The quantum theory of a closed string.

Johan Noldus

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Abstract

The Virasoro problem of string theory is traced back to the non-intrinsic character of the dynamics of string theory, meaning that the dynamics depends too much upon the normal directions to the string. This is the disadvantage of the worldsheet formulation of Polyakov as well as Nambu and Goto and bcomes particularly clear in the context of covariant quantum theory.

1 Introduction.

The Virasoro problem in string theory arises most clearly in the covariant quantization where one has hermitian generators L_n with $n \in \mathbb{Z}$ which have to be regarded as constraints; that is physical states have to satisfy $L_n|\Psi\rangle = 0$ for $n \neq 0$ and $L_0|\Psi\rangle = a|\Psi\rangle$ with $a \neq 0$. The Virasoro algebra without central anomalies $c(n)$,

$$[L_n, L_m] = i(n-m)L_{n+m} + c(n-m)1$$

makes this impossible given that

$$0 = [L_n, L_{-n}]\,|\Psi\rangle = 2inL_0|\Psi\rangle = 2ina|\Psi\rangle$$

which contradicts $a \neq 0$. The "fix" of the problem is to keep the constraints $L_n|\psi\rangle = 0$ for $n > 0$ while dropping the others. This leads to physical operators changing particle species, spin and angular momentum causing all known conservation laws of particle physics to fail (but not largely in practice). The downside is that the geometrical description of the theory is totally lost at the quantum level even in a Minkowski background and that everything becomes therefore gauge dependent. This is not expected given that quantum theory works perfectly fine for flat geometries and we shall trace back the problem to the non-geometric character of quantum theory itself. In that context, the worldsheet formulation evaporates and only reparametrisations of the type $t'(t)$ and $s'(s)$ can be made such that the Virasoro problem dissapears giving rise to two mutually commuting symmetry algebra's as the *full* symmetry algebra.

2 Strings from the viewpoint of covariant quantum theory.

Given a closed string worldsheet $\gamma(t, \theta)$, we define two vectorfields $\mathbf{V} = \partial_t\gamma(t, s)$ and $\mathbf{Z} = \partial_s\gamma(t, s)$ where $t \in [0, T]$ and $s \in [0, 2\pi]$ with periodic boundary conditions; obviously $[\mathbf{V}, \mathbf{Z}] = 0$.

The law one is looking for clearly is of the kind

$$\nabla_{\mathbf{V}} \mathbf{V} = \mathbf{F}(\mathbf{V}, \mathbf{Z}, \nabla_{\mathbf{Z}} \mathbf{Z}, \mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{R}(\mathbf{Z}, \mathbf{V})\mathbf{Z}, \mathbf{h})$$

where $\mathbf{A} = \nabla_{\mathbf{Z}} \mathbf{Z}$ is a kind of acceleration, $\mathbf{h}$ the, possibly degenerate, metric on the string and all Riemann curvature terms involve the intrinsic geometry of the string. The problem so far is that the velocity field $\mathbf{V}$ is randomly chosen and that therefore it is desirable to impose constraints on $\nabla_{\mathbf{V}} \mathbf{Z}$. We have basically two types: (a) one involving the extrinsic geometry and the latter only the intrinsic geometry. In other words, we have (I)

$$\nabla_{\mathbf{V}} \mathbf{Z} = \mathbf{G}(\mathbf{V}, \mathbf{Z}, \nabla_{\mathbf{Z}} \mathbf{Z}, \mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{R}(\mathbf{Z}, \mathbf{V})\mathbf{Z}, \mathbf{h})$$

or (II)

$$\mathbf{g}(\nabla_{\mathbf{V}} \mathbf{Z}, \mathbf{V}) = P(\mathbf{V}, \mathbf{Z}, \nabla_{\mathbf{Z}} \mathbf{Z}, \mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{R}(\mathbf{Z}, \mathbf{V})\mathbf{Z}, \mathbf{h})$$

whereas a condition of the kind

$$\mathbf{g}(\nabla_{\mathbf{V}} \mathbf{Z}, \mathbf{Z}) = Q(\mathbf{V}, \mathbf{Z}, \nabla_{\mathbf{Z}} \mathbf{Z}, \mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{R}(\mathbf{Z}, \mathbf{V})\mathbf{Z}, \mathbf{h})$$

is meaningless given that consistency would bring it down to an algebraic condition on $\mathbf{g}(\mathbf{Z}, \mathbf{Z})$. Such theories are usually empty and therefore not interesting at all.

One has to demand now that the dynamics preserves the constraint; that is

$$\nabla_{\mathbf{V}} \nabla_{\mathbf{V}} \mathbf{Z} = \nabla_{\mathbf{V}} \mathbf{G} = \mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V} + \nabla_{\mathbf{Z}} \mathbf{F}$$

a consistency condition. Note that

$$\nabla_{\mathbf{Z}}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}) = (\nabla_{\mathbf{Z}} \mathbf{R})(\mathbf{V}, \mathbf{Z})\mathbf{V} - \mathbf{R}(\mathbf{G}, \mathbf{Z})\mathbf{V} + \mathbf{R}(\mathbf{V}, \mathbf{A})\mathbf{V} - \mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{G}$$

which can be reduced to, by means of the second Bianchi identity to

$$\nabla_{\mathbf{Z}}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}) = (\nabla_{\mathbf{V}} \mathbf{R})(\mathbf{V}, \mathbf{Z})\mathbf{Z} + \mathbf{R}(\mathbf{Z}, \mathbf{G})\mathbf{V} + \mathbf{R}(\mathbf{V}, \mathbf{A})\mathbf{V} - \mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{G}.$$

On the other hand, a similar compuation gives that

$$\nabla_{\mathbf{V}}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{Z}) = (\nabla_{\mathbf{V}} \mathbf{R})(\mathbf{V}, \mathbf{Z})\mathbf{Z} + \mathbf{R}(\mathbf{F}, \mathbf{Z})\mathbf{Z} + \mathbf{R}(\mathbf{V}, \mathbf{G})\mathbf{Z} + \mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{G}$$

where no second Bianchi identity has been used and the other terms do not allow for comparison between $\mathbf{F}$ and $\mathbf{G}$ by means of the latter identity. Contractions with the spacetime metric do allow for further use of the first Bianchi identity and gives rise to a larger margin to construct stringy laws. Hence, in light of the conservation law for the constraint,

$$\mathbf{F}(\mathbf{V}, \mathbf{Z}, \nabla_{\mathbf{Z}} \mathbf{Z}, \mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{h})$$

and

$$\mathbf{G}(\mathbf{V}, \mathbf{Z}, \nabla_{\mathbf{Z}} \mathbf{Z}, \mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{Z}, \mathbf{h})$$

with

$$\frac{\delta \mathbf{F}}{\delta \mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}} = \frac{\delta \mathbf{G}}{\delta \mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{Z}}.$$

We also have that
$$\frac{\delta \mathbf{G}}{\delta \mathbf{Z}} \triangle \mathbf{G} + \frac{\delta \mathbf{G}}{\delta \mathbf{V}} \triangle \mathbf{F} + \frac{\delta \mathbf{G}}{\delta \mathbf{A}} \triangle \nabla_{\mathbf{V}} \mathbf{A}$$
$$= \frac{\delta \mathbf{F}}{\delta \mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V}} \triangle (-\mathbf{R}(\mathbf{G},\mathbf{Z})\mathbf{V} + \mathbf{R}(\mathbf{V},\mathbf{A})\mathbf{V} - \mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{G} - \mathbf{R}(\mathbf{F},\mathbf{Z})\mathbf{Z} - \mathbf{R}(\mathbf{V},\mathbf{G})\mathbf{Z} - \mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{G})$$
$$+ \frac{\delta \mathbf{F}}{\delta \mathbf{Z}} \triangle \mathbf{A} - \frac{\delta \mathbf{F}}{\delta \mathbf{V}} \triangle \mathbf{G} + \frac{\delta \mathbf{F}}{\delta \mathbf{A}} \triangle \nabla_{\mathbf{Z}} \mathbf{A} + \mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V}.$$
From a generalist point of view, this would suggest
$$-\mathbf{R}(\mathbf{G},\mathbf{Z})\mathbf{V} + \mathbf{R}(\mathbf{V},\mathbf{A})\mathbf{V} - \mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{G} - \mathbf{R}(\mathbf{F},\mathbf{Z})\mathbf{Z} - \mathbf{R}(\mathbf{V},\mathbf{G})\mathbf{Z} - \mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{G} = 0$$
as well as
$$\frac{\delta \mathbf{G}}{\delta \mathbf{Z}} \triangle \mathbf{G} + \frac{\delta \mathbf{G}}{\delta \mathbf{V}} \triangle \mathbf{F} + \frac{\delta \mathbf{G}}{\delta \mathbf{A}} \triangle \nabla_{\mathbf{V}} \mathbf{A} = \frac{\delta \mathbf{F}}{\delta \mathbf{Z}} \triangle \mathbf{A} - \frac{\delta \mathbf{F}}{\delta \mathbf{V}} \triangle \mathbf{G} + \frac{\delta \mathbf{F}}{\delta \mathbf{A}} \triangle \nabla_{\mathbf{Z}} \mathbf{A} + \mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V}.$$
It is immediately seen that, in general and independent of this ansatz,
$$\frac{\delta \mathbf{F}}{\delta \mathbf{A}} = 0$$
given that higer spatial derivatives do not occur elsewhere in the formula and therefore
$$\mathbf{F}(\mathbf{V},\mathbf{Z},\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V})$$
given that we have already neglected $\mathbf{h}$. On the other hand
$$\nabla_{\mathbf{V}} \mathbf{A} = \mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{Z} + \nabla_{\mathbf{Z}} \mathbf{G}$$
which implies that
$$\frac{\delta \mathbf{G}}{\delta \mathbf{A}} = 0$$
due to consistency given that no algebraic relations are allowed for between higher spatial derivatives. Hence,
$$\mathbf{G}(\mathbf{V},\mathbf{Z},\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{Z})$$
and we conclude from the remaining master equation that only intrinsic contractions of the Riemann tensor with $\mathbf{V},\mathbf{Z}$ are allowed for to eliminate the nasty
$$\mathbf{R}(\mathbf{V},\mathbf{A})\mathbf{V}$$
term. This however happens in two different ways $\mathbf{g}(\mathbf{R}(\mathbf{V},\mathbf{A})\mathbf{V},\mathbf{V}) = 0$ identically whereas contractions of the kind
$$\mathbf{g}(\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V},\mathbf{Z})$$
require a balancing between
$$\frac{\delta \mathbf{F}}{\delta \mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V}} \triangle \mathbf{R}(\mathbf{V},\mathbf{A})\mathbf{V}$$
and
$$\frac{\delta \mathbf{F}}{\delta \mathbf{Z}} \triangle \mathbf{A}$$

in the sense that they have to be equal to one and another due to the first Bianchi identity. As a conclusion, we further specify that

$$\mathbf{F} = X(\mathbf{V}, \mathbf{Z}, \mathbf{g}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{Z}))\mathbf{V} + Y(\mathbf{V}, \mathbf{Z}, \mathbf{g}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{Z}))\mathbf{Z}$$

which automatically satifies this requirement by means of symmetries of the Riemann tensor. This further limits

$$\mathbf{G} = R(\mathbf{V}, \mathbf{Z}, \mathbf{g}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{Z}, \mathbf{Z}))\mathbf{V} + S(\mathbf{V}, \mathbf{Z}, \mathbf{g}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{Z}, \mathbf{V}))\mathbf{Z}$$

with

$$\frac{\delta X}{\delta \mathbf{g}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{Z})} = -\frac{\delta R}{\delta \mathbf{g}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{Z}, \mathbf{Z})}$$

and

$$\frac{\delta Y}{\delta \mathbf{g}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{Z})} = -\frac{\delta S}{\delta \mathbf{g}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{Z}, \mathbf{Z})}.$$

This shows that ∇ cannot be the Christoffel connection of a Riemannian metric and $\mathbf{R}$ its associated Riemann tensor. Although the Riemann tensor of any connection satisfies the second Bianchi identies, the first Bianchi identies and the associated symmetries of the Riemann tensor follow from the metric and torsionless character. Therefore, the connection needs torsion for the subsequent analysis to hold.

Given that one would expect only curvature to occur in the acceleration law of the string and moreover that the acceleration is of the geodesic type so that the sring t coordinate is nothing but a rescaling of the geodesic time, reparamentrization invariance has to be given up in the light of the fact that no $g(\mathbf{Z}, \mathbf{Z})$ or $g(\mathbf{Z}, \mathbf{V})$ terms may occur due to an inappropriate appearance of $\mathbf{A}$ in the

$$\frac{\delta \mathbf{F}}{\delta \mathbf{Z}} \triangle \mathbf{A}$$

term. Therefore,

$$\nabla_{\mathbf{V}} \mathbf{V} = \frac{c}{L^3} \mathbf{g}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{Z})\mathbf{V}$$

where c is the speed of light and L has units of meters. This is the correct way of looking at it given that the curves are ordinary geodesics again but then reparametrized in a way as to balance the tidal forces; t can be reparametrized but generally speaking only *one* worldline of a point of the circle can have unit time parametrization. This is a salient feature given that strings will not induce superluminal effects in this way by means of its nonlocal character. In particular, we have that if x is a point past to the string and $\mathbf{V}$ is a future pointing timelike vectorfield, then the entire string will remain within $I^+(x)$. Finally,

$$\nabla_{\mathbf{V}} \mathbf{Z} = K(\mathbf{g}(\mathbf{V}, \mathbf{Z}), \mathbf{g}(\mathbf{V}, \mathbf{V}), \mathbf{g}(\mathbf{Z}, \mathbf{Z}))\mathbf{V} +$$

$$-\frac{c}{L^3} \mathbf{g}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{Z}, \mathbf{Z}))\mathbf{V} + L(\mathbf{g}(\mathbf{V}, \mathbf{Z}), \mathbf{g}(\mathbf{V}, \mathbf{V}), \mathbf{g}(\mathbf{Z}, \mathbf{Z}))\mathbf{Z}.$$

The consistency equation has now been reduced to

$$\frac{\delta \mathbf{G}}{\delta \mathbf{Z}} \triangle \mathbf{G} + \frac{\delta \mathbf{G}}{\delta \mathbf{V}} \triangle \mathbf{F} =$$

$$-\frac{\delta \mathbf{F}}{\delta \mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V}}(\mathbf{R}(\mathbf{G},\mathbf{Z})\mathbf{V}+\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{G}+\mathbf{R}(\mathbf{F},\mathbf{Z})\mathbf{Z}+\mathbf{R}(\mathbf{V},\mathbf{G})\mathbf{Z}+\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{G})$$

$$-\frac{\delta \mathbf{F}}{\delta \mathbf{V}}\triangle \mathbf{G}+\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V}.$$

As expected one page ago, this equation can only have solution in case $\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V}$ equals its projection on the string worldsheet determined by the $\mathbf{V},\mathbf{Z}$ plane which is in general impossible except for Einstein spaces. Therefore, it might be possible to develop a type I string theory for Einstein spaces with torsion but given such restriction it is utterly clear that type II is the only physical case.

Here, we might try to arrive at a theory with equation of motion

$$\nabla_{\mathbf{V}}\mathbf{V}=\mathbf{F}$$

and constraint equations

$$\begin{aligned}
\mathbf{g}(\mathbf{V},\nabla_{\mathbf{V}}\mathbf{Z}) &= \alpha\mathbf{g}(\mathbf{V},\mathbf{G})\\
\mathbf{g}(\mathbf{E}_i,\nabla_{\mathbf{V}}\mathbf{Z}) &= \alpha\mathbf{g}(\mathbf{E}_i,\mathbf{G})\\
\frac{1}{2}\mathbf{g}(\nabla_{\mathbf{V}}\mathbf{Z},\nabla_{\mathbf{V}}\mathbf{Z}) &= \alpha\mathbf{g}(\nabla_{\mathbf{V}}\mathbf{Z},\mathbf{G})
\end{aligned}$$

where E_i is a $n-2$ bein orhogonal to $\mathbf{V},\mathbf{Z}$. In vector language, this gives

$$\nabla_{\mathbf{V}}\mathbf{Z}-\alpha\mathbf{G}=\mathbf{W}$$

with $\mathbf{W}$ perpendicular to the $n-1$ plane defined by $\mathbf{V},\mathbf{E}_i$. Moreover,

$$\mathbf{g}(\mathbf{W}-\alpha\mathbf{G},\nabla_{\mathbf{V}}\mathbf{Z})=0.$$

Hence,

$$\mathbf{g}(\mathbf{W},\mathbf{W})=\alpha^2\mathbf{g}(\mathbf{G},\mathbf{G}).$$

The structure of these equations is as such that they are preserved during time evolution. Time evolution of the first gives

$$\mathbf{g}(\mathbf{F},\nabla_{\mathbf{V}}\mathbf{Z})+\mathbf{g}(\mathbf{V},\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V})+\mathbf{g}(\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{F})=$$

$$\alpha\mathbf{g}(\mathbf{F},\mathbf{G})+\alpha\mathbf{g}(\mathbf{V},\nabla_{\mathbf{V}}\mathbf{G})$$

which generically leads to

$$\mathbf{g}(\mathbf{F},\nabla_{\mathbf{V}}\mathbf{Z}-\alpha\mathbf{G})=0,\ \ \mathbf{g}(\mathbf{V},\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V}+\nabla_{\mathbf{Z}}\mathbf{F}-\alpha\nabla_{\mathbf{V}}\mathbf{G})=0.$$

The other equations are

$$\mathbf{g}(\mathbf{E}_i,\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V}+\nabla_{\mathbf{Z}}\mathbf{F}-\alpha\nabla_{\mathbf{V}}\mathbf{G})=0$$

and

$$\mathbf{g}(\nabla_{\mathbf{V}}\mathbf{Z},\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V}+\nabla_{\mathbf{Z}}\mathbf{F}-\alpha\nabla_{\mathbf{V}}\mathbf{G})=0$$

supplemented with

$$\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{F}+\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V},\mathbf{G})=0$$

which obviously gives the same problems as before. It appears some more delicate analysis is necessary: clearly, one would like

$$\nabla_{\mathbf{V}}\mathbf{V} = \mathbf{F}(\mathbf{V}, \mathbf{Z}, \mathbf{A}, \mathbf{g}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{Z}))$$

given that $\mathbf{Z}$ is chosen according to arc length and evolution should only depend upon the intrinsic geometry and only as far on the directions perpendicular to the infinitesimal string surface as the acceleration goes. That is

$$\mathbf{g}(\mathbf{Z}, \mathbf{A}) = 0$$

and one would like to preserve this property under evolution, in either keep it as a constraint. Time evolution gives

$$\mathbf{g}(\nabla_{\mathbf{V}}\mathbf{Z}, \mathbf{A}) + \mathbf{g}(\mathbf{Z}, \nabla_{\mathbf{Z}}\nabla_{\mathbf{V}}\mathbf{Z}) = 0 = \nabla_{\mathbf{Z}}(\mathbf{g}(\mathbf{Z}, \nabla_{\mathbf{V}}\mathbf{Z})).$$

Therefore, we should add as constraint

$$\mathbf{g}(\mathbf{Z}, \nabla_{\mathbf{V}}\mathbf{Z}) = \nabla_{\mathbf{Z}}\mathbf{g}(\mathbf{Z}, \mathbf{V}) - \mathbf{g}(\mathbf{A}, \mathbf{V}) = 0$$

which follows from

$$\mathbf{g}(\mathbf{Z}, \mathbf{V}) = \mathbf{g}(\mathbf{A}, \mathbf{V}) = 0.$$

In ordinary string theory in flat Minkowski $\mathbf{F} = \mathbf{A}$ for a Lorentzian flat worldsheet metric and $\mathbf{F} = -\mathbf{A}$ for a Riemannian worldsheet metric and the former two conditions give by means of the equation of motion

$$\mathbf{g}(\mathbf{Z}, \mathbf{V}) = \nabla_{\mathbf{V}}\mathbf{g}(\mathbf{V}, \mathbf{V}) = 0.$$

The first of those is the usual Virasoro constraint

$$\partial_t\gamma.\partial_s\gamma = 0$$

whereas the second equals

$$\partial_t(\partial_t\gamma.\partial_t\gamma) = 0$$

which is the time derivative of one of the other constraints. Our original constraint was

$$\partial_s(\partial_s\gamma.\partial_s\gamma) = 0$$

which is the space derivative of the last Virasoro constraint. It is now possible to *impose* the constraints

$$\mathbf{g}(\mathbf{Z}, \mathbf{V}) = 0 = \mathbf{g}(\mathbf{V}, \mathbf{V}) = \mathbf{g}(\mathbf{A}, \mathbf{V})$$

where we have eliminated one integration function depending upon s only. One could leave a positive integration constant

$$\mathbf{g}(\mathbf{V}, \mathbf{V}) = \gamma$$

so that strings would move on timelike curves excluding therefore massless particles in their description. Similarly, we could demand that

$$\mathbf{g}(\mathbf{Z}, \mathbf{Z}) = 0$$

where we have eliminated a space integration constant β arising from

$$\nabla_{\mathbf{Z}}\mathbf{g}(\mathbf{Z},\mathbf{Z}) = 2\mathbf{g}(\mathbf{A},\mathbf{Z}) = 0.$$

We show now that the remaining three constraints close under time evolution

$$\nabla_{\mathbf{V}}\mathbf{g}(\mathbf{V},\mathbf{Z}) = \mathbf{g}(\mathbf{F},\mathbf{Z}) + \mathbf{g}(\mathbf{V},\nabla_{\mathbf{V}}\mathbf{Z}) = \mathbf{g}(\mathbf{F},\mathbf{Z})$$

where we have used the torsionless character of the Levi Civita connection and the commuting of the coordinate fields. This does not impose any constraints on the $\mathbf{F}$ field given the constraints. Finally

$$\nabla_{\mathbf{V}}\mathbf{g}(\mathbf{V},\mathbf{V}) = 2\mathbf{g}(\mathbf{V},\mathbf{F}) = 0$$

for similar reasons and

$$\nabla_{\mathbf{V}}\nabla_{\mathbf{V}}\mathbf{g}(\mathbf{V},\mathbf{V}) = 2\nabla_{\mathbf{V}}\mathbf{g}(\mathbf{F},\mathbf{V}) = 2\mathbf{g}(\nabla_{\mathbf{V}}\mathbf{F},\mathbf{V}) + 2\mathbf{g}(\mathbf{F},\mathbf{F}) = 0$$

where the last equality only holds in case

$$\mathbf{g}(\kappa(\mathbf{g}(\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V},\mathbf{Z}),\mathbf{A})\nabla_{\mathbf{V}}\mathbf{Z},\mathbf{V}) + \mathbf{g}(\delta(\mathbf{g}(\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V},\mathbf{Z}),\mathbf{A})\nabla_{\mathbf{V}}\mathbf{A},\mathbf{V}) + \delta^2(\mathbf{g}(\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V},\mathbf{Z}),\mathbf{A})\mathbf{g}(\mathbf{A},\mathbf{A}) = 0.$$

In particular, the equation reduces by means of the torsionless character to

$$\delta(\mathbf{g}(\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V},\mathbf{Z}),\mathbf{A})\,[\mathbf{g}(\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{Z},\mathbf{V}) + \mathbf{g}(\nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{V})] + \delta^2(\mathbf{g}(\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V},\mathbf{Z}),\mathbf{A})\mathbf{g}(\mathbf{A},\mathbf{A}) = 0$$

where

$$\mathbf{g}(\nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{V}) = -\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{V}) = 2\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{V})\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{K}) + 2\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{Z})\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{L})$$

$$-\sum_{i=1}^{n-4}\eta_{ii}(\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{E}_i))^2$$

where η_{ij} is an $n-4$ dimensional Lorentzian vielbein in the remaining orthogonal space assuming a spacetime metric with three time directions and linearly independent $\mathbf{V},\mathbf{Z}$. Obviously, the helicity components only remain in the sense that

$$\mathbf{g}(\nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{V}) = -\sum_{i=1}^{n-4}\eta_{ii}(\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{E}_i))^2.$$

There is something extremely important about $\mathbf{g}(\nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{V})$ which we expect to happen and that is that it equals

$$\pm\mathbf{g}(\mathbf{A},\mathbf{A}).$$

In general, there is a deep connection with the equation of motion so that it is wise to suggest that

$$\delta(\mathbf{A},\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{V}),\mathbf{g}(\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V},\mathbf{Z}))$$

which we are free to since it does not intervene with our previous analysis. That is, we have found a consistent theory with constraints

$$\mathbf{g}(\mathbf{Z},\mathbf{Z}) = \mathbf{g}(\mathbf{V},\mathbf{Z}) = \mathbf{g}(\mathbf{V},\mathbf{V}) = \mathbf{g}(\mathbf{F},\mathbf{V}) = 0$$

supplied with the condition

$$\mathbf{F}(\mathbf{A}, \mathbf{g}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{Z}), \mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{V})) = \frac{\mathbf{g}(\mathbf{R}(\mathbf{V}, \mathbf{Z})\mathbf{V}, \mathbf{Z}) + \mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{V})}{\mathbf{g}(\mathbf{A}, \mathbf{A})}.$$

Classically, these equations cannot be solved for in a spacetime with a Lorentzian signature unless $\mathbf{V} \sim \mathbf{Z}$ which is rather boring and actually occurs in "string theory". It therefore appears clear that standard string theory would require at least two independent time directions which would endager the whole edifice of causality and make no sense at all unless those time directions are compactified of some sort and far beyond our scale of observation. Hence, a fibre structure is needed for the spacetime manifold with a four dimensional Lorentzian base manifold and Lorentzian fibre. In standard quantum theory of the string with a Lorentzian world sheet and spacetime metric, one solves for right and left moving strings which should be kept strictly separate to impose the constraints. Alas, such line of reasoning is inconsistent with the Heisenberg commutation relations given that $\mathbf{V}$ and $\mathbf{Z}$ should fluctuate independently; hence, the Virasoro problem. In our setup, there are two possibilities, either one keeps $\beta < 0$ so that $\mathbf{g}(\mathbf{Z}, \mathbf{Z}) = \beta$ and therefore $\mathbf{Z}$ is always spacelike involving $\mathbf{F}(\mathbf{V}, \mathbf{A})$ *or* one goes over to the higher time formalism such that the projection of $\mathbf{Z}$ on the base manifold is spacelike and varying in case the fibre is one dimensional and the standard Virasoro picture with a more general force field may hold.

We now discuss these things in the next section. The real reason why in a general spacetime the left and right moving modes cannot be defined is due to the dependency of the metric on the string world sheet coordinates and hence world sheet coordinates must be endowed with a spacetime geometric meaning which is precisely what happens here. In the standard Minkowski quantization, this feature does not arise and therefore the standard procedure gives the wrong results due to the wrong signature of spacetime. Finally, *if* one would insist upon *one* string theory to describe the entire particle spectrum, then it is utterly clear that the ultrahyperbolic fibre picture with a Riemannian flat world sheet metric is obliged for. It is this picture we shall further develop in subsequent chapter.

3 Quantization of the string.

In ordinary particle theory, we look for the little group of the momentum vector which for massive particles equals $SO(3)$ and for massless particles E_2, the Euclidean group in two dimensions at least if the spacetime dimension equals four. To have a similar thing in string theory, we need to go to $7 = 2 + 5$ dimensions where the little group is $SO(3) \times \mathbb{R}^3 = E_3$ taking into account that the helicity has to be perpendicular to $\mathbf{V}$ as well as $\mathbf{Z}$. This provides one with a richer particle spectrum and suggests that massive particles can travel at the speed of light in case $\mathbf{Z}$ resides exclusively in the fibre. The velocity field, being timelike, then has the speed of light in the base four dimensional spacetime which is a contradiction to standard particle theory. In plain words, the string is entirely in the fibre and behaves as a point particle from the point of view of the observer; this is a violation to Einstein's mass formula and in conflict to special relativity. It is clear that the string veloity needs to have a timelike component in the fibre manifold for a massive particle to arise; clearly, mass quantization can only occur when the bulk momenta are quantized which necessitates closed timelike curves in the fibre. Therefore, mass and in particular the mass gap, are dynamical quantities closely related to the microscopic structure of the timelike fibre which are in turn determined by the string length which suggests that the fibre is incapable of blowing up given a fixed string length scale. On the other hand, given that the fibre winding number cannot increase and the string length is fixed, formation of singularities of the bulk is excluded unless the string expands drastically in the base manifold in which case it beoomes extremely heavy as an ordinary base spacetime particle (the mass increases much more if there are extra spatial dimensions in the fibre added). Therefore, local mass eigenstates, in the sense explained below, appear to be stable unless the dynamics forces the strings to blow up in which case the mass runs astray.

In order to go over to the quantum theory, let us first define the suitable strings $\zeta(t,s)$ where $t \in \mathbb{R}^+$ and $\zeta(0,s) \sim S^1$. Let $\mathbf{R}$ be the projection of $\mathbf{Z}$ on the plane perpendicular to $\mathbf{T} = \partial_t \zeta(t,s)$. Such hyperplane is always an $n-1$ dimensional Lorentzian, ultrahyperbolic or of a null-Lorentzian geometry. We propose now the following dragging law

$$\nabla_{\mathbf{T}}\mathbf{T} = \mathbf{F}(\mathbf{T},\mathbf{Z},\mathbf{A},\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V}))$$

with

$$\mathbf{g}(\mathbf{V},\mathbf{T}) = \mathbf{g}(\mathbf{F}(\mathbf{T},\mathbf{Z},\mathbf{A},\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V})),\mathbf{V}) =$$

$$\mathbf{g}(\mathbf{Z},\mathbf{Z}) = \mathbf{g}(\mathbf{T},\mathbf{Z}) = \mathbf{g}(\mathbf{Z},\mathbf{V}) = \mathbf{g}(\mathbf{V},\mathbf{V}) = \mathbf{g}(\mathbf{T},\mathbf{A}) = 0$$

and

$$\nabla_{\mathbf{T}}\mathbf{V} = \mathbf{F}(\mathbf{V},\mathbf{R},\mathbf{A},\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V})).$$

Also, it is necessary to make $\mathbf{g}(\mathbf{T},\mathbf{T}) = \lambda$ into a constant with respect to $\mathbf{T}$ and $\mathbf{Z}$. Before we check that these constraints are preserved under evolution $\nabla_{\mathbf{T}}$, we must remark that it requires at least a $3+5$ picture of spacetime or a four dimensional Lorentzian base space with a three dimensional negative Lorentzian fibre. This may sound unappealing but it has some potential to explain the origin of time in our universe as a kind of symmetry breaking due to small extra time dimensions.

First of all, it is easy to convince oneself that *all* those constraints are necessary; in case one chooses the constraint

$$\mathbf{g}(\mathbf{V},\mathbf{T}) = 0$$

to start with, then time derivation gives

$$\mathbf{g}(\mathbf{F}(\mathbf{V},\mathbf{R},\mathbf{A},\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V})),\mathbf{T}) + \mathbf{g}(\mathbf{V},\mathbf{F}(\mathbf{T},\mathbf{Z},\mathbf{A},\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V}))) = 0$$

which suggests

$$\mathbf{g}(\mathbf{V},\mathbf{Z}) = \mathbf{g}(\mathbf{V},\mathbf{F}(\mathbf{T},\mathbf{Z},\mathbf{A},\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V}))) = 0$$

and

$$\mathbf{g}(\mathbf{T},\mathbf{A}) = 0$$

as well. To be precise, the string we have so far is a worldsheet $\zeta(t,s)$ where $\mathbf{T} = \partial_t \zeta(t,s)$ and $\mathbf{Z} = \partial_s \zeta(t,s)$; $\mathbf{V}$ is treated as a vectorfield along $\zeta(t,s)$ and serves to infinitesimally thicken the worldsheet $\zeta(t,s,r)$ where $\mathbf{V} = \partial_r \zeta(t,s,r)$ for $r \in (-\epsilon, \epsilon)$ and all previous equations only hold for $r = 0$. We are now in position to compute the time evolution of $\mathbf{g}(\mathbf{V},\mathbf{Z}) = 0$ which results in

$$\mathbf{g}(\mathbf{F}(\mathbf{V},\mathbf{R},\mathbf{A},\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V})),\mathbf{Z}) + \mathbf{g}(\mathbf{V},\nabla_{\mathbf{T}}\mathbf{Z}) = 0$$

which suggests $\mathbf{F}$ not to depend upon $\mathbf{Z}$, $\mathbf{g}(\mathbf{A},\mathbf{Z}) = 0$ and $\mathbf{g}(\mathbf{V},\nabla_{\mathbf{T}}\mathbf{Z}) = 0$. The preservation in time of the former condition gives

$$\mathbf{g}(\nabla_{\mathbf{Z}}\nabla_{\mathbf{T}}\mathbf{Z},\mathbf{Z}) + \mathbf{g}(\mathbf{A},\nabla_{\mathbf{T}}\mathbf{Z}) = 0$$

which results again in

$$\nabla_{\mathbf{Z}}\mathbf{g}(\nabla_{\mathbf{T}}\mathbf{Z},\mathbf{Z}) = 0.$$

This constraint is as usual replaced by considering that

$$\mathbf{g}(\nabla_{\mathbf{T}}\mathbf{Z},\mathbf{Z}) = 0$$

giving rise to

$$\mathbf{g}(\mathbf{T},\mathbf{Z}) = \rho$$

which is equivalent to

$$\nabla_{\mathbf{T}}\mathbf{g}(\mathbf{Z},\mathbf{Z}) = 0.$$

Time evolution of the former constraint gives rise to $\rho = 0$ or a further functional restriction to $\mathbf{F}(\mathbf{A},\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V}))$ and $\mathbf{g}(\mathbf{T},\mathbf{T}) = \lambda$. This constraint is only preserved only in case $\mathbf{g}(\mathbf{T},\mathbf{T}) = 0$ or in case the former restriction on the force field holds which is the minimal case in a sense. The former condition is clearly nonsensical so that *quantum theory* imposes a restriction on $\mathbf{F}$ which was not available classically; that is,

$$\mathbf{F}(\mathbf{A},\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V})).$$

We add as constraint $\mathbf{g}(\mathbf{V},\nabla_{\mathbf{T}}\mathbf{Z}) = 0$ which is preserved in time if and only if

$$\mathbf{g}(\mathbf{F}(\mathbf{A},\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V})),\nabla_{\mathbf{T}}\mathbf{Z}) +$$

$$\mathbf{g}(\mathbf{V},\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{T}) + \mathbf{g}(\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{F}(\mathbf{A},\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V})))$$

which reduces further to

$$\delta(\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V}))\mathbf{g}(\mathbf{A},\nabla_{\mathbf{T}}\mathbf{Z}) + \mathbf{g}(\mathbf{V},\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{T}) +$$

$$\mathbf{g}(\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{F}(\mathbf{A},\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V}))).$$

Further calculation gives the following equation

$$\mathbf{g}(\mathbf{A},\delta(\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V}))(\nabla_{\mathbf{T}}\mathbf{Z} + \nabla_{\mathbf{V}}\mathbf{Z})) + \mathbf{g}(\mathbf{V},\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{T}) = 0.$$

Using the thickening of the string away from $r = 0$ we can consider the above formula as a constraint for the definition of $\nabla_{\mathbf{V}}\mathbf{Z}$.

Remains to further investigate the consistency of the two remaining constraints

$$\nabla_{\mathbf{T}}\mathbf{g}(\mathbf{T},\mathbf{A}) = 0 = \delta(\mathbf{g}(\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V}),\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{V}))\mathbf{g}(\mathbf{A},\mathbf{A}) + \mathbf{g}(\mathbf{T},\mathbf{R}(\mathbf{T},\mathbf{Z})\mathbf{Z}) + \mathbf{g}(\mathbf{T},\nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{T}).$$

Here,

$$\mathbf{g}(\mathbf{T}, \nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{T}) = -\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T}, \nabla_{\mathbf{Z}}\mathbf{T}) = -\nabla_{\mathbf{T}}\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T}, \mathbf{Z}) + \mathbf{g}(\nabla_{\mathbf{T}}\nabla_{\mathbf{Z}}\mathbf{T}, \mathbf{Z}) = \mathbf{g}(\nabla_{\mathbf{T}}\nabla_{\mathbf{Z}}\mathbf{T}, \mathbf{Z})$$

which equals

$$\mathbf{g}(\mathbf{R}(\mathbf{T}, \mathbf{Z})\mathbf{T}, \mathbf{Z}) + \mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{F}(\mathbf{A}, \mathbf{g}(\mathbf{R}(\mathbf{T}, \mathbf{Z})\mathbf{Z}, \mathbf{V}), \mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T}, \nabla_{\mathbf{Z}}\mathbf{V})), \mathbf{Z}) =$$

$$\mathbf{g}(\mathbf{R}(\mathbf{T}, \mathbf{Z})\mathbf{T}, \mathbf{Z}) - \delta(\mathbf{g}(\mathbf{R}(\mathbf{T}, \mathbf{Z})\mathbf{Z}, \mathbf{V}), \mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T}, \nabla_{\mathbf{Z}}\mathbf{V}))\mathbf{g}(\mathbf{A}, \mathbf{A}).$$

Therefore,

$$\nabla_{\mathbf{T}}\mathbf{g}(\mathbf{T}, \mathbf{A}) = 0.$$

Finally,

$$\nabla_{\mathbf{T}}\mathbf{g}(\mathbf{V}, \mathbf{F}(\mathbf{A}, \mathbf{g}(\mathbf{R}(\mathbf{T}, \mathbf{Z})\mathbf{Z}, \mathbf{V}), \mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T}, \nabla_{\mathbf{Z}}\mathbf{V}))) = 0$$

is fully equivalent for the defining equation of $\delta(\mathbf{g}(\mathbf{R}(\mathbf{T}, \mathbf{Z})\mathbf{T}, \mathbf{Z}))$, namely

$$\delta^2(\mathbf{g}(\mathbf{R}(\mathbf{T}, \mathbf{Z})\mathbf{Z}, \mathbf{V}), \mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T}, \nabla_{\mathbf{Z}}\mathbf{V}))\mathbf{g}(\mathbf{A}, \mathbf{A}) +$$

$$\mathbf{g}(\mathbf{V}, \nabla_{\mathbf{T}}\mathbf{F}(\mathbf{A}, \mathbf{g}(\mathbf{R}(\mathbf{T}, \mathbf{Z})\mathbf{Z}, \mathbf{V}), \mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T}, \nabla_{\mathbf{Z}}\mathbf{V}))) = 0.$$

Here, the last term reduces to

$$\delta(\mathbf{g}(\mathbf{R}(\mathbf{T}, \mathbf{Z})\mathbf{Z}, \mathbf{V}), \mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T}, \nabla_{\mathbf{Z}}\mathbf{V}))\mathbf{g}(\mathbf{V}, \nabla_{\mathbf{T}}\mathbf{A})$$

where

$$\mathbf{g}(\mathbf{V}, \nabla_{\mathbf{T}}\mathbf{A}) = \mathbf{g}(\mathbf{V}, \mathbf{R}(\mathbf{T}, \mathbf{Z})\mathbf{Z}) + \mathbf{g}(\nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{T}, \mathbf{V}) = -\mathbf{g}(\mathbf{Z}, \mathbf{R}(\mathbf{T}, \mathbf{Z})\mathbf{V}) - \mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{T}, \nabla_{\mathbf{Z}}\mathbf{V}).$$

This gives $\mathbf{F}$ back its old functional description which shows the adequacy of our approach. Clearly, in order to recuperate string theory we need to put $\mathbf{g}(\mathbf{Z}, \mathbf{Z}) = 0 = \mathbf{g}(\mathbf{V}, \mathbf{V})$ as well as $\rho = 0$ and we notice that the former did need to be put zero in the classical string theory by hand too which explains the enlarged theory.

References

[1] J. Noldus, Foundations of a theory of quantum gravity, Vixra:1106.0028