$|00\rangle + |11\rangle = |01\rangle + |10\rangle$?

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Consider four-dimensional Hilbert space H over $\mathbb{C}$ as a direct product of two two-dimensional Hilbert spaces over $\mathbb{C}$, in which two binary quantum states are represented, respectively, that is, $|00\rangle$ = (1, 0, 1, 0), $|10\rangle$ = (0, 1, 1, 0), $|01\rangle$ = (1, 0, 0, 1) and $|11\rangle$ = (0, 1, 0, 1). Then, $|00\rangle + |11\rangle = |01\rangle + |10\rangle$ = (1, 1, 1, 1). Existence of such linear dependency is obvious considering three binary quantum states in six dimensional Hilbert space, because there are eight combinations of 0 and 1 in a six dimensional space. Moreover, though it may be surprising that basis set $\{|00\rangle. |10\rangle, |01\rangle. |11\rangle\}$ is not enough to cover H, considering degree of freedom of quantum state spaces represented in four and two dimensional Hilbert spaces over $\mathbb{C}$ counted by $\mathbb{R}$ are 7 and 3, respectively, and 7-3*2=1, it is also obvious. The natural basis set of H is {(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)}. Considering practical communication with two binary quantum channels, (1, 0, 0, 0), which is a valid quantum state even traditionally, should mean that the first two-dimensional Hilbert space represent $|0\rangle$ is sent and the second one represent no photons sent, that is, vacuum . Moreover, (0, 0, 0, 0) should also represent a valid quantum state that no photons are sent in either channel, that is, total vacuum. Violation of Bell's inequality not by quantum entanglement is discussed in a separate paper [PHASE].

REFERENCES

[PHASE] M. Ohta, "Applied Physical Understanding on Phase", To Appear