to describe the natural unit of a Planck mass, he said little about what it represented, except that it was likely related to something fundamental. Even now, what the Planck mass is related to and whether or not it means something special is still a mystery in modern physics. We might also wonder if a Planck mass particle even exists.

Lloyd Motz, while working at the Rutherford Laboratory in 1962, [14, 15, 16] suggested that there was probably a very fundamental particle with a mass equal to the Planck mass. Motz named this particle the *Uniton*.[1] Motz suggested that the Uniton could be the most fundamental of all particles and that all other particles were initially made of Unitons. Motz acknowledged that his Unitons (Planck mass particles) had far too much mass compared to known subatomic masses. He tried to get around this issue by claiming that Unitons had radiated most of their energy away:

> According to this point of view, electrons and nucleons are the lowest bound states of two or more Unitons that have collapsed down to the appropriate dimensions gravitationally and radiated away most of their energy in the process. – Lloyd Motz

Others have suggested that there were plenty of Planck mass type particles around just after the Big Bang; see [17], but that most of the mass of these super-heavy particles has radiated away. Modern physics has also suggested a hypothetical Planck particle that has $\sqrt{\pi}$ more mass than the Uniton suggested by Motz. Some physicists including Motz and Hawking have suggested such particles could be micro-black holes [18, 19, 22]. Planck mass particles have even been suggested as being candidates for cosmological dark matter, [23, 13]. We are actually quite skeptical towards dark matter, but that is beyond the scope of this paper.

In 1979, Motz and Epstein [19] suggested the possible existence of a fundamental particle with half the Planck mass that could be essential to solving the mysteries of gravity. Motz and Epstein may have been the first to suggest a fundamental particle with this mass. Although, the Planck mass is still considered more or less to be an unsolved problem today, we think that the recent rapid advancement of mathematical atomism may provide a compelling answer to this longstanding challenge. Based on atomism, Haug [11] [24] has suggested that there is indeed is an essential half Planck mass particle. [2] Under atomism, this is an indivisible particle always moving at the speed of light as measured with Einstein-Poincaré synchronized clocks. When moving at the speed of light, it is mass-less. Only when colliding with another indivisible does it have mass, or we could say that it is mass then. Actually, an indivisible particle's collision with another indivisible particle constitutes the only true mass, and when not colliding, it is energy. The mass of two colliding indivisible particles is the Planck mass particle. Since the Planck mass particle consists of two indivisible particles, then a single indivisible particle is half of this, and therefore has half the Planck mass as rest-mass. The so-called reduced Compton wavelength of a Planck mass particle is the Planck length

$$\bar{\lambda} = \frac{\hbar}{m_p c} = l_p \tag{3}$$

And naturally we can also use the reduced Compton wavelength in combination with the Planck constant and the speed of light to calculate the rest-mass of a particle

$$m_p = \frac{\hbar}{\bar{\lambda}}\frac{1}{c} = \frac{\hbar}{l_p}\frac{1}{c} \tag{4}$$

The diameter of the indivisible particle is the Planck length, and the distance from center to center between two indivisible particles corresponds to the reduced Compton wavelength of the mass in question. This means the minimum reduced Compton wavelength is, in this theory, the Planck length.

Again, two indivisible particles need to collide to create this mass. The collision is the pure mass. Each indivisible, therefore, has a rest-mass of half of this pure mass. This means that even if an indivisible particle does not have a reduced Compton wavelength, it must have an equivalent reduced Compton wavelength of $2l_p$. This is a very important point.

For example, an electron has a reduced Compton wavelength of $\bar{\lambda}_e$. This reduced Compton wavelength is enormous compared to the Planck mass particle's reduced Compton wavelength. The electron is, under this theory, simply two indivisible particles each traveling back and forth over the reduced Compton wavelength and colliding occasionally. This means we have $\frac{c}{\bar{\lambda}_e} \approx 7.76344 \times 10^{20}$ collisions per second in an electron. This is very similar to Schrödinger's [25] hypothesis in 1930 of a Zitterbewegung ("trembling motion" in German) in the electron that he indicated was $\frac{2mc^2}{\hbar} \approx 1.55269 \times 10^{21}$. This is actually exactly twice our number. This just to point out that our theory even if coming in from a very different and new perspective seems to have many similarities hypothesis as modern quantum mechanics. Each collision is a Planck mass, but each collision only lasts for one Planck second. The mass of the electron can be written as

[1] See also [26] who introduces a similar particle that he calls Maximons.

[2] It is also important to understand that we thought and wrote about this long before we tried to derive a quantum gravity theory.

$$\frac{c}{\bar{\lambda}_e} m_p \frac{l_p}{c} = \frac{\hbar}{\bar{\lambda}_e} \frac{1}{c} \approx 9.10938 \times 10^{-31} \text{ kg} \tag{5}$$

Since the electron consists of two indivisible particles, the shortest distance we can have between them is the Planck length. The maximum distance between two indivisible particles making up an electron is $2\bar{\lambda}_e$ and the minimum distance is l_p (the latter is what we could call the collision point wavelength embedded in any mass). The average distance is its reduced Compton wavelength.

Further, when an electron is moving, it is the reduced Compton wavelength that undergoes length contraction as observed with Einstein-Poincaré synchronized clocks from the stationary frame. The indivisible particles with diameter equal to the Planck length cannot undergo length contraction.[3]

Clearly, the Planck length plays an important role in relation to certain aspects of all matter.

- The Planck length is the shortest reduced Compton wavelength any elementary particle can have.
- Any elementary particle traveling at its maximum Haug velocity, see [4, 27], will in the limit get a reduced Compton wavelength equal to the Planck length. Actually Lorentz symmetry is broken at the Planck scale. Something that also possibly explains why gravity not is symmetrical. If two people measure time dilation, the first person from the highest peak of a mountain and the second person from the deepest trough in a valley, both will agree that the clock at the top of the mountain is going faster. Gravity is, at the depth of reality, high energy physics, but over incredibly short time intervals, e.g. one Planck second.
- At the very depth of reality there is actually only one type of mass: the Planck mass particle. This is the building block of all other elementary particles. The Planck mass particle only lasts for one Planck second; it is more correct to call it 1.17337×10^{-51}kg. The Planck mass particle is the only mass that is observationally time dependent when we operate with our definition of mass. The Planck mass particle is one Planck mass when the observational time window is one Planck second; see [28]. The Planck mass particle is surprisingly the mass-gap.

The Planck length is reduced Compton wavelength of all fundamental particles when traveling at their maximum velocity. The Planck length is also what we can call the contracted "wavelength" of all elementary particles, such as the electron. In our theory there is only one pure mass, which is the Planck mass particle that only lasts for one Planck second. All non-Planck mass particles are, in this model, rapidly fluctuating between being energy and being mass. All things that "normally" have rest-mass therefore have a contracted "wavelength" equal to the Planck length. This could pay a central role in building a new theory of quantum gravity.

There is only one exception here, which concerns individual indivisible particles (light is a series of these traveling after each other). These particles have an equivalent reduced Compton wavelength of $2l_p$. Be aware that this must hold for any photon, despite different wavelengths of light. The reduced Compton wavelength of light is actually $2l_p$ and is independent of the wavelength of light. This plays an important role in understanding our quantum gravity theory. For example, light with different wavelengths doesn't bend differently. Light under atomism corresponds to the old Newton view, that light is composed of indivisible particles traveling one after another at the speed of light. A photon with a wavelength must be at a minimum two indivisible particles traveling after each other. No matter if the distance between the two indivisible particles, for example, where 500 nanometers or one femtometer, the reduced Compton wavelength of light is always twice the Planck length. We could also call it the light particle's mass length to distinguish it from the wavelength of light (the distance between indivisible particles traveling in the same direction after each other).

What is most important to keep in mind from this section is simply that all masses are somehow related to the Planck length, while light is linked to twice the Planck length.

4 Modifying Newton's Composite Constant Based on Atomism

So our hypothesis is simply that Newton's gravitational constant is a composite constant. And when we understand this we can look at the different parts of this composite constant and see if any of them need adjustments for "special situations." These adjustments should not be based on merely fudging the parameters. In other words, we should not simply manipulate the different parts of the composite constant without having a fundamental reason for doing so.

In 1979, Motz and Epstein [19] suggested that there could be a discontinuity in Newton's gravitational constant when crossing the boundary of a subatomic particle, and they hypothesized that this could lead to a strong gravitational force. This is exactly what we will suggest here, but the analysis is rooted in our composite gravitational constant. Our approach is still speculative, but adds a different perspective from the Motz and

[3] The relationship is simply $\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\hbar c}{\bar{\lambda}\sqrt{1-\frac{v^2}{c^2}}}$

Epstein suggestion, as they had not decomposed the gravitational constant. Motz and Epstein correctly point out that even an ad-hoc hypothesis can lead to profound and revolutionary consequences. As an example, they mention length contraction, which was initially suggested by Fitzgerald [20] and Lorentz [21] to explain the Michelson–Morley experiment. The ad-hoc hypothesis of Lorentz contraction later became a central part of Einstein's special relativity theory. Based on such reasoning, Motz and Epstein claim

> *We believe that introducing a discontinuity in G is such an ad hoc hypothesis and that a full understanding can only come from an analysis of the physical operational meaning of the constant.*

This is exactly what this paper is about; we have found strong evidence that Newton's gravitational constant is a composite constant. By decomposing it and knowing its parts, it is much easier to suggest which parts should be changed for special situations. In our theory of quantum gravity, we suggest that there is indeed such a discontinuity in the gravitational constant when working with matter versus light instead of matter against matter.

Based on this we suggest that when working with matter against matter the gravitational constant should be the normal one, but still a composite

$$G_m = \frac{l^2c^3}{\hbar} = G \approx 6.67384 \times 10^{-11} m^3 \cdot kg^{-1} \cdot s^{-2} \tag{6}$$

As we have seen in the section above, the reduced Compton wavelength of individual indivisible particles (light) is always twice the Planck length. This because its rest-mass or its potential rest-mass when it is moving is half the Planck mass. So we will claim this leads to a gravitational constant of

$$G_m = \frac{2l_pl_pc^3}{\hbar} = 2G \approx 1.33477 \times 10^{-10} m^3 \cdot kg^{-1} \cdot s^{-2} \tag{7}$$

Again the reason for the modification when working with matter against light is that we claim gravity has to do with the collision point between indivisible particles that are, in our view, the ultimate building blocks of subatomic particles.

5 Bending of Light

In 1881 and 1884, Soldner predicted the following deflection of light, based on Newton's classical mechanics (see [29, 30])

$$\delta_S = \frac{2Gm}{c^2r} \tag{8}$$

In 1911, Einstein obtained the same formula for the bending of light when he derived it from Newtonian gravitation, see [31]. The angle of deflection in Einstein's general relativity theory [32] is twice what one gets from Newtonian gravity

$$\delta_{GR} = \frac{4Gm}{c^2r} \tag{9}$$

The solar eclipse experiment of Dyson, Eddington, and Davidson performed in 1919 confirmed [33] the idea that the deflection of light was very close to that predicted by Einstein's general relativity theory. That is 1.75 arc-seconds compared to the 0.875 as predicted by Soldner's 1884 formula.[4] This was one of the main reasons general relativity took off and partly replaced or rather extended Newtonian gravitation. A drawback with general relativity theory is that it is very complex to understand and it does not seem to be consistent with the quantum world.

Sato and Sato [34] have suggested that it looks like the 2 factor (double of Newton) in observed light deflection likely would be due to an unknown property of the photon rather than the bending of space-time. This is exactly what we get from atomism and leads to our Newtonian quantum gravity theory.

The relationship between the angle of the asymptote to the hyperbole of eccentricity ϵ is given by

$$\cos(\beta) = \frac{1}{\epsilon} \tag{10}$$

and the angle of deflection of light in Newton's theory must be given by (see Figure 1.)

$$\delta = \pi - 2\beta = \pi - 2\left(\frac{1}{\epsilon}\right) \tag{11}$$

[4] In 1881, Soldner calculated the light deflection to be 0.84 arc-seconds based on less accurate knowledge of the mass of the Sun and speed of light than we have today.

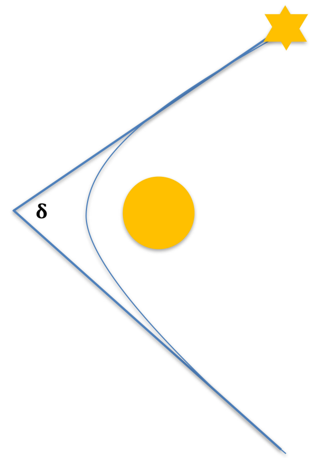

Figure 1: This figure illustrate the bending of light around the Sun. The figure is strongly exaggerated for illustration purpose.

In other words, we need to find the orbital eccentricity. The orbital eccentricity in some of Newton's deflection of light calculations [35] is given by

$$\epsilon = \sqrt{1+\frac{2EL^2}{G_m^2 M_S^2 m^3}} \tag{12}$$

but we have to understand how this particular form of eccentricity comes into being. Pay particular attention to the 2 factor in the formula. This type of orbital eccentricity can be found from

$$h^2 = GMa(1-\epsilon^2) \tag{13}$$

where h is the specific angular momentum, $h = \frac{L}{m}$, and a is the is the length of the semi-major axis, and G is the gravitational constant. Solved with respect to the eccentricity ϵ we get

$$\epsilon = \sqrt{1-\frac{h^2}{GMa}} \tag{14}$$

Next we will use the argument that the gravitational energy can be described as[5]

$$E = -\frac{GMm}{2a} \tag{15}$$

Again we see the 2 factor, which basically corresponds to the energy for low velocity orbital objects (somehow similar to the kinetic energy approximation of $E \approx \frac{1}{2}mv^2$). Putting this energy formula into formula 14 we get the known formula 12.

$$\begin{aligned}\epsilon &= \sqrt{1-\frac{h^2E}{GMaE}}\\ \epsilon &= \sqrt{1+\frac{h^2E}{GMa\frac{GMm}{2a}}}\\ \epsilon &= \sqrt{1+\frac{2h^2E}{G^2M^2m}} = \sqrt{1+\frac{2EL^2}{G^2M^2m^3}}\end{aligned} \tag{16}$$

However, for a photon we claim that it must be wrong to use the gravitational energy formula above and thereby the eccentricity formula above, because it is rooted in low velocity objects. Instead, we should use the following version when dealing with "orbital" velocity objects moving at significant speed compared to that of light. A photon is clearly doing so, as it moves at the speed of light, so in this case we must have

$$E = -\frac{GMm}{a} \tag{17}$$

[5]See for example http://scienceworld.wolfram.com/physics/Eccentricity.html

This gives an eccentricity formula of

$$\epsilon = \sqrt{1 + \frac{Eh^2}{G^2M^2m}} = \sqrt{1 + \frac{EL^2}{G^2M^2m^3}} \tag{18}$$

and with respect to our composite gravitational constant we must have

$$\epsilon = \sqrt{1 + \frac{EL^2}{G_m^2M^2m^3}} \tag{19}$$

Thus, in our view, the energy to be used in the formula when dealing with bending of light must be the photon rest-mass energy minus the gravitational energy.

$$E = mc^2 - G_m\frac{Mm}{R} \tag{20}$$

This lies in contrast to the kinetic energy approximation that only holds for low velocities, which is used in an otherwise very interesting paper [6] by Soares [35]:

$$E = \frac{1}{2}mc^2 - G_m\frac{Mm}{R} \tag{21}$$

Since $\frac{1}{2}mc^2$ is a kinetic energy formula approximation for very low velocity, it should not be used in this context where we deal with photons. Further, we claim that the (full) exact kinetic energy formula cannot be used for photons either.

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2 - G_m\frac{Mm}{R} \tag{22}$$

setting $v = c$ we would get an infinite kinetic energy. So both the kinetic energy approximation (that only holds for low velocities) and the full kinetic energy formula do not seem to make sense when we work with photons. The kinetic energy formula is made for something that has rest-mass in its normal constitution, and not for light. Here we are considering a photon that is traveling at the speed of light, but we will claim it is actually the rest-mass energy of the photon that is relevant here, which is $E = mc^2$. This is simply because, for a photon all the rest-mass energy is actually kinetic energy. The kinetic energy for a photon is, according to atomism, the very collision point between indivisible (light) particles, where the collision lasts for one Planck second. The collision is only changing the direction of the light particle, that is to say, the kinetic energy of light is a somewhat special case. Why is this? This is because all the rest-mass energy in a light particle is its kinetic energy as well.

Further, we have the angular momentum of the photon, which is

$$L = mcR \tag{23}$$

We will claim that only a light particle can have momentum equal to mc, and this is actually directly linked to the Planck momentum, as the Planck mass momentum is always m_pc.

A single light particle has an angular momentum of

$$L = mcR = \frac{\hbar}{2l_p}\frac{1}{c}cR = \frac{1}{2}m_pcR \tag{24}$$

this gives

$$\begin{aligned}
\epsilon &= \sqrt{1 + \frac{EL^2}{G_m^2M^2mm^2}} \\
\epsilon &= \sqrt{1 + \frac{\left(mc^2 - G_m\frac{Mm}{R}\right)m^2c^2R^2}{G_m^2M^2m^3}} \\
\epsilon &= \sqrt{1 + \frac{\left(c^2 - G_m\frac{M}{R}\right)c^2R^2}{G_m^2M^2}}
\end{aligned} \tag{25}$$

Bear in mind that all the small m above actually are $= \frac{\hbar}{2l_p}\frac{1}{c}$, but we see they cancel each other out; this has an impact on our G_m, which is something we will get back to later on. Since for the Sun we must have $G_m\frac{M_s}{R} << c^2$, we have

[6] To a large degree we have based our calculations on this paper by Soares.

$$\epsilon \approx \sqrt{1+\frac{c^4R^2}{G_m^2M_S^2}} \tag{26}$$

and since $\frac{c^4R^2}{G_m^2M_S^2} >> 1$, we can approximate this very well as

$$\epsilon \approx \frac{c^2R}{G_mM_S} \tag{27}$$

The bending of light is given by

$$\delta = \pi - 2\arccos\left(\frac{G_mM_S}{c^2R}\right) = \pi - 2\arccos\left(\frac{\frac{l_p 2 l_p c_3}{\hbar}M_S}{c^2R}\right) \tag{28}$$

and the reason we have $G_m = \frac{2l_pl_pc^3}{\hbar} = 2G$ instead of $G_m = \frac{l_pl_pc^3}{\hbar} = G$ is because here we are working with a standard mass (the Sun) versus light (the light beam). The light particles making up a photon each have half a Planck mass in rest-mass and therefore even if it doesn't have a reduced Compton wavelength per se we think it is relevant to claim it must have an equivalent reduced Compton wavelength of $2l_p$. Remember that this rest-mass only lasts for one Planck second at collision, so it is more "correct" to think of this mass as only 5.86×10^{-52} kg.

Further, we can expand arccos(x) since $x = \frac{G_mM_S}{c^2R} << 1$ using a Taylor series expansion, this gives

$$\arccos(x) = \frac{\pi}{2} - \arcsin(x) = \frac{\pi}{2} - \left(x - \frac{x^3}{6} + \frac{3x^5}{40} + \cdots\right) \tag{29}$$

Using only the first part of the Taylor series expansion we get

$$\delta \approx \pi - 2\left(\frac{\pi}{2} - \frac{\frac{2l_p^2c^3}{\hbar}M_S}{c^2R}\right) = \frac{4\frac{l_p^2c^3}{\hbar}M_S}{c^2R} = \frac{4GM_S}{c^2R} \approx 1.75 \text{ arc seconds} \tag{30}$$

This in strong contrast to Soares [35] who uses the same approach in deriving Newton deflection, but his work is based on the non-modified G. He correctly gets the traditional Newton bending of light, which is half of the above. Still, we will claim that Soares made a technical mistake in his derivation using the kinetic energy approximation that clearly should not be used for light, as it is an approximation that only holds when $v << c$. This is partly understandable, as the full kinetic energy formula cannot be used either, since it returns infinity for light. The light particle is special and we must return to atomism to get the proper insight to understand the rest-mass of light particles. However, even when he is using the wrong kinetic energy formula, Soares has obtained the right answer for Newtonian bending of light, as he also indirectly has used the low energy approximation for the orbital eccentricity (the 2 factor in his eccentricity formula seems to come from this). It would appear that the two errors cancel each other out.

Soares' approach is indirectly hinting that there is something special about the photon, since the normal kinetic energy framework cannot be used, nor the slow velocity approximation, nor the exact relativistic kinetic energy formula. It is clear that the kinetic energy approximation formula and the full exact kinetic energy formula for standard rest-mass cannot be used for pure energy (light).

For a photon neither can the kinetic energy approximation formula be used, nor the full kinetic energy formula. The rest-mass energy must be used. Light is special, it has rest-mass when colliding for one Planck second, in other words, all of its rest-mass is actually kinetic energy.

It seems like the atomism understanding of energy and matter combined with understanding that the Newton gravitational constant is a composite gives the correct bending of light prediction. From our derivation above, our theory also seems to predict that the bending of light in a highly accelerated electron beam would be only the Newtonian bending of light. Has this been tested? If not, then it should be: a high energy beam of electrons could be sent from Earth, and a spaceship with a measurement device could try to measure this. In short, our theory predicts something different than GR that likely could be tested.

6 Bending of Light from the Linear Equivalent Speed in Gravitational Fields and a Snell's-Type Law

There also exists an alternative way to calculate and study the bending of light that gives the same result as above. The idea of trying to combine Snell's law with gravity goes back to at least 1968, see [36], but has not led to any dramatic conclusions. This is not so strange, because light is actually not slowed down in a gravitational field; it simply has to take a longer path between two points since it is bent by gravity. When we understand this and combine it with key ideas from atomism concerning rest-mass, half Planck mass, and Planck-second durations, we can take a step forward. Here, we will introduce a quantized Snell's-type law that relies on what we will call the linear equivalent speed of light in a gravitational field.

In a gravitational field, light is bent. If we take the time in which light travels along this bent path and divide the shortest distance between two points on the path by this time, we will obtain what we can call the linear equivalent speed of the light. To be clear, the light beam is still moving at the speed of light c, as measured with Einstein-Poincaré synchronized clocks. However, we are calculating a virtual straight path for the light beam that is very useful for evaluating the bending of light in a new and interesting way. The linear equivalent speed of light in a gravitational field must be

$$\begin{aligned}
\bar{v} &= c\sqrt{\frac{2}{1+\frac{G_m M}{c^2 r}}-\frac{1}{\left(1+\frac{G_m M}{c^2 r}\right)^2}} \\
\bar{v} &= c\sqrt{\frac{2}{1+\frac{\frac{l_p 2 l_p c^3}{\hbar}Nm_p}{c^2 r}}-\frac{1}{\left(1+\frac{\frac{l_p 2 l_p c^3}{\hbar}Nm_p}{c^2 r}\right)^2}} \\
\bar{v} &= c\sqrt{\frac{2}{1+\frac{2\frac{l_p^2 c^3}{\hbar}N\frac{\hbar}{l_p}\frac{1}{c}}{c^2 r}}-\frac{1}{\left(1+\frac{2\frac{l_p^2 c^3}{\hbar}N\frac{\hbar}{l_p}\frac{1}{c}}{c^2 r}\right)^2}} \\
\bar{v} &= c\sqrt{\frac{2}{1+2N\frac{l_p}{r}}-\frac{1}{\left(1+2N\frac{l_p}{r}\right)^2}}
\end{aligned} \tag{31}$$

where l_p is the Planck length, c is the speed of light in vacuum, r is the radius from the center of a spherical mass where the light beam is passing, and N is the number of Planck masses in the gravitational mass.

Next we will introduce what can be called the gravitational refraction index, which we define as

$$n = \frac{c}{\bar{v}} \geq 1 \tag{32}$$

To calculate the bending of light, we can use something similar to Snell's law for this quantized linear equivalent speed of light

$$\delta = \arccos\left(\frac{n_1}{n_2}\right) \tag{33}$$

Since we normally talk about going from a zero gravity field to passing in and out of a gravity field, n_1 will typically be equal to 1. This is because in a zero gravitational field there is no bending of light.

For example, the mass of the Sun is approximately 1.989×10^{30}, which is equivalent to approximately 9.13848×10^{37} Planck masses. The radius of the Sun is about 696,342,000 meters. This means the straight line equivalent speed of light passing the Sun at the radius of the Sun is

$$\bar{v} = c\sqrt{\frac{2}{1+2\times 9.13848\times 10^{37}\times\frac{l_p}{696{,}342{,}000}}-\frac{1}{\left(1+2\times 9.13848\times 10^{37}\times\frac{l_p}{696{,}342{,}000}\right)^2}} \approx 299792457.9892 \text{ m/s}$$

This leads to a gravitational refraction index of the Sun of

$$n = \frac{299792458}{299792457.9892} \approx 1.00000000003599$$

Next we take the arccosine of that and get the deflection angle of light around the Sun in radians

$$\delta = \arccos\left(\frac{1}{1.00000000003599}\right) \approx 8.48404 \times 10^{-6}$$

This must be multiplied by $\frac{648000}{\pi}$ to get the answer in arcseconds, as is most normally reported in relation to the bending of light; this gives

$$\delta = 8.48404 \times 10^{-6} \times \frac{648000}{\pi} \approx 1.75 \text{ arc seconds}$$

That is about 1.75 arc seconds, which is basically the same prediction as given by Einstein's general relativity theory and lies well within the range of what has been observed. Here it is simply calculated from a different quantized perspective.

On the surface of Earth, when we talk about light speed experiments, we must be aware that the path taken is not linear. So here we also have a linear equivalent speed, which is the virtual speed light has attained on the linear path. The mass of the Earth is approximately 5.972×10^{24} kg and about 2.74384×10^{32} Planck masses. Further, the radius of the Earth is about 6,371,000 meters. This gives a linear equivalent speed of light of

$$\begin{aligned} \bar{v} &= c\sqrt{\frac{2}{1+2\times 2.74384\times 10^{32}\times \frac{l_p}{6{,}371{,}000}} - \frac{1}{\left(1+2\times 2.74384\times 10^{32}\times \frac{l_p}{6{,}371{,}000}\right)^2}} \\ &\approx 299792457.99999999970949 \text{ m/s} \end{aligned}$$

As expected, this "effect" is mini-scule in the weak gravitational field of the Earth. The main consequences are naturally that when one assumes light travels in a straight path in a gravity field: a) it is actually not traveling in a straight path (well known), and b) the equivalent straight-line speed is slower than the speed of light in vacuum.

To our knowledge, only Wålin [37] has tried to do something similar. First of all, his velocity formula is different, and he mistakenly assumes that it is the real velocity of light in a gravitational field, rather than a linear equivalent virtual light speed, as we understand here. The speed of light that Wålin assumes is a slowing down from a gravitational field is also far too slow based on his formula. His prediction would easily have been detected on Earth already in a series of experiments. Still, he is able to get the right amount of light bending. We think that there is too much fudging to his approach, but it is still a great paper, as it inspired us to come up with the idea of linear equivalent speed of light. His formula for the speed of light in gravitational field is given by

$$v = \frac{c}{1+\frac{2GM}{c^2}} \tag{34}$$

At the surface of Earth, this would lead to a light speed of

$$v = \frac{c}{1+\frac{2GM_E}{c^2 r}} \approx 299,792,457.583 \tag{35}$$

This obviously cannot be correct, as it would have been detected long ago if the speed of light on the surface of Earth was very different from the speed of light in outer space. We have simply calculated linear velocity from the linear distance between two points divided by the time it takes for the bent light path to travel between those points.

7 Gravitational Red-Shift and Gravitational Time Dilation

Gravitational red-shift and time dilation are basically just a function of the escape velocity. The escape velocity from Newtonian gravity is typically always calculated by solving the equation below with respect to v

$$\begin{aligned} \frac{1}{2}mv^2 - G\frac{Mm}{R} &= 0 \\ v^2 - 2G\frac{M}{R} &= 0 \\ v &= \sqrt{2\frac{GM}{R}} \end{aligned} \tag{36}$$

This is also the same escape velocity one gets from Einstein's general relativity using the Schwarzschild metric; see [38]. Please notice that the equation above is solved from the kinetic energy approximation that only holds when $v << c$. Light is moving at speed c, so this formula should clearly not hold for light. Also, the full kinetic energy formula cannot be used, as it would lead to infinity when $v = c$, as it must be for light. We have to understand that light is special; we claim it is the rest-mass formula that must be used for light, so in other words we must have

$$\begin{aligned} mc^2 - G_m\frac{Mm}{R} &= 0 \\ v^2 - G_m\frac{M}{R} &= 0 \\ v &= \sqrt{\frac{G_m M}{R}} \end{aligned} \tag{37}$$

However, because we are working with light here, we have $G_m = \frac{2l_p l_p c^3}{\hbar} = 2G$ rather than $G_m = \frac{l_p l_p c^3}{\hbar} = G$, as we would have for any non-light moving object. Under our theory, we get the same escape velocity as before

$$v = \sqrt{\frac{G_m M}{R}} = \sqrt{\frac{l_p 2 l_p c^3}{\hbar}\frac{M}{R}} = \sqrt{2\frac{GM}{R}} \tag{38}$$

Only the escape velocity is needed to calculate gravitational time dilation, red-shift, and gravitational acceleration. This means we get the same outcomes as Newton and general relativity here.

8 A probability interpretation of Gravity when below Planck mass size

Haug [7] has suggested that when re-writing Newton and Einstein gravitational formulas based on the view that the gravitational constant likely is a composite constant that we at the subatomic scale has to do with a type of gravity probability. We are also presenting this view here. The classical Newtonian gravity is then given by

$$F = G\frac{m_{\mathbf{P}} m_{\mathbf{P}}}{r^2}\ m^3 \cdot kg^{-1} \cdot s^{-2} \tag{39}$$

where $m_{\mathbf{P}}$ is the proton mass, not to be confused with the Planck mass notation m_p. This is about 10^{38} times weaker than the strong force. One of the most significant open questions in modern physics is why the strong force is so much more powerful than the gravity force. However, we can rewrite the formula above based on the view that the gravitational constant is a composite of the form $G = \frac{l_p^2 c^3}{\hbar}$, this gives

$$F = G_m\frac{m_{\mathbf{P}} m_{\mathbf{P}}}{r^2} = \frac{l_p^2 c^3}{\hbar}\frac{\frac{\hbar}{\bar{\lambda}_P}\frac{1}{c}\frac{\hbar}{\bar{\lambda}_P}\frac{1}{c}}{r^2} = \frac{\hbar c}{r^2}\frac{l_p^2}{\bar{\lambda}_P^2} \tag{40}$$

We will here as Haug has suggested in several papers claim that $\frac{l_p^2}{\bar{\lambda}_P^2}$ can be seen as a quantum probability factor linked to a one Planck second observational time window. This is in other words an expected gravity, and should be written as

$$E[F] = G_m\frac{m_{\mathbf{P}} m_{\mathbf{P}}}{r^2} = \frac{l_p^2 c^3}{\hbar}\frac{\frac{\hbar}{\bar{\lambda}_P}\frac{1}{c}\frac{\hbar}{\bar{\lambda}_P}\frac{1}{c}}{r^2} = \frac{\hbar c}{r^2}\frac{l_p^2}{\bar{\lambda}_P^2} \tag{41}$$

Let us for a moment exclude the probability factor in the Proton gravity, $\frac{l_p^2}{\bar{\lambda}^2}$, in the formula above. Then we get a gravity force that is approximately 1.69×10^{38} times the force that would be predicted using the conventional gravity formula today, still that is without taking into account the probability factor. When going below Planck masses probability starts to kick in, and when down to the proton size, the probabilities are totally dominating in relation to gravity. The gravity force is discrete at Planck time, either there is the full Planck gravity force $G\frac{m_p m_p}{l_p^2}$ lasting for only one Planck second or there is no gravity. For a proton, the gravity kicks in and out $\frac{c}{\bar{\lambda}_P} \approx 1.4254 \times 10^{24}$ times per second. However each gravity event only last for one Planck second, so the very strong Planck gravity force is if we are observing at a time interval of longer than 7.0151×10^{-25} seconds observed as Planck gravity force that only lasted for one Planck second is smoothed (divided) out over the reduced Compton time of the proton, an idea not so different than that have been suggested by Motz and Epstein [19]. The very strong gravity force appears weak even if it is as strong as the strong force. This because it only lasts for one Planck second for every reduced Compton time interval in the proton ($\frac{\bar{\lambda}}{c}$). All of this is consistent with a new probabilistic Heisenberg theory recently derived by Haug, [39] and also a gravity theory derived from the Heisenberg principle, first for masses fully divisible by the Planck mass derived by McCulloch in 2014 [8] and then extended to include masses below the Planck mass as well as not divisible by the Planck mass by Haug in 2018 [40] (actually that last development happened after the first version of this paper).

9 Modified Gravitational Acceleration Field

The acceleration field is unrealistically low under classical Newtonian physics at the Schwarzschild radius. And yet the escape velocity at the Schwarzschild radius is always the speed of light, as we think it should be. Assume a super-massive object that is 10^{14} solar masses. The gravitational acceleration field at the Schwarzschild radius is, under Newton's universal gravitation, only

$$g = \frac{GM}{R^2} = \frac{GM}{2\frac{GM}{c^2}} \approx 0.152 \text{ m}/s^2 \tag{42}$$

How can the escape velocity be c and at the same time is the surface gravity field much weaker than that on Earth, where it is about 9.8 m/s^2? According to Einstein's General Relativity theory, the gravitational acceleration field under the Schwarzschild metric is different than that of Newton. We will suggest that no acceleration field can be stronger than the Planck acceleration field:

$$a_p = \frac{c^2}{l_p} \approx 5.56092 \times 10^{51} \text{ m}/s^2 \tag{43}$$

If the shortest possible time interval during which something can undergo acceleration is one Planck second, then if an object undergoes Planck acceleration for this time interval, it will reach the speed of light:

$$a_p t_p = \frac{c^2}{l_p}\frac{l_p}{c} = c \tag{44}$$

As matter cannot travel at the speed of light, in our interpretation this means only a Planck mass particle can undergo this acceleration. As shown by Haug in a series of papers, the Planck particle is likely at absolute rest and is within one Planck second dissolving into pure energy. This also explains why the mass can accelerate from rest-mass to speed c within a Planck second; it has to dissolve into pure energy in this time-frame. And from mathematical atomism only the Planck mass particle can do this within a Planck second. Anyway we will assume the Planck acceleration is what we have at the Schwarzschild radius. Further, we will assume the inverse square rule basically holds for a radius going out from the Schwarzschild radius rather than from the very center of the mass. Based on this our suggested somewhat ad-hoc modified formula for gravitational acceleration field is

$$\begin{aligned} g &\approx \frac{GM}{r^2 - \left(\frac{2GM}{c^2}\right)^2 + \left(\frac{GM}{c^2}\right) l_p} \\ g &\approx \frac{c^2 N l_p}{r^2 - (2Nl_p)^2 + Nl_p^2} \end{aligned} \tag{45}$$

The acceleration field now for a 10^{14} solar mass object at the Schwarzschild radius, $r = \frac{2GM}{c^2}$ gives

$$g \approx \frac{GM}{r^2 - \left(\frac{2GM}{c^2}\right)^2 + \left(\frac{GM}{c^2}\right) l_p} = \frac{GM}{\left(\frac{2GM}{c^2}\right)^2 - \left(\frac{2GM}{c^2}\right)^2 + \left(\frac{GM}{c^2}\right) l_p} = \frac{c^2}{l_p} \approx 5.56092 \times 10^{51} \text{ m}/s^2 \tag{46}$$

Next, the mass of the Earth is approximately 2.74388×10^{32} Planck masses. Further, the radius of the Earth is 6,371,000; this gives an acceleration field of the Earth at the surface of Earth equal to

$$g \approx \frac{c^2 N l_p}{r^2 + N l_p^2 (1-N)} = \frac{c^2 \times 2.74388 \times 10^{32} \times l_p}{6371000^2 + (2 \times 2.74388 \times 10^{32} \times l_p)^2 - 2.74388 \times 10^{32} \times l_p} \approx 9.8194 \ m/s^2$$

Still, this formula always gives the Planck acceleration at the modified Schwarzschild radius. We have not investigated this adjustment in depth yet, and it should be investigated further for possible weaknesses.

10 Perihelion of Mercury

Supposedly one of the greatest achievements of general relativity was to predict the perihelion of Mercury correctly. At least in its original form, the Newtonian theory does not seem to predict this. However, several smaller modifications of Newton's gravitational theory have been proposed that make it compatible with the observed perihelion of Mercury. Friedman and Steiner [41], for example, have recently suggested a relativistic correction for Newton that seems to work in this regard. Sato and Sato [34] have speculated that the perihelion precession of Mercury is caused by an gravitationally-induced electromagnetic "Bremsstrahlung." Abramowicz et.al [42, 43] have suggested that an "enhanced" Newtonian theory is consistent with bending of space and thereby with the perihelion of Mercury. Other researchers have pointed out that it is not completely clear that Newton is truly incompatible with the observed perihelion of Mercury; see [44], for example.

How these theories and ideas may be compatible with the Newtonian quantum gravity theory suggested here is a question we will hold for future research.

11 Summary

Here we will shortly summarize some of our findings.

- In Newtonian quantum gravity theory, the gravitational "constant" takes the same value as it does today, as long as we are working with two objects larger than or equal to the Planck mass.
- Newtonian quantum gravity theory surprisingly predicts the same gravitational bending as GR for light, but it is based on a much simpler model with postulates on what the ultimate building blocks are. We have $\delta = 2\frac{G_m M}{c^2 R} = 4\frac{GM}{c^2 R}$ when dealing with light. That is for photon beams we have $G_m = \frac{l_p 2 l_p c^3}{\hbar} = 2G$.
- Newtonian quantum gravity theory seems to predict that the bending of non-photon beams will be $\delta = \frac{G_m M}{c^2 R} = 2\frac{GM}{c^2 R}$. This is a prediction not given by GR. This could possibly be tested out by electron beams or proton beams. For non-photon beams we have $G_m = \frac{l_p^2 c^3}{\hbar} = G$.
- When working below Planck masses or with masses not fully divisible by the Planck mass we will claim uncertainty kicks in for gravity and we have suggested that there is a probability factor that kicks in when below the Planck mass size. There are also probabilities when working with Planck mass objects, but the partial probabilities then add up to one, as recently shown by Haug [39].
- Newtonian quantum gravity theory seems to give the same predictions as GR for time dilation, gravitational red-shift, and so forth.
- When it comes to the gravitational acceleration field, we have suggested a modification that makes the field equal to the Planck acceleration at the Schwarzschild radius. For weak gravitational fields we still get the same as predicted by Newton.
- In Newtonian quantum gravity, the speed of gravity moves at the speed of light. This is already hidden inside Newton's gravitational constant. We could say that Newton's gravitational theory is consistent with the speed of gravity being the speed of light, but this is made clear when we have modified it into a Newtonian quantum gravity theory.

Newtonian quantum gravity is much simpler than general relativity theory and is rooted in a theory of atomism. Table 1 illustrates the main differences between this new gravity theory and Newtonian gravity and general relativity theory.

Case:	**Prediction**
Speed of gravity	Speed of light
Time dilation	Same as GR (and Newton).
Red-shift	Same as GR (and Newton).
Gravitational acceleration field	Different than GR and Newton.
Deflection light beam	Same as GR, twice of Newton.
Deflection electron beam	Same as Newton for light, half of GR.

Table 1: The table of a series of measurements that can be observed and measured in relation to gravity, and the gravitational force that we cannot observe or measure.

Case:	**Result**
When working with two rest-masses	$G_m = \frac{l_p l_p c^3}{\hbar} = G.$
When working with light	$G_m = \frac{2 l_p^2 c^3}{\hbar} = 2G.$

Table 2: The table of a series of measurements that can be observed and measured in relation to gravity, and the gravitational force that we cannot observe or measure.

Table 2 illustrates how we think Newton's composite gravitational constant should be used for different "special" situations.

12 Conclusion

We have pointed out that Newton's gravitational constant is most likely a composite constant. Based on new ideas about energy and mass at the most fundamental level, we have also suggested how the composite gravitational constant must be modified when we have to deal with light and other special situations.

Our Newtonian quantum gravity theory gives the same prediction of bending of light as Einstein's theory of general relativity. For non-light beams such as electrons, our theory seems to predict that they only would bend half as much as light. That is to say, we get a Newton bending of light for electron beams, but a GR bending when we deal with light.

Further, we have suggested a modification of the gravitational acceleration field equation. Our modified formula always gives Planck acceleration at the Schwarzschild radius, and at the same time it gives the same values as Newton when we are in weak gravitational fields.

In addition, our theory suggests that the gravitational force becomes uncertain when working with masses below the Planck mass size. The closer we get to the subatomic world the more uncertainty. The difference between the strong force and the gravity force can likely be explained by a quantum probability factor that we have looked at briefly in this paper and in much more detail in other recent working papers.

We admit that our theory is somewhat speculative. However, as with any emerging scientific theory, we strongly recommend studying recent developments in mathematical atomism closely before coming to any conclusions about this new theory of quantum gravity that can be derived from the Heisenberg uncertainty principle.

References

[1] H. Cavendish. Experiments to determine the density of the Earth. *Philosophical Transactions of the Royal Society of London, (part II)*, 88, 1798.

[2] M. Planck. *Naturlische Masseinheiten*. Der Königlich Preussischen Akademie Der Wissenschaften, p. 479, 1899.

[3] M. Planck. *Vorlesungen über die Theorie der Wärmestrahlung*. Leipzig: J.A. Barth, p. 163, see also the English translation "The Theory of Radiation" (1959) Dover, 1906.

[4] E. G. Haug. The gravitational constant and the Planck units: A simplification of the quantum realm. *Physics Essays Vol 29, No 4*, 2016.

[5] E. G. Haug. Planck quantization of Newton and Einstein gravitation. *International Journal of Astronomy and Astrophysics*, 6(2), 2016.

[6] E. G. Haug. Can the Planck length be found independent of big G ? *Applied Physics Research*, 9(6), 2017.

[7] E. G. Haug. Newton and Einstein's gravity in a new perspective for Planck masses and smaller sized objects. *International Journal of Astronomy and Astrophysics*, 8, 2018.

[8] M. E. McCulloch. Gravity from the uncertainty principle. *Astrophysics and Space Science*, 349(2), 2014.

[9] M. E. McCulloch. Quantised inertia from relativity and the uncertainty principle. *EPL (Europhysics Letters)*, 115(6), 2016.

[10] E. G. Haug. A suggested boundary for Heisenberg's uncertainty principle. *http://vixra.org/abs/1701.0497*, 2017.

[11] E. G. Haug. *Unified Revolution: New Fundamental Physics*. Oslo, E.G.H. Publishing, 2014.

[12] E. G. Haug. Modern physics incomplete absurd relativistic mass interpretation. and the simple solution that saves Einstein's formula. *http://vixra.org/abs/1612.0249*, 2016.

[13] V. Dokuchaev and Y. N. Eroshenko. Black hole atom as a dark matter particle candidate. *aXxiv.org/abs/1403.1375*, 2014.

[14] L. Motz. Gauge invariance and the structure of charged particles. *Il Nuovo Cimento*, 26(4), 1962.

[15] L. Motz. A gravitational theory of the mu meson and leptons in general. *Rutherford Observatory, Columbia University*, 1966.

[16] L. Motz. The quantization of mass. *Rutherford Observatory, Columbia University*, 1971.

[17] S. S. De. Quantum creation of highly massive particles in the very early universe. *International Journal of Theoretical Physics*, 2001.

[18] S. Hawking. Gravitationally collapsed objects of very low mass. *Monthly Notices of the Royal Astronomical Society*, 152, 1971.

[19] L. Motz and J. Epstein. The gravitational charge $1/2\sqrt{\hbar c}$ as a unifying principle in physics. *Il Nuovo Cimento*, 51(1), 1979.

[20] G. F. FitzGerald. The ether and the Earth's atmosphere. *Science*, 13(328):390, 1889.

[21] H. A. Lorentz. The relative motion of the Earth and the aether. *Zittingsverlag Akad. V. Wet.*, (1):74–79, 1892.

[22] G. M. Obermair. Primordial Planck mass black holes (ppmbhs) as candidates for dark matter? *Journal of Physics, Conference Series*, 442, 2013.

[23] J. H. Macgibbon. Can Planck-mass relics of evaporating black holes close the universe? *Nature*, 329, 1987.

[24] E. G. Haug. The Planck mass particle finally discovered! The true God particle! Good bye to the point particle hypothesis! *http://vixra.org/abs/1607.0496*, 2016.

[25] E. Schrödinger. Über die kräftefreie bewegung in der relativistischen quantenmechanik. *Sitzungsberichte der Preuischen Akademie der Wissenschaften. Physikalisch-mathematische Klasse*, 1930.

[26] M. Markov. Elementary particles of maximally large masses (quarks and maximons). *Soviet Physics JPT*, 24(3), 2014.

[27] E. G. Haug. The ultimate limits of the relativistic rocket equation: The Planck photon rocket. *Acta Astronautica*, 136, 2017.

[28] E. G. Haug. The mass gap, kg, the Planck constant, and the gravity gap. *http://vixra.org*, 2017

[29] L. Soldner. On the deflection of a light ray from its rectilinear motion, by the attraction of a celestial body at which it nearly passes by. *Berliner Astronomisches Jahrbuch:*, 1881.

[30] L. Soldner. On probability theory and probabilistic physics – axiomatics and methodology. *Foundations of Physics*, 3(4), 1884.

[31] A. Einstein. Über den einfluss der schwercraft auf die ausbreitung des lichtes. *Annalen der Physik*, 4, 1911.

[32] A. Einstein. Näherungsweise integration der feldgleichungen der gravitation. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften Berlin*, 1916.

[33] F. Dyson, A. Eddington, and C. Davidson. A determination of the deflection of light by the Sun's gravitational field, from observations made at the total eclipse of May 29, 1919. *Philosophical Translation Royal Society*, 1920.

[34] M. Sato and H. Sato. Light bending by gravity: Back to space and time from spacetime. *Physics Essays*, 29, 2016.

[35] D. S. L. Soares. Newtonian gravitational deflection of light revisited. *arXiv:physics/0508030*, 2009.

[36] F. R. Tangherlini. On Snell's law and the gravitational deflection of light. *American Journal of Physics 36*, 30, 1968.

[37] L. Wåhlin. The bending of light by gravity. *Unpublished Working Paper*, 2005.

[38] A. T. Augousti and A. Radosz. An observation on the congruence of the escape velocity in classical mechanics and general relativity in a Schwarzschild metric. *European Journal of Physics*, 376:331–335, 2006.

[39] E. G. Haug. Heisenberg quantum probabilities
God does not throw dice at the planck scale, but below! *Working Paper Norwegian University of Life Science*, 2018.

[40] E. G. Haug. Heisenberg probabilistic quantum gravity that holds at the subatomic and the macroscopic scale. *http://vixra.org/pdf/1803.0642*, 2018.

[41] Y. Friedman and J. M. Steiner. Predicting Mercury's precession using simple relativistic Newtonian dynamics. *ArXiv:1603.02560*, 2016.

[42] M. A. Abramowicz. The perihelion of Mercury advance calculated in Newton's theory. *ArXiv:1212.0264*, 2012.

[43] M. A. Abramowicz, G. F. R. Ellis, H. J. , and M. Wielgus. The perihelion of Mercury advance and the light bending calculated in (enhanced) Newton's theory. *General Relativity and Gravitation*, 46, 2014.

[44] W. Harper. Newton's methodology and Mercury's perihelion before and after Einstein. *Philosophy of Science*, 74, 2007.

[45] V. Stenger. Myths of physics: 2. Gravity is much weaker than electromagnetism. *Huffington Post, Blog by Victor Stenger*, 2014.