Zeno’s Paradox and the Planck Length
The Fine Structure Constant and The Speed of Light
137 and 144 and 64
Espen Gaarder Haug*  
January 30, 2018

Abstract

In this short paper we look at some interesting relationships between Zeno’s paradox, the Planck length, the speed of light, and the fine structure constant. Did you know it takes 144 Zeno steps to reduce the speed of light to Planck length speed? Did you know it takes 137 Zeno steps to go from $\alpha c$ to Planck length speed? And interestingly it is 7 steps between 144 and 137. We assume this is merely a coincidence, but it is often assumed that there are likely a maximum of 137 elements, the last one being the “Feynmanium.” And there are considered to be 7 orbital shells.
We do not claim this explains anything particular in physics, but we also do not exclude the possibility that there could be something meaningful in it. Are these patterns purely numerically coincidences, or could they be a link to deeper understanding of other patterns related to physics, like the periodic table? Whatever the answer may be, it is interesting to look at Zeno’s paradox in relation to the Planck length.

Key words: Zeno’s paradox, Zeno steps, the Planck length, the speed of light, the fine structure constant.

1 Zeno Steps: From the Speed of Light to the Planck Length

Zeno of Elea (ca. 490 – 430 BC) introduced what today is known as “Zeno’s paradox.” Assume you are 100 meters from a door. You walk half the distance, that is 50 meters, next you walk half of that, that is 25 meters, next you walk half of that, that is 12.5 meters. You will soon find out that in continuous space you will need to take an infinite number of steps to get to the door; in other words, you actually will never get to the door. However, assume there is an absolute minimum indivisible distance, that once reached can no longer be divided. Then there will be a finite number of steps, and there is no longer Zeno’s paradox. This is intriguing, but we will not go further in that direction for now. Instead, this paper looks at how half-steps are linked to the Planck length [1, 2], the speed of light, and the fine structure constant. Zeno’s paradox has also been linked to quantum physics in the past, see for example [3, 4]. In particular, there are many papers on the quantum Zeno effect, which is partly linked to Zeno’s paradox. Even if we are not experts on this topic, we do not think our simple calculations here have been shown before. In any case, it is mostly done for fun.

Each time we divide a distance (or a velocity) in half, we will call this a Zeno step. Let’s assume this indivisible distance is the Planck length. The idea that the Planck length possibly resolves the Zeno’s paradox is not new, see for example [5, 6]. That is to say, when we get to this length, we can no longer take a step. The Planck length is incredibly short, only about $1.6162 \times 10^{-35}$ meter. Until recently it was thought that the Planck length could only be found by first knowing the gravitational constant and then calculating the Planck length from the gravitational constant, plus the speed of light and the Planck constant, as first shown by Max Planck. Haug has recently shown that the Planck length can be measured without any knowledge of Newton’s gravitational constant [7]. Many physicists assume the Planck length is the shortest distance we can measure hypothetically. We think so too. In addition, according to modern atomism as described by [8, 9], the Planck length is the diameter of an indivisible particle.

Now assume we have the speed of light, typically described with the letter $c$; it is exactly 299,792,458 meters per second. This is by definition, as we have put the uncertainty over in the second and the meter. Assume there also is a minimum velocity equal to the Planck length. Now if we start to cut the speed of light in half, and then cut the resulting speed in half, how many Zeno steps do we need to take before we reach the Planck length? Simple calculations show that it takes 144 steps, where the very last step is a 1.66 step, instead of a 2 step (half-step) to arrive exactly at the Planck length. Granted that there is considerable uncertainty in exactly

*e-mail espenhaug@mac.com. Thanks to Victoria Terces for helping me edit this manuscript.
what the Planck length is, but inside any uncertainty limit there will indeed be 144 steps. Of course if there is no minimum velocity or a velocity-gap, then there will be an infinite number of Zeno steps between the speed of light and zero, but if the velocity-gap (the lowest velocity above zero is one Planck length per time unit) then the number of Zeno steps goes from infinite to just 144 in this case. Quite amazing, what the world’s shortest length can do!

2 Zeno Steps and the Fine Structure Constant

Arnold Sommerfeld [10] introduced the fine structure constant in relation to spectral lines. The fine structure constant \( \alpha \approx 0.0072973525664 \) (2014 CODATA recommended values) plays an important role in modern physics.

Further, the fine structure constant is often linked to the orbital velocity of the electron, which, in a hydrogen atom for example, is assumed to move at velocity \( v = \alpha c \). The number of Zeno steps to go from \( v = \alpha c \) to the Planck length speed is 137 steps. Again, the very last step is not quite a half-step to reach the Planck length exactly.

Also notice (and this is well-known) that when one has \( 137 \alpha c \), one is just below the speed of light, and \( 138 \alpha c \) would be forbidden, as this would mean a velocity faster than the speed of light. In quantum physics this is linked to the number of elements in the periodic table. Feynman stated that there was possibly a maximum of 137 elements. For this reason, element 137 is called Feynmanium.

3 Zeno Steps and Reduced Compton Wavelength of an Electron

There are several other interesting Zeno-step speculations to consider in physics. It takes 75 Zeno steps to go from the reduced Compton wavelength of the electron to the Planck length. And it takes 64 Zeno steps to climb down the reduced Compton wavelength of the proton to the Planck length. Further, to climb down the Higgs boson’s reduced Compton wavelength to the Planck length is 57 Zeno steps (assuming a reduced Compton wavelength of approximately \( 1.577 \times 10^{-18} \) meters. And the muon staircase appears to be 67 Zeno steps.

4 Conclusion

We have pointed out that it takes 144 Zeno steps to go from the speed of light to the Planck length velocity. Further, we have shown that it takes 137 Zeno steps to go from the velocity of an electron in the inner orbital shell, \( \alpha c \), to Planck length velocity. We do not claim there is necessarily any deeper connection to the rules of fundamental physics in the creation of the Zeno step unit, but we do not exclude this possibility either.

References