Test of the General Relativity Theory by Investigating the Relativistic Free Fall in the Uniform Gravitational Field

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Abstract: This paper investigates the possibility of testing the General Relativity Theory (GRT) by studying the relativistic free fall of a small test body in a uniform gravitational field. The constant improvements in technology lead to increased precision of measurements, which opens up new possibility of testing the GRT. The paper compares the free fall predictions obtained from the Newtonian physics theory, the GRT, and the Metric Theory of Gravity (MTG). It is found that it might be possible to distinguish between the GRT and the MTG theories with a reasonable confidence and thus determine by experimental methods which theory is actually correct.

Introduction: The theories describing the free fall are well understood in both; the Newtonian physics and in the General Relativity. In the GRT the inertial mass and the gravitational mass are assumed identical with identical dependencies on velocity. In the MTG, on the other hand, the gravitational mass depends on the velocity differently than the inertial mass. It is thus simple to derive equations describing the free fall velocity of a small test body in dependence on time in either theory and make comparisons with possible measurement results.

Theories: In the Newtonian physics the relation between the velocity and time is described as follows:

$$v_{nt}(t) = g \cdot t \tag{1}$$

where the symbol g is the gravitational acceleration, which is constant in the studied case. In the GRT the relation between the velocity v and time is more complicated and is derived as follows:

$$\frac{d}{dt}\left[\frac{m_0 v}{\sqrt{1-v^2/c^2}}\right] = g\,\frac{m_0}{\sqrt{1-v^2/c^2}} \tag{2}$$

where m_0 is the rest mass and c the speed of light in a vacuum. The left hand side of Eq.2 is the relativistic formula for the inertial force and the right hand side is the formula for the gravitational force that includes the gravitational force dependence on velocity. The formula in Eq.2 can be rearranged and simplified resulting in the following relation:

$$\frac{dv}{dt} = g(1-v^2/c^2) \tag{3}$$

This equation can be easily integrates as follows:

$$2gt = \int_0^v \frac{dv}{(1-v/c)} + \int_0^v \frac{dv}{(1+v/c)} = c\ln\left(\frac{c+v}{c-v}\right) \tag{4}$$

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The result can be rearranged and simplified assuming that *gt* is much smaller than *c* as follows:

$$v = c\frac{e^{2gt/c}-1}{e^{2gt/c}+1} \cong \frac{2gt}{2+2gt/c} \cong gt\left(1-\frac{gt}{c}\right) \tag{5}$$

This is an interesting result that might be reachable by today's experiments. For example; for the fall time of *t=10.2 sec* and the Earth's gravitational acceleration of $g=9.81m/sec^2$ the term *gt=100m/sec* and *gt/c* is approximately equal to $(1/3)10^{-6}$. This can perhaps be relatively easily measured today with a radar and laser interferometer.

However, there is now also a possibility to verify that the gravitational mass is identical to the inertial mass independent of velocity. This is sometimes called the Einstein's Weak Equivalence Principle (WEP). The author of this paper has shown in previous publications [1, 2] that this is not true, that the WEP is false, and that the gravitational mass depends on velocity differently than the inertial mass.

$$m_i = \frac{m_0}{\sqrt{1-v^2/c^2}} \tag{6}$$

$$m_g = m_0\sqrt{1-v^2/c^2} \tag{7}$$

Substituting this dependency for the gravitational mass on velocity into the right hand side of Eq.2 the equivalent of Eq.3 will become:

$$\frac{dv}{dt} = g(1-v^2/c^2)^2 \tag{8}$$

This equation can also be integrated:

$$gt = \int_0^v \frac{dv}{\left(1-v^2/c^2\right)^2} \tag{9}$$

with the following result:

$$4gt - \frac{2v}{\left(1-v^2/c^2\right)} = c\ln\left(\frac{c+v}{c-v}\right) \tag{10}$$

Assuming again that *v* is much smaller than *c* this expression can be simplified by neglecting the second order terms in *v/c* to read:

$$\frac{c+v}{c-v} \cong e^{4gt/c}\left(1-2\frac{v}{c}\right) \tag{11}$$

and simplified further to expression:

$$\left(1+2\frac{v}{c}\right) \cong e^{4gt/c}\left(1-2\frac{v}{c}\right) \tag{12}$$

to finally obtain the simple result:

$$v \cong gt\left(1-2\frac{gt}{c}\right) \tag{13}$$

To summarize the results we therefore have the following relations between the velocity and time in a small test body free fall experiment:

$$v_{nt} = gt \tag{14}$$

$$v_{grt} \cong gt\left(1 - \frac{gt}{c}\right) \tag{15}$$

$$v_{mtg} \cong gt\left(1 - 2\frac{gt}{c}\right) \tag{16}$$

The differences between the velocities predicted by the discussed theories can be best shown graphically and by subtracting the Newtonian physics velocity. This is shown in a graph in Fig.1.

Test result predictions for the GRT and MTG Free Fall Experiment in Uniform Gravitational Field

$g = 9.807\frac{m}{s^2}$ $c = 2.998\times 10^8\frac{m}{s}$ $t := 0\cdot sec, 0.1\cdot sec..20\cdot sec$

$vnt(t) := g\cdot t$ $vsr(t) := g\cdot t\cdot\left(1 - g\cdot\frac{t}{c}\right)$ $vgm(t) := g\cdot t\cdot\left(1 - 2\cdot g\cdot\frac{t}{c}\right)$

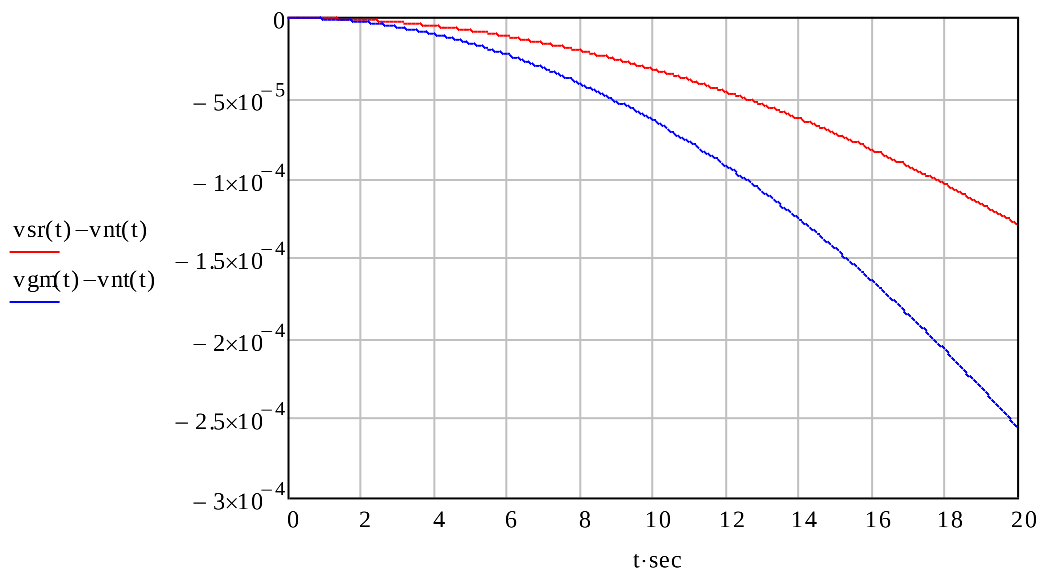

FIG.1. the graphs show the departure of the fall time in the small test body free fall experiment from the Newtonian physics prediction of *v=gt.* The red line is the prediction from the GRT where the inertial mass and the gravitational mass of the falling body have the same dependency on velocity. The blue graph is the prediction of the MTG where the gravitational mass depends on velocity differently than the inertial mass and the WEP does not hold true.

Conclusions: The paper derived simple relations between the velocity and time for a free fall experiment in the uniform gravitational field. The velocity could be continuously measured during the fall by, for example, the radar interferometer. The fall time could also be similarly measured with a high precision.

From the derived results it is clear that it might be possible to prove the correctness of either the GRT or the MTG if sufficient precision of measurement is actually achieved. Such a measurement could, for example, be carried out in the Bremen Drop Tower [3].

The experiment would have significant consequences for the GRT, because it could prove its correctness. On the other hand if the GRT is experimentally proven incorrect this finding would have a significant impact on all the theories based on the GRT such as the Big Bang and similar models of the Universe.

References:

[1] http://gsjournal.net/Science-Journals/Research%20Papers/View/7070

[2] http://physicsessays.org/browse-journal-2/product/904-8-jaroslav-hynecek-remarks-on-the-equivalence-of-inertial-and-gravitational-masses-and-on-the-accuracy-of-einstein-s-theory-of-gravity.html

[3] https://www.bremen-tourism.de/drop-tower