Investigation of Relativistic Free Fall in the Uniform Gravitational Field

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Abstract: This paper investigates the possibility of testing the General Relativity Theory (GRT) by studying the relativistic free fall of a small test body in a uniform gravitational field. The constant improvements in technology lead to increased precision of measurements, which opens up new possibility of testing the GRT. The paper compares the free fall predictions obtained from the Newtonian physics theory, the GRT, and the Metric Theory of Gravity (MTG). It is found that it might perhaps be possible to distinguish between the GRT and the MTG theories with a reasonable confidence and thus determine by experimental means which theory is actually correct.

Introduction: The theories describing the free fall are well understood in both; the Newtonian physics and in the General Relativity. In the GRT the inertial mass and the gravitational mass are assumed identical with identical dependencies on velocity. In the MTG, on the other hand, the gravitational mass depends on the velocity differently than the inertial mass. It is thus simple to derive equations describing the free fall velocity of a small test body in dependence on time in either theory and make comparisons with possible measurement results.

Theories: In the Newtonian physics the relation between the velocity and time is described as follows:

$$v_{nt}(t) = g \cdot t \tag{1}$$

where the symbol g is the gravitational acceleration, which is constant in the studied case. In the GRT the relation between the velocity v and time is more complicated and is derived as follows:

$$\frac{d}{dt}\left[\frac{m_0 v}{\sqrt{1-v^2/c^2}}\right] = g\,\frac{m_0}{\sqrt{1-v^2/c^2}} \tag{2}$$

where m_0 is the rest mass and c the speed of light in a vacuum. The left hand side of Eq.2 is the relativistic formula for the inertial force and the right hand side is the formula for the gravitational force that includes the gravitational force dependence on velocity. The formula in Eq.2 can be rearranged and simplified resulting in the following equation for acceleration:

$$\frac{dv}{dt} = g(1-v^2/c^2) \tag{3}$$

This equation can be integrated to obtain the formula for the falling time in dependence on velocity:

$$2gt = \int_0^v \frac{dv}{(1-v/c)} + \int_0^v \frac{dv}{(1+v/c)} = c\ln\left(\frac{c+v}{c-v}\right) \tag{4}$$

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This result can be rearranged once more, the velocity calculated, and finally simplified as follows:

$$v = c\frac{e^{2gt/c}-1}{e^{2gt/c}+1} = c\cdot\tanh\left(\frac{gt}{c}\right) \cong gt\left(1-\frac{1}{3}\left(\frac{gt}{c}\right)^2\right) \tag{5}$$

This is an interesting result that might be reachable by today's experiments. For example; for the fall time of *t=10.2 sec* and the Earth's gravitational acceleration of $g=9.81m/sec^2$ the term *gt=100m/sec* and *gt/c* is approximately equal to $(1/3)10^{-6}$. This can perhaps be measured today with a radar and a laser interferometer. An interesting point of exact portion of Eq.5 is that the limiting velocity is equal to *c*.

However, there is now also a possibility to verify that the gravitational mass is identical to the inertial mass independent of velocity. This is sometimes called the Einstein's Weak Equivalence Principle (WEP). The author of this paper has shown in previous publications [1, 2] that this is not true, that the WEP is false, and that the gravitational mass depends on velocity differently than the inertial mass.

$$m_i = \frac{m_0}{\sqrt{1-v^2/c^2}} \tag{6}$$

$$m_g = m_0\sqrt{1-v^2/c^2} \tag{7}$$

Substituting this formula for the gravitational mass dependence on velocity into the right hand side of Eq.2 the equivalent of Eq.3 becomes:

$$\frac{dv}{dt} = g(1-v^2/c^2)^2 \cong g(1-2v^2/c^2) \tag{8}$$

This equation can also be integrated:

$$gt = \int_0^v \frac{dv}{\left(1-v^2/c^2\right)^2} \tag{9}$$

with the following result:

$$4gt = c\ln\left(\frac{c+v}{c-v}\right) + \frac{2v}{\left(1-v^2/c^2\right)} \tag{10}$$

However, it is not easy to calculate the velocity from Eq.10, so it is more accurate to evaluate the fall time as a function of the velocity, since Eq.4 and Eq.10 provide the exact solutions. Equation used in FIG.2 for the velocity in the MTG theory is derived from the approximation introduced in Eq.8 where the fourth order terms in *v/c* were neglected.

$$2gt = \int_0^v \frac{dv}{(1-\sqrt{2}v/c)} + \int_0^v \frac{dv}{(1+\sqrt{2}v/c)} = \frac{c}{\sqrt{2}}\ln\left(\frac{1+\sqrt{2}v/c}{1-\sqrt{2}v/c}\right) \tag{11}$$

This formula can be rearranged and solved for velocity similarly as in Eq. 5:

$$v = \frac{c}{\sqrt{2}}\frac{e^{2\sqrt{2}g/c}-1}{e^{2\sqrt{2}g/c}+1} = \frac{c}{\sqrt{2}}tanh\left(\frac{\sqrt{2}gt}{c}\right) \cong gt\left(1-\frac{2}{3}\left(\frac{gt}{c}\right)^2\right) \tag{12}$$

There is also another way how to show that the mass dependence on velocity is different than what is claimed in the GRT. From the conservation of energy during the fall it is possible to write:

$$\frac{m_0 c^2}{\sqrt{1-v^2/c^2}} = m_0 c^2 + m_0 g \cdot z \tag{13}$$

By differentiating this equation with respect to time we obtain the following formula for acceleration:

$$\frac{dv}{dt} = g\left(1-\frac{v^2}{c^2}\right)^{3/2} \tag{14}$$

This equation can be rewritten in another way to be form equivalent to Eq.2 and thus reveal the gravitational force:

$$\frac{d}{dt}\left(\frac{m_0 v}{\sqrt{1-v^2/c^2}}\right) = m_0 g \tag{15}$$

This clearly indicates that the gravitational force and therefore the gravitational mass, as appearing on the right hand side of Eq.15, do not depend on velocity. This is contrary to what is claimed in the GRT.

By solving Eq.14 for time as a function of velocity will result in:

$$gt = \int \frac{dv}{\left(1-v^2/c^2\right)^{3/2}} = \frac{v}{\sqrt{1-v^2/c^2}} \tag{16}$$

After rearranging this result and simplifying it, the velocity formula becomes equal to:

$$v = \frac{gt}{\sqrt{1+\left(gt/c\right)^2}} \cong gt\left(1-\frac{1}{2}\left(\frac{gt}{c}\right)^2\right) \tag{17}$$

We, therefore, have the following four equations for velocities as functions of time:

$$v_{nt} = gt \tag{18}$$

$$v_{gr} = gt\left(1-\frac{1}{3}\left(\frac{gt}{c}\right)^2\right) \tag{19}$$

$$v_{ec} = gt\left(1-\frac{1}{2}\left(\frac{gt}{c}\right)^2\right) \tag{20}$$

$$v_{mg} = gt\left(1-\frac{2}{3}\left(\frac{gt}{c}\right)^2\right) \tag{21}$$

The differences between the fall times and velocities predicted by the discussed theories can be best shown graphically and by subtracting the Newtonian physics fall time velocity. This is shown in graphs in FIG.1 and FIG.2. The results were calculated for the higher velocities in FIG.1, because the Mathcad software could not handle the required precision for the smaller velocity values.

GRT test for a free fall (time is measured in dependence on the velocity)

$$g = 9.80665\,\frac{m}{s^2} \qquad v(x) := x\cdot c \qquad x := 10^{-20}, 3\cdot 10^{-8} .. 10^{-4.7} \qquad tnt(x) := \frac{c\cdot x}{g}$$

$$tgr(x) := \frac{c}{2\cdot g}\cdot \ln\left(\frac{1+x}{1-x}\right) \qquad tec(x) := \frac{c}{g}\cdot\frac{x}{\sqrt{1-x^2}} \qquad tmg(x) := \frac{c}{4\cdot g}\cdot\left(\ln\left(\frac{1+x}{1-x}\right) + \frac{2\cdot x}{1-x^2}\right)$$

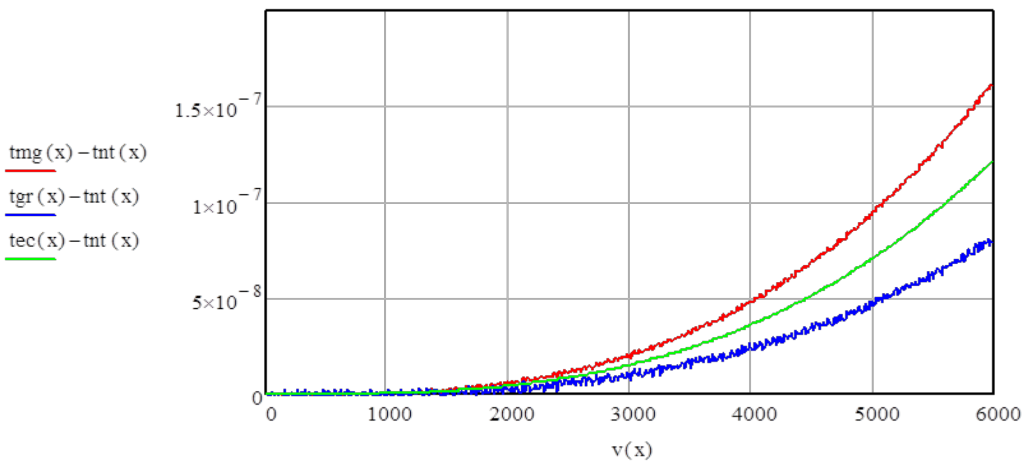

FIG.1 the graphs show the departures of the falling time in the free fall experiments from the Newtonian physics prediction *t=v/g* in dependence on velocity. The MTG theory is the red trace, the GRT theory is the blue trace, and the Energy conservation law is the green trace.

GRT test for a fee fall (velocity is measured as a function of time)

$$g = 9.807\,\frac{m}{s^2} \qquad t := 0\cdot sec, 0.2\cdot sec .. 30\cdot sec \qquad vnt(t) := g\cdot t$$

$$vsr(t) := g\cdot t\cdot\left[1 - \frac{1}{3}\left(\frac{g\cdot t}{c}\right)^2\right] \qquad vgm(t) := g\cdot t\cdot\left[1 - \frac{2}{3}\cdot\left(\frac{g\cdot t}{c}\right)^2\right] \qquad vec(t) := g\cdot t\cdot\left[1 - \frac{1}{2}\cdot\left(\frac{g\cdot t}{c}\right)^2\right]$$

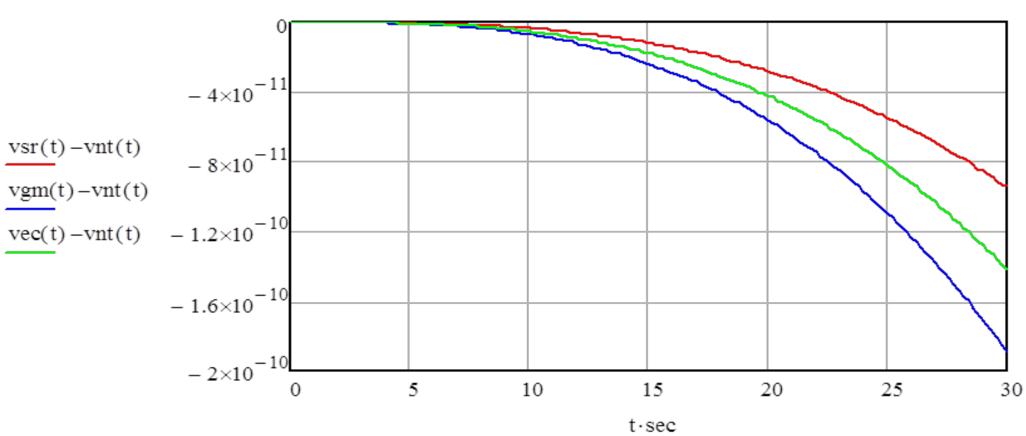

FIG.2 the graphs show the velocity departures from the Newtonian physics prediction of *v=gt* in dependence on time for the MTG theory, the blue trace, the GRT theory, the red trace, and the Energy conservation law, the green trace.

From the practical point of view it may be easier to measure time as a function of distance. The reason for this is that the distance of a fall is precisely known and the time of fall can also be precisely measured from the moment of body release to the moment when it hits the sensor at the end stop. Equations derived for this dependency are as follows:

$$t_{nt} = \sqrt{\frac{2z}{g}} \tag{22}$$

$$t_{gr} = \sqrt{\frac{2z}{g}}\left(1 - \frac{1}{6}\frac{gz}{c^2}\right) \tag{23}$$

$$t_{ec} = \sqrt{\frac{2z}{g}}\left(1 - \frac{1}{4}\frac{gz}{c^2}\right) \tag{24}$$

$$t_{mg} = \sqrt{\frac{2z}{g}}\left(1 - \frac{1}{3}\frac{gz}{c^2}\right) \tag{25}$$

The graphs for these dependencies with the Newton time subtracted are shown in FIG.3.

GRT test for a fee fall (time is measured as a function of distance)

$$z := 0 \cdot m, 0.1 \cdot m .. 300 \cdot m \qquad g = 9.807 \frac{m}{s^2}$$

$$tnt(z) := \sqrt{\frac{2 \cdot z}{g}} \qquad \Delta tgr(z) := \frac{1}{6} \cdot \sqrt{\frac{2 \cdot z}{g}} \cdot \frac{g \cdot z}{c^2} \qquad \Delta tec(z) := \frac{1}{4} \cdot \sqrt{\frac{2 \cdot z}{g}} \cdot \frac{g \cdot z}{c^2} \qquad \Delta tmg(z) := \frac{1}{3} \cdot \sqrt{\frac{2 \cdot z}{g}} \cdot \frac{g \cdot z}{c^2}$$

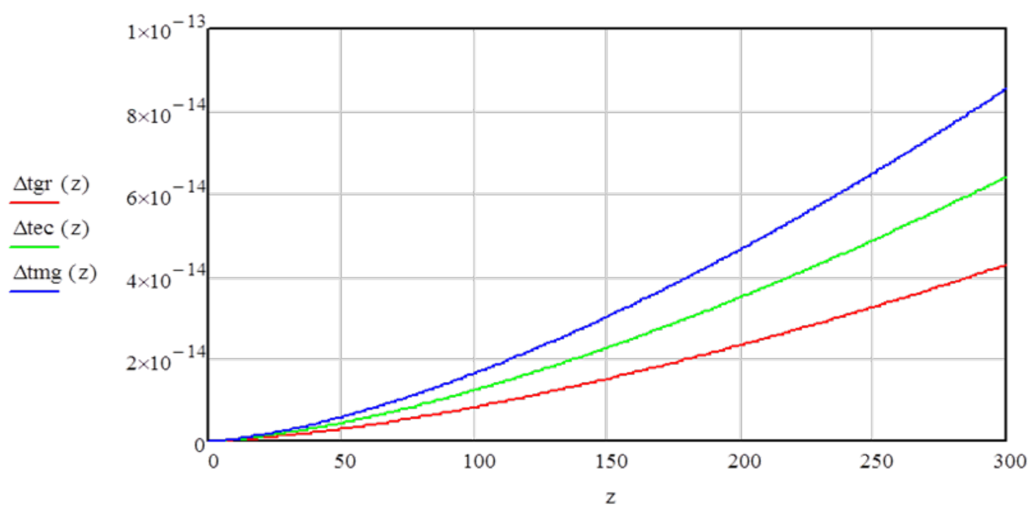

FIG.3 the graphs show the fall time departures from the Newtonian physic predictions in dependence on distance of fall for the MTG theory, the blue trace, the GRT theory, the red trace, and the Energy conservation law, the green trace.

The conservation of energy curve is probably the closest one to reality and the graphs demonstrate again that the GRT is not the correct theory of gravity.

Conclusions: The paper derived simple relations between the velocity and time for a free fall experiment in the uniform gravitational field. The velocity could be continuously measured during the fall by, for example, the radar interferometer. The fall time could also be similarly measured with a high precision.

From the derived results it is clear that it might be possible to prove the correctness of the GRT, the MTG, or the conservation of energy approach if a sufficient precision of measurement is actually achieved. Such a measurement could, for example, be carried out in the Bremen Drop Tower [3].

The experiment would have significant consequences for the GRT, because it could prove its correctness. On the other hand if the GRT is experimentally proven incorrect this finding would have a significant impact on all the theories based on the GRT such as the Big Bang and similar models of the Universe.

Finally, these results could be used to investigate the conservation of energy in the free fall and the possible loss of it due to a gravitational radiation resulting from the falling body acceleration. This topic is deferred to a future study.

References:

[1] http://gsjournal.net/Science-Journals/Research%20Papers/View/7070

[2] http://physicsessays.org/browse-journal-2/product/904-8-jaroslav-hynecek-remarks-on-the-equivalence-of-inertial-and-gravitational-masses-and-on-the-accuracy-of-einstein-s-theory-of-gravity.html

[3] https://www.bremen-tourism.de/drop-tower