Test of the General Relativity Theory by Investigating the Relativistic Free Fall in the Uniform Gravitational Field

By Jaroslav Hynecek[1]

Abstract: This paper investigates the possibility of testing the General Relativity Theory (GRT) by studying the relativistic free fall of a small test body in a uniform gravitational field. The constant improvements in technology lead to increased precision of measurements, which opens up new possibility of testing the GRT. The paper compares the free fall predictions obtained from the Newtonian physics theory, the GRT, and the Metric Theory of Gravity (MTG). It is found that it might be possible to distinguish between the GRT and the MTG theories with a reasonable confidence and thus determine by experimental methods which theory is actually correct.

Introduction: The theories describing the free fall are well understood in both; the Newtonian physics and in the General Relativity. In the GRT the inertial mass and the gravitational mass are assumed identical with identical dependencies on velocity. In the MTG, on the other hand, the gravitational mass depends on the velocity differently than the inertial mass. It is thus simple to derive equations describing the free fall velocity of a small test body in dependence on time in either theory and make comparisons with possible measurement results.

Theories: In the Newtonian physics the relation between the velocity and time is described as follows:

$$v_{nt}(t) = g \cdot t \tag{1}$$

where the symbol *g* is the gravitational acceleration, which is constant in the studied case. In the GRT the relation between the velocity *v* and time is more complicated and is derived as follows:

$$\frac{d}{dt}\left[\frac{m_0 v}{\sqrt{1-v^2/c^2}}\right] = g\,\frac{m_0}{\sqrt{1-v^2/c^2}} \tag{2}$$

where m_0 is the rest mass and *c* the speed of light in a vacuum. The left hand side of Eq.2 is the relativistic formula for the inertial force and the right hand side is the formula for the gravitational force that includes the gravitational force dependence on velocity. The formula in Eq.2 can be rearranged and simplified resulting in the following relation:

$$\frac{dv}{dt} = g(1 - v^2/c^2) \tag{3}$$

This equation can be easily integrated to obtain the formula for the falling time in dependence on velocity:

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[1] Email: jhynecek@netscape.net ;

$$2gt=\int_0^v \frac{dv}{(1-v/c)}+\int_0^v \frac{dv}{(1+v/c)}=c\ln\left(\frac{c+v}{c-v}\right) \tag{4}$$

This result can be rearranged, the velocity calculated, and finally simplified as follows:

$$v=c\frac{e^{2gt/c}-1}{e^{2gt/c}+1}=c\cdot\tanh\left(\frac{gt}{c}\right)\cong gt\left(1-\frac{1}{3}\left(\frac{gt}{c}\right)^2\right) \tag{5}$$

This is an interesting result that might be reachable by today's experiments. For example; for the fall time of *t=10.2 sec* and the Earth's gravitational acceleration of $g=9.81m/sec^2$ the term *gt=100m/sec* and *gt/c* is approximately equal to $(1/3)10^{-6}$. This can perhaps be measured today with a radar and a laser interferometer.

However, there is now also a possibility to verify that the gravitational mass is identical to the inertial mass independent of velocity. This is sometimes called the Einstein's Weak Equivalence Principle (WEP). The author of this paper has shown in previous publications [1, 2] that this is not true, that the WEP is false, and that the gravitational mass depends on velocity differently than the inertial mass.

$$m_i=\frac{m_0}{\sqrt{1-v^2/c^2}} \tag{6}$$

$$m_g=m_0\sqrt{1-v^2/c^2} \tag{7}$$

Substituting this dependency for the gravitational mass on velocity into the right hand side of Eq.2 the equivalent of Eq.3 will become:

$$\frac{dv}{dt}=g(1-v^2/c^2)^2 \tag{8}$$

This equation can also be integrated:

$$gt=\int_0^v \frac{dv}{\left(1-v^2/c^2\right)^2} \tag{9}$$

with the following result:

$$4gt=c\ln\left(\frac{c+v}{c-v}\right)+\frac{2v}{\left(1-v^2/c^2\right)} \tag{10}$$

It is not easy, however, to calculate the velocity from Eq.10, so it is more accurate to evaluate the fall time as a function of velocity, since Eq.4 and Eq.10 provide the exact solutions. The equation in FIG.2 for the velocity in the MTG theory is derived semi empirically based on the exact formula for time.

The differences between the fall times predicted by the discussed theories can be best shown graphically and by subtracting the Newtonian physics fall time. This is shown in graphs in Fig.1 and Fig2. The results are calculated for the higher velocities in FIG.1, because the Mathcad software could not handle the required accuracy for the smaller velocity values. The velocity calculated accurately for the GRT from Eq.5 is shown in FIG.2.

GRT test in a Free Fall (time is measured in dependence on the velocity)

$$g = 9.80665\frac{m}{s^2} \qquad c = 2.997925\times 10^8 \frac{m}{s} \qquad v(x) := x\cdot c \qquad x := 10^{-9}, 2\cdot 10^{-8} .. 10^{-4.7}$$

$$tgr(x) := \frac{c}{2\cdot g}\cdot \ln\left(\frac{1+x}{1-x}\right) \qquad tnt(x) := \frac{c\cdot x}{g} \qquad tmg(x) := \frac{c}{4\cdot g}\cdot\left(\ln\left(\frac{1+x}{1-x}\right) + \frac{2\cdot x}{1-x^2}\right)$$

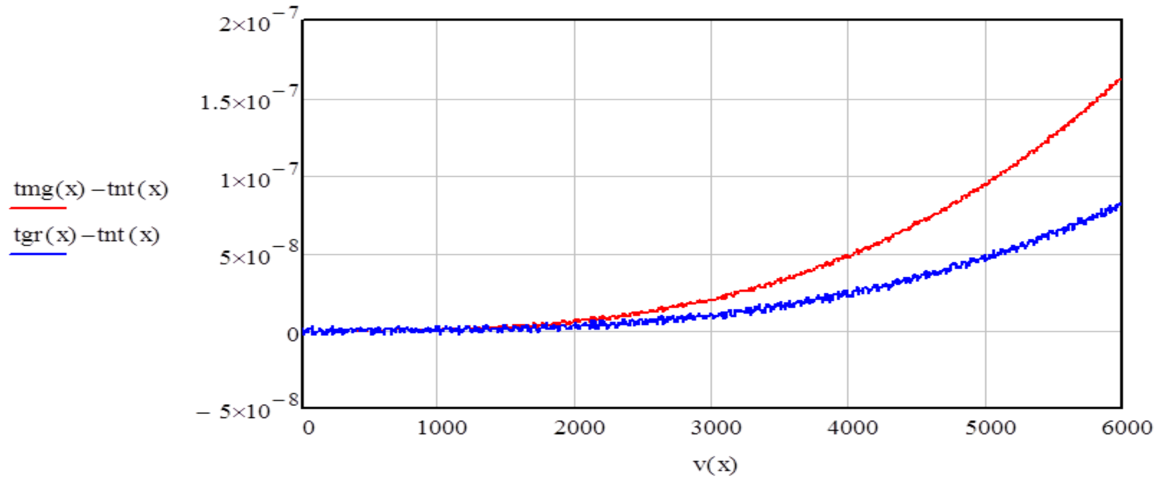

FIG.1. the graphs show the departures of the falling time in the small test body free fall experiment from the Newtonian physics prediction t=v/g. The red trace is for the MTG, the blue trace is for the GRT.

GRT test in a Free Fall (the velocity is measured in dependence on time)

$$g = 9.807\frac{m}{s^2} \qquad c = 2.998\times 10^8 \frac{m}{s} \qquad t := 0\cdot sec, 0.2 sec .. 30 sec$$

$$vnt(t) := g\cdot t \qquad vgr(t) := g\cdot t\cdot\left[1 - \frac{1}{3}\left(\frac{g\cdot t}{c}\right)^2\right] \qquad vmg(t) := g\cdot t\cdot\left[1 - \frac{2}{3}\cdot\left(\frac{g\cdot t}{c}\right)^2\right]$$

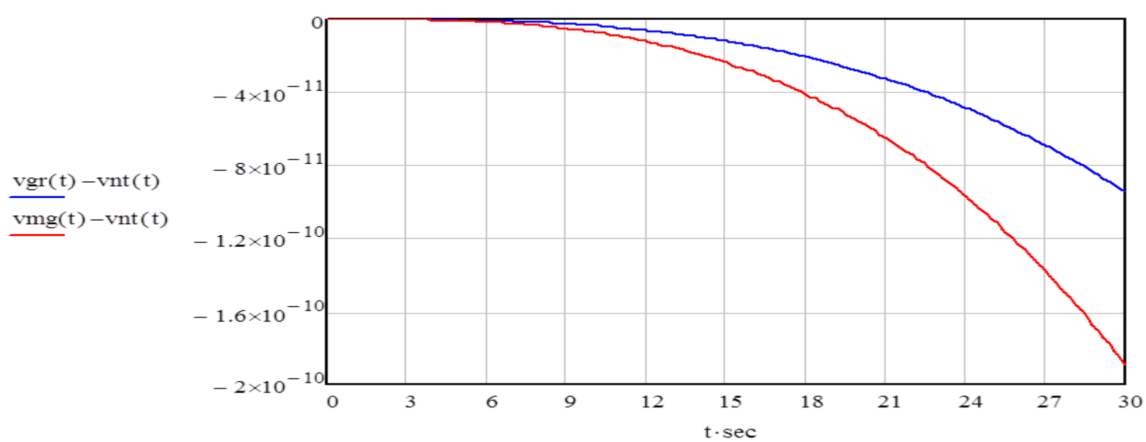

FIG.2. the graphs show the velocity departures from the Newtonian physics prediction of *v=gt* in dependence on time for the MTG (the red trace) and the GRT (the blue trace) theories.

Conclusions: The paper derived simple relations between the velocity and time for a free fall experiment in the uniform gravitational field. The velocity could be continuously measured during the fall by, for example, the radar interferometer. The fall time could also be similarly measured with a high precision.

From the derived results it is clear that it might be possible to prove the correctness of either the GRT or the MTG if a sufficient precision of measurement is actually achieved. Such a measurement could, for example, be carried out in the Bremen Drop Tower [3].

The experiment would have significant consequences for the GRT, because it could prove its correctness. On the other hand if the GRT is experimentally proven incorrect this finding would have a significant impact on all the theories based on the GRT such as the Big Bang and similar models of the Universe.

Finally, these results could be used to investigate the conservation of energy in the free fall and the possible loss of it due to a gravitational radiation resulting from the falling body acceleration. This topic is deferred to a future study.

References:

[1] http://gsjournal.net/Science-Journals/Research%20Papers/View/7070

[2] http://physicsessays.org/browse-journal-2/product/904-8-jaroslav-hynecek-remarks-on-the-equivalence-of-inertial-and-gravitational-masses-and-on-the-accuracy-of-einstein-s-theory-of-gravity.html

[3] https://www.bremen-tourism.de/drop-tower