Meth8 validation of Bayes rule

Bayes rule from: cs.cornell.edu/home/kleinber/networks-book/networks-book-ch16.pdf, information cascades

Section 1. We ask:

"Can we validate Bayes rule as defined in the captioned textbook link?"

We assume the notation of Meth8 and Pr[...] from the text as Probability of [...], which is ignored for our purposes here because Pr[...] precedes each term of the formulas of the text.

We assume the apparatus of Meth8 modal logic model checker, implementing our resuscitation of the Łukasiewicz four-valued logic as system variant VŁ4. The 16-valued truth tables are horizontal.

LET: p q [A B, from the text], (q>p) [A|B], (p>q) [B|A]
vt Validated tautology, nvt Not validated tautology,
Designated truth value: T Tautology (F Contradiction)

The text defines A given B, that is, if B then A:

(q>p)=((p&q)\q) ; nvt ; TTFF TTFF TTFF TTFF (1)

Because Eq 1 is not vt, as expected from the text, we test the main connective for > Imply instead of = Equivalent.

(q>p)>((p&q)\q) ; nvt ; TTTF TTTF TTTF TTTF (1.1)

The text defines B given A, that is, if A then B:

(p>q)=((q&p)\p) ; nvt ; TFTF TFTF TFTF TFTF (2)

Because Eq 2 is not vt, as expected from the textbook, we test the main connective for > Imply instead of = Equivalent.

(p>q)>((q&p)\p) ; nvt ; TTTF TTTF TTTF TTTF (2.1)

Eq 1 and Eq 2 are supposed to be vt but are not. We note that Eq 1.1 is equivalent to Eq 2.1 where the respective main connectives are > Imply, not = Equivalent.

((q>p)>((p&q)\q)) = ((p>q)>((q&p)\p)) ; vt ; TTTT TTTT TTTT TTTT (3)

Because Eqs 1 and 2 are nvt, we could terminate validation at this point.

Section 2. We ask:

"Can the argument from the text be resuscitated in the process of continuing to evaluate it?"

The text rewrites Eqs 1 and 2 by multiplying both sides of the formula by the denominator in the respective consequent. In Eqs 1 and 2 the respective multiplier terms are q and p. The idea is to clear the denominator in the respective consequents.

((q>p)&q) = (((p&q)\q)&q) ; nvt ; TTFF TTFF TTFF TTFF (4)
((p>q)&p) = (((q&p)\p)&p) ; nvt ; TFTF TFTF TFTF TFTF (5)

We test the main connective in Eqs 4 and 5 for > Imply instead of = Equivalent, with the same result as in Eqs 1.1,2.1, and 3.

Because (p&q) = (q&p), the text rewrites Eq 5 but Eq 4 is carried over as unchanged.

((q>p)&q) = (((p&q)\q)&q) ; nvt ; TTFF TTFF TTFF TTFF (6)
((p>q)&p) = (((p&q)\p)&p) ; nvt ; TFTF TFTF TFTF TFTF (7)

The text rewrites Eqs 6 and 7 by simplifying the consequents.

((q>p)&q) = (p&q) ; vt ; TTTT TTTT TTTT TTTT (8)
((p>q)&p) = (p&q) ; vt ; TTTT TTTT TTTT TTTT (9)

The text sets Eq 8 equal to Eq 9.

((q>p)&q) = ((p>q)&p) ; vt ; TTTT TTTT TTTT TTTT (10)

For Eq 10 the text divides both antecedent and consequent by the term q to reduce the antecedent then rewrites.

(q>p) = (((p>q)&p)\q) ; nvt ; TTFF TTFF TTFF TTFF (11)

This produces the intended definition of the text for the expression Pr[(A|B] (16.4) as Bayes rule.

Bayes rule as Eq 11 is nvt. We note the text begins with Eqs 1 and 2, both nvt.

This leads us to consider Eq 3 vt as the basis from which to obtain Bayes rule.

((q>p)>((p&q)\q)) = ((p>q)>((q&p)\p)) ; vt ; TTTT TTTT TTTT TTTT (3)

From Eq 3, we seek to find the definition of (q>p), or as an alternative approach of (p>q).

In the case of the term (q>p) we seek to remove from the antecedent in Eq 3 the term ((p&q)\q). The procedure is to apply the expression <((p&q)\q) to the antecedent and consequent.

(((q>p)>((p&q)\q))<((p&q)\q)) = (((p>q)>((q&p)\p))<((p&q)\q)) ; vt ;
TTTT TTTT TTTT TTTT (12)

We simplify and rewrite Eq 12.

(q>p) = (((p>q)>((q&p)\p))<((p&q)\q)) ; nvt ; FFTF FFTF FFTF FFTF (13)

In the case of the term (p>q) we seek to remove from the consequent in Eq 3 the term ((q&p)\p)). The procedure is to apply the expression <((q&p)\p) to the consequent and antecedent.

(((q>p)>((p&q)\q))<((q&p)\p)) = (((p>q)>((q&p)\p))<((q&p)\p)) ; vt ;
TTTT TTTT TTTT TTTT (14)

We simplify and rewrite Eq 14.

(p>q) = (((q>p)>((p&q)\q))<((q&p)\p)) ; nvt ; FTFF FTFF FTFF FTFF (15)

The textbook definitions of Bayes rule are not validated as tautologous and cannot be resuscitated from the textbook.

Section 3. As an experiment, we ask:

> "Are the definitions of Bayes rule derivable from Eq 3, the only expression tautologous, from Section 1; in other words, can Meth8 produce a correct Bayes rule because Section 1 failed to do so?"

We reiterate Eq 3 from above and rename it for this section as Eq 3'.

((q>p)>((p&q)\q)) = ((p>q)>((q&p)\p)) ; vt (3')

LET r=((p&q)\q), s=((q&p)\p) and rewrite Eq 3' with those definitions by substitution.

((r=((p&q)\q))&(s=((q&p)\p)))> ((((q>p)>r)-s) = (((p>q)>s)-r)) ; vt (4')

Our approach is to manipulate the term ((q>p)>r)-s) so that (q>p) is the antecedent of an equality.

This means finding the correct method to represent (q>p) as a separate term in ((q>p)>r)-s), or as an alternative approach to represent (p>q) as a separate term in ((p>q)>s)-r), or both.

We use the template A>B = ~A+B where A is (q>p) and B is r, so ((q>p)>r)-s becomes (~(q>p)+r)-s.

((r=((p&q)\q))&(s=((q&p)\p)))> (((~(q>p)+r)-s) = (((p>q)>s)-r)) ; vt (5')

This successfully removed from the antecedent term of interest the second > Imply connective to leave connectives + Or and - Not Or.

We use the same template as C>D = ~C+D where C is (p>q) and D is s, so that ((p>q)>s)-r becomes (~(p>q)+s)-r.

((r=((p&q)\q))&(s=((q&p)\p)))> (((~(q>p)+r)-s) = ((~(p>q)+s)-r)) ; vt (6')

This successfully removed from the consequent term of interest the second > Imply connective to leave connectives + Or and - Not Or.

We cannot extract either (q>p) or (p>q) as separate terms from Eq. 6'. Therefore we abandon seeking these terms as those claimed for Pr[A|B] or Pr[B|A] in the text for Bayes rule.