Mass Transformation between Inertial Reference Frames

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An isolated physical system of elastic collision between two identical objects is chosen to manifest the conservation of momentum in two inertial reference frames. In the first reference frame, the center of mass (COM) is stationary. In the second reference frame, one object is at rest. The second frame is created by a temporary acceleration from the first frame. By applying both velocity transformation and conservation of momentum to this isolated system, mass transformation is derived precisely. The result shows that the mass of an object is independent of its motion.

I. INTRODUCTION

Elastic collision between two identical objects is an excellent physics system to demonstrate the conservation of momentum. Two preferred reference frames are the center of mass (COM) frame and the rest frame of one object. Conservation of momentum is expected to hold in both frames. The velocity of each object depends on the choice of reference frame. Therefore, the mass of each object may also depend on the reference frame. The exact expression of mass will be derived from the expression of velocity and momentum.

The concept of relativistic mass becomes less popular in modern physics. Momentum of an object is represented by either $\gamma(v) * m(0) * v$ or $m(v) * v$. Both representations are equivalent to each other mathematically. In this paper, $m(v)*v$ is chosen for its simpler expression.

II. PROOF

Consider one-dimensional motion.

A. Elastic Collision

Two identical objects move toward each other to make head-on collision. In the COM frame (Center Of Mass), both objects move at identical speed but opposite direction. At the moment both objects make contact, there is a repulsive force between them. Both objects eventually slow down to stand still. This repulsive force continues to push them away until there is no contact between two objects.

B. Center of Mass

Let a reference frame F_1 be stationary relatively to this COM frame. Let the mass of an object depends on its velocity.

TABLE I. Velocity and Mass in Reference Frame

Object	Frame	Value
The velocity of object 1, O_1, in	F_1	is V
The velocity of object 2, O_2, in	F_1	is -V
The mass of O_1 in	F_1	is m(V)
The mass of O_2 in	F_1	is m(-V)

C. Acceleration

Let another reference frame F_2 be stationary relatively to F_1.

The velocity of F_1 relative to F_2 is 0

Apply constant accleration of A for a duration of T to F_2 relatively to F_1. According to the definition of acceleration, all objects in F_1 gain a velocity difference of -A*T in F_2.

TABLE II. Relative Velocity After Acceleration

Object	Frame	Velocity
The velocity of O_1 in	F_1	is V
The velocity of O_2 in	F_1	is -V
The velocity of O_1 in	F_2	is V-A*T
The velocity of O_2 in	F_2	is -V-A*T
The velocity of F_1 relative to	F_2	is 0-A*T

Choose this acceleration so that O_2 becomes stationary relatively to F_2.

$$V = -A * T \tag{1}$$

TABLE III. COM Frame and Rest Frame

Object	Frame	Velocity
The velocity of O_1 in	F_1	is V
The velocity of O_2 in	F_1	is -V
The velocity of O_1 in	F_2	is 2V
The velocity of O_2 in	F_2	is 0
The velocity of F_1 relative to	F_2	is V

For elastic collision, both objects in F_1 will come to stand still before moving away from each other. At the moment when both objects are stationary in F_1, both objects move at the same velocity in F_2.

TABLE IV. Both Objects Are Stationary to Each Other

Object	Frame	Velocity
The velocity of O_1 in	F_1	is 0
The velocity of O_2 in	F_1	is 0
The velocity of O_1 in	F_2	is V
The velocity of O_2 in	F_2	is V
The velocity of F_1 relative to	F_2	is V

D. Conservation of Momentum

Total momentum in F_2 before collision is

$$m(2V) * (2V) + m(0) * 0 = 2V * m(2V) \tag{2}$$

Total momentum in F_2 during collision when both objects move at the same velocity is

$$m(V) * V + m(V) * V = 2V * m(V) \tag{3}$$

Conservation of Momentum demands, (from equations (2) and (3)),

$$2V * m(2V) = 2V * m(V) \tag{4}$$

$$m(2V) = m(V) \tag{5}$$

Let x be a dummy variable.

$$\frac{d(m(V))}{dV} = \frac{d(m(x))}{dx} = \frac{d(m(2V))}{d(2V)} = \frac{1}{2} * \frac{d(m(2V))}{dV} \tag{6}$$

$$\frac{d(m(V))}{dV} = \frac{1}{2} * \frac{d(m(2V))}{dV} \tag{7}$$

From equations (5) and (7)

$$\frac{d(m(V))}{dV} = \frac{1}{2} * \frac{d(m(2V))}{dV} = \frac{1}{2} * \frac{d(m(V))}{dV} \tag{8}$$

$$\frac{1}{2} * \frac{d(m(V))}{dV} = 0 \tag{9}$$

m(V) is independent of V.

III. CONCLUSION

The mass of an object is independent of its motion. Consequently, mass is independent of inertial reference frame. The mass of an object is identical in all inertial reference frames.

This is a direct property from the requirement of conservation of momentum in any inertial reference frame.

Therefore, the concept of relativistic mass from Special Relativity[1][2] is invalid in physics.

Lorentz Transformation was proposed on the assumption that the speed of light is independent of inertial reference frame.

As the result of this incorrect assumption[3], Lorentz Transformation violates Translation Symmetry[4] in physics. Translation Symmetry requires conservation of simultaneity[5], conservation of distance[6], and conservation of time[7]. All three conservation properties are broken by Lorentz Transformation. Therefore, Lorentz Transformation is not a proper transformation in physics.

Consequently, any theory based on Lorentz Transformation is incorrect in physics. For example, Special Relativity[2][8]

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