# Research Project Primus

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## 1 Theorems and Conjectures

**Theorem 1.1.** *A natural number $n > 2$ is a prime iff* $\prod_{k=1}^{n-1} k \equiv n-1 \pmod{\sum_{k=1}^{n-1} k}$.

**Theorem 1.2.** *Let $p \equiv 5 \pmod 6$ be prime then , $2p+1$ is prime iff $2p+1 \mid 3^p - 1$ .*

**Theorem 1.3.** *Let $p_n$ be the nth prime , then*

$$p_n = 1 + \sum_{k=1}^{2\cdot(\lfloor n\ln(n)\rfloor+1)} \left(1 - \left\lfloor \frac{1}{n} \cdot \sum_{j=2}^{k} \left\lceil \frac{3 - \sum_{i=1}^{j} \left\lfloor \frac{\lfloor \frac{j}{i} \rfloor}{\lceil \frac{j}{i} \rceil} \right\rfloor}{j} \right\rceil \right\rfloor\right)$$

**Theorem 1.4.** *Let $P_j(x) = 2^{-j} \cdot \left(\left(x - \sqrt{x^2-4}\right)^j + \left(x + \sqrt{x^2-4}\right)^j\right)$ , where $j$ and $x$ are nonnegative integers . Let $N = k \cdot 2^m - 1$ such that $m > 2$ , $3 \mid k$ , $0 < k < 2^m$ and*

$$\begin{cases} k \equiv 1 \pmod{10} \; with \; m \equiv 2,3 \pmod 4 \\ k \equiv 3 \pmod{10} \; with \; m \equiv 0,3 \pmod 4 \\ k \equiv 7 \pmod{10} \; with \; m \equiv 1,2 \pmod 4 \\ k \equiv 9 \pmod{10} \; with \; m \equiv 0,1 \pmod 4 \end{cases}$$

*Let $S_i = S_{i-1}^2 - 2$ with $S_0 = P_k(3)$ , then $N$ is prime iff $S_{m-2} \equiv 0 \pmod N$*

**Theorem 1.5.** *Let $P_j(x) = 2^{-j} \cdot \left(\left(x - \sqrt{x^2-4}\right)^j + \left(x + \sqrt{x^2-4}\right)^j\right)$ , where $j$ and $x$ are nonnegative integers . Let $N = k \cdot 2^m - 1$ such that $m > 2$ , $3 \mid k$ , $0 < k < 2^m$ and*

$$\begin{cases} k \equiv 3 \pmod{42} \; with \; m \equiv 0,2 \pmod 3 \\ k \equiv 9 \pmod{42} \; with \; m \equiv 0 \pmod 3 \\ k \equiv 15 \pmod{42} \; with \; m \equiv 1 \pmod 3 \\ k \equiv 27 \pmod{42} \; with \; m \equiv 1,2 \pmod 3 \\ k \equiv 33 \pmod{42} \; with \; m \equiv 0,1 \pmod 3 \\ k \equiv 39 \pmod{42} \; with \; m \equiv 2 \pmod 3 \end{cases}$$

*Let $S_i = S_{i-1}^2 - 2$ with $S_0 = P_k(5)$ , then $N$ is prime iff $S_{m-2} \equiv 0 \pmod N$*

**Theorem 1.6.** *Let* $P_j(x) = 2^{-j} \cdot \left( \left(x-\sqrt{x^2-4}\right)^j + \left(x+\sqrt{x^2-4}\right)^j \right)$ *, where* $j$ *and* $x$ *are nonnegative integers . Let* $N = k \cdot 2^m + 1$ *such that* $m > 2$ *,* $0 < k < 2^m$ *and*

$$\begin{cases} k \equiv 1 \pmod{42} \textit{ with } m \equiv 2,4 \pmod{6} \\ k \equiv 5 \pmod{42} \textit{ with } m \equiv 3 \pmod{6} \\ k \equiv 11 \pmod{42} \textit{ with } m \equiv 3,5 \pmod{6} \\ k \equiv 13 \pmod{42} \textit{ with } m \equiv 4 \pmod{6} \\ k \equiv 17 \pmod{42} \textit{ with } m \equiv 5 \pmod{6} \\ k \equiv 19 \pmod{42} \textit{ with } m \equiv 0 \pmod{6} \\ k \equiv 23 \pmod{42} \textit{ with } m \equiv 1,3 \pmod{6} \\ k \equiv 25 \pmod{42} \textit{ with } m \equiv 0,2 \pmod{6} \\ k \equiv 29 \pmod{42} \textit{ with } m \equiv 1,5 \pmod{6} \\ k \equiv 31 \pmod{42} \textit{ with } m \equiv 2 \pmod{6} \\ k \equiv 37 \pmod{42} \textit{ with } m \equiv 0,4 \pmod{6} \\ k \equiv 41 \pmod{42} \textit{ with } m \equiv 1 \pmod{6} \end{cases}$$

*Let* $S_i = S_{i-1}^2 - 2$ *with* $S_0 = P_k(5)$ *, then* $N$ *is prime iff* $S_{m-2} \equiv 0 \pmod{N}$ *.*

**Theorem 1.7.** *Let* $P_j(x) = 2^{-j} \cdot \left( \left(x-\sqrt{x^2-4}\right)^j + \left(x+\sqrt{x^2-4}\right)^j \right)$ *, where* $j$ *and* $x$ *are nonnegative integers . Let* $N = k \cdot 2^m + 1$ *such that* $m > 2$ *,* $0 < k < 2^m$ *and*

$$\begin{cases} k \equiv 1 \pmod{6} \textit{ and } k \equiv 1,7 \pmod{10} \textit{ with } m \equiv 0 \pmod{4} \\ k \equiv 5 \pmod{6} \textit{ and } k \equiv 1,3 \pmod{10} \textit{ with } m \equiv 1 \pmod{4} \\ k \equiv 1 \pmod{6} \textit{ and } k \equiv 3,9 \pmod{10} \textit{ with } m \equiv 2 \pmod{4} \\ k \equiv 5 \pmod{6} \textit{ and } k \equiv 7,9 \pmod{10} \textit{ with } m \equiv 3 \pmod{4} \end{cases}$$

*Let* $S_i = S_{i-1}^2 - 2$ *with* $S_0 = P_k(8)$ *, then* $N$ *is prime iff* $S_{m-2} \equiv 0 \pmod{N}$ *.*

**Theorem 1.8.** *Let* $N = k \cdot 2^n + 1$ *with* $n > 1$ *,* $k$ *is odd ,* $0 < k < 2^n$ *,* $3 \mid k$ *and*

$$\begin{cases} k \equiv 3 \pmod{30}, & \textit{with } n \equiv 1,2 \pmod{4} \\ k \equiv 9 \pmod{30}, & \textit{with } n \equiv 2,3 \pmod{4} \\ k \equiv 21 \pmod{30}, & \textit{with } n \equiv 0,1 \pmod{4} \\ k \equiv 27 \pmod{30}, & \textit{with } n \equiv 0,3 \pmod{4} \end{cases}$$

*then* $N$ *is prime iff* $5^{\frac{N-1}{2}} \equiv -1 \pmod{N}$ *.*

**Theorem 1.9.** *Let* $N = k \cdot 2^n + 1$ *with* $n > 1$ *,* $k$ *is odd ,* $0 < k < 2^n$ *,* $3 \mid k$ *and*

$$\begin{cases} k \equiv 3 \pmod{42}, & \textit{with } n \equiv 2 \pmod{3} \\ k \equiv 9 \pmod{42}, & \textit{with } n \equiv 0,1 \pmod{3} \\ k \equiv 15 \pmod{42}, & \textit{with } n \equiv 1,2 \pmod{3} \\ k \equiv 27 \pmod{42}, & \textit{with } n \equiv 1 \pmod{3} \\ k \equiv 33 \pmod{42}, & \textit{with } n \equiv 0 \pmod{3} \\ k \equiv 39 \pmod{42}, & \textit{with } n \equiv 0,2 \pmod{3} \end{cases}$$

*then* $N$ *is prime iff* $7^{\frac{N-1}{2}} \equiv -1 \pmod{N}$ *.*

**Theorem 1.10.** *Let* $N = k \cdot 2^n + 1$ *with* $n > 1$ *,* $k$ *is odd ,* $0 < k < 2^n$ *,* $3 \mid k$ *and*

$$\begin{cases} k \equiv 3 \pmod{66}, & \text{with } n \equiv 1,2,6,8,9 \pmod{10} \\ k \equiv 9 \pmod{66}, & \text{with } n \equiv 0,1,3,4,8 \pmod{10} \\ k \equiv 15 \pmod{66}, & \text{with } n \equiv 2,4,5,7,8 \pmod{10} \\ k \equiv 21 \pmod{66}, & \text{with } n \equiv 1,2,4,5,9 \pmod{10} \\ k \equiv 27 \pmod{66}, & \text{with } n \equiv 0,2,3,5,6 \pmod{10} \\ k \equiv 39 \pmod{66}, & \text{with } n \equiv 0,1,5,7,8 \pmod{10} \\ k \equiv 45 \pmod{66}, & \text{with } n \equiv 0,4,6,7,9 \pmod{10} \\ k \equiv 51 \pmod{66}, & \text{with } n \equiv 0,2,3,7,9 \pmod{10} \\ k \equiv 57 \pmod{66}, & \text{with } n \equiv 3,5,6,8,9 \pmod{10} \\ k \equiv 63 \pmod{66}, & \text{with } n \equiv 1,3,4,6,7 \pmod{10} \end{cases}$$

*then* $N$ *is prime iff* $11^{\frac{N-1}{2}} \equiv -1 \pmod{N}$ *.*

**Theorem 1.11.** *A positive integer* $n$ *is prime iff* $\varphi(n)! \equiv -1 \pmod{n}$

**Theorem 1.12.** *For* $m \geq 1$ *number* $n$ *greater than one is prime iff*

$$(n^m - 1)! \equiv (n-1)^{\left\lceil \frac{(-1)^{m+1}}{2} \right\rceil} \cdot n^{\frac{n^m - mn + m - 1}{n-1}} \pmod{n^{\frac{n^m - mn + m + n - 2}{n-1}}}$$

**Theorem 1.13.** *Sequence* $S_i$ *is defined as* $S_i = \begin{cases} 8 & \text{if } i = 0; \\ (S_{i-1}^2 - 2)^2 - 2 & \text{otherwise} . \end{cases}$ *then ,* $F_n = 2^{2^n} + 1, (n \geq 2)$ *is a prime if and only if* $F_n$ *divides* $S_{2^{n-1}-1}$ *.*

**Theorem 1.14.** *Let* $p \equiv 1 \pmod{6}$ *be prime and let* $5 \nmid 4p + 1$ *, then* $4p + 1$ *is prime iff* $4p + 1 \mid 2^{2p} + 1$ *.*

**Theorem 1.15.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2 - 4}\right)^m + \left(x + \sqrt{x^2 - 4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $F_n(b) = b^{2^n} + 1$ *such that* $n \geq 2$ *and* $b$ *is even number . Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_b(6)$*, thus If* $F_n(b)$ *is prime, then* $S_{2^n-1} \equiv 2 \pmod{F_n(b)}$*.*

**Theorem 1.16.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2 - 4}\right)^m + \left(x + \sqrt{x^2 - 4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $E_n(b) = \frac{b^{2^n}+1}{2}$ *such that* $n > 1$*,* $b$ *is odd number greater than one . Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_b(6)$*, thus If* $E_n(b)$ *is prime, then* $S_{2^n-1} \equiv 6 \pmod{E_n(b)}$*.*

**Theorem 1.17.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2 - 4}\right)^m + \left(x + \sqrt{x^2 - 4}\right)^m \right)$ *.*

*Let* $N_p(b) = \frac{b^p+1}{b+1}$ *, where* $p$ *is an odd prime and* $b$ *is an odd natural number greater than one .*

*CASE(1).* $b \equiv 1,9 \pmod{12}$ *, or* $b \equiv 3,7 \pmod{12}$ *and* $p \equiv 1 \pmod{4}$ *, or* $b \equiv 5 \pmod{12}$ *and* $p \equiv 1,7 \pmod{12}$ *, or* $b \equiv 11 \pmod{12}$ *and* $p \equiv 1,11 \pmod{12}$ *.*

*CASE(2).* $b \equiv 3,7 \pmod{12}$ *and* $p \equiv 3 \pmod{4}$ *, or* $b \equiv 5 \pmod{12}$ *and* $p \equiv 5,11 \pmod{12}$ *, or* $b \equiv 11 \pmod{12}$ *and* $p \equiv 5,7 \pmod{12}$ *.*

*Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_b(4)$ *. Suppose* $N_p(b)$ *is prime , then :*

- $S_{p-1} \equiv P_b(4) \pmod{N_p(b)}$ *if Case(1) holds ;*
- $S_{p-1} \equiv P_{b+2}(4) \pmod{N_p(b)}$ *if Case(2) holds ;*

**Theorem 1.18.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ .

*Let* $M_p(a) = \frac{a^p-1}{a-1}$ *, where* $p$ *is an odd prime and* $a$ *is an odd natural number greater than one*

*CASE(1).* $a \equiv 3, 11 \pmod{12}$ *, or* $a \equiv 5, 9 \pmod{12}$ *and* $p \equiv 1 \pmod{4}$ *, or* $a \equiv 7 \pmod{12}$ *and* $p \equiv 1, 7 \pmod{12}$ *, or* $a \equiv 1 \pmod{12}$ *and* $p \equiv 1, 11 \pmod{12}$ .

*CASE(2).* $a \equiv 5, 9 \pmod{12}$ *and* $p \equiv 3 \pmod{4}$ *, or* $a \equiv 7 \pmod{12}$ *and* $p \equiv 5, 11 \pmod{12}$ *, or* $a \equiv 1 \pmod{12}$ *and* $p \equiv 5, 7 \pmod{12}$ .

*Let* $S_i = P_a(S_{i-1})$ *with* $S_0 = P_a(4)$ *. Suppose* $M_p(a)$ *is prime , then :*

- $S_{p-1} \equiv P_a(4) \pmod{M_p(a)}$ *if Case(1) holds ;*
- $S_{p-1} \equiv P_{a-2}(4) \pmod{M_p(a)}$ *if Case(2) holds ;*

**Conjecture 1.1.** *Let* $b_n = b_{n-2} + lcm(n-1, b_{n-2})$ *with* $b_1 = 2$ *,* $b_2 = 2$ *and* $n > 2$ *. Let* $a_n = b_{n+2}/b_n - 1$ *, then*

*1. Every term of this sequence* $a_i$ *is either prime or* $1$ *. 2. Every odd prime number is member of this sequence . 3. Every new prime in sequence is a next prime from the largest prime already listed .*

**Conjecture 1.2.** *Let* $b_n = b_{n-1} + lcm(\lfloor\sqrt{n^3}\rfloor, b_{n-1})$ *with* $b_1 = 2$ *and* $n > 1$ *. Let* $a_n = b_{n+1}/b_n - 1$ *, then*

*1. Every term of this sequence* $a_i$ *is either prime or* $1$ *. 2. Every odd prime of the form* $\lfloor\sqrt{n^3}\rfloor$ *is member of this sequence . 3. Every new prime of the form* $\lfloor\sqrt{n^3}\rfloor$ *in sequence is a next prime from the largest prime already listed .*

**Conjecture 1.3.** *Let* $b_n = b_{n-1} + lcm(\lfloor\sqrt{2}\cdot n\rfloor, b_{n-1})$ *with* $b_1 = 2$ *and* $n > 1$ *. Let* $a_n = b_{n+1}/b_n - 1$ *, then*

*1. Every term of this sequence* $a_i$ *is either prime or* $1$ *. 2. Every prime of the form* $\lfloor\sqrt{2}\cdot n\rfloor$ *is member of this sequence . 3. Every new prime of the form* $\lfloor\sqrt{2}\cdot n\rfloor$ *in sequence is a next prime from the largest prime already listed .*

**Conjecture 1.4.** *Let* $b_n = b_{n-1} + lcm(\lfloor\sqrt{3}\cdot n\rfloor, b_{n-1})$ *with* $b_1 = 3$ *and* $n > 1$ *. Let* $a_n = b_{n+1}/b_n - 1$ *, then*

*1. Every term of this sequence* $a_i$ *is either prime or* $1$ *. 2. Every prime of the form* $\lfloor\sqrt{3}\cdot n\rfloor$ *is member of this sequence . 3. Every new prime of the form* $\lfloor\sqrt{3}\cdot n\rfloor$ *in sequence is a next prime from the largest prime already listed .*

**Conjecture 1.5.** *Let* $b$ *and* $n$ *be a natural numbers ,* $b \geq 2$ *,* $n > 2$ *and* $n \neq 9$ *. Then* $n$ *is prime if and only if* $\sum\limits_{k=1}^{n-1} \left(b^k - 1\right)^{n-1} \equiv n \pmod{\frac{b^n-1}{b-1}}$

**Conjecture 1.6.** *Let* $a$ *,* $b$ *and* $n$ *be a natural numbers ,* $b > a > 1$ *,* $n > 2$ *and* $n \notin \{4, 9, 25\}$ *. Then* $n$ *is prime iff* $\prod\limits_{k=1}^{n-1} \left(b^k - a\right) \equiv \frac{a^n-1}{a-1} \pmod{\frac{b^n-1}{b-1}}$

**Conjecture 1.7.** *Let* $a$ *,* $b$ *and* $n$ *be a natural numbers ,* $b > a > 0$ *,* $n > 2$ *and* $n \notin \{4, 9, 25\}$ *. Then* $n$ *is prime iff* $\prod\limits_{k=1}^{n-1} \left(b^k + a\right) \equiv \frac{a^n+1}{a+1} \pmod{\frac{b^n-1}{b-1}}$

**Conjecture 1.8.** *Let $P_m(x)=2^{-m}\cdot\left(\left(x-\sqrt{x^2-4}\right)^m+\left(x+\sqrt{x^2-4}\right)^m\right)$, where $m$ and $x$ are nonnegative integers. Let $N=k\cdot b^n-1$ such that $k>0$ , $3\nmid k$ , $k<2^n$ , $b>0$ , $b$ is even number, $3\nmid b$ and $n>2$ . Let $S_i=P_b(S_{i-1})$ with $S_0=P_{kb/2}(P_{b/2}(4))$ , then $N$ is prime iff $S_{n-2}\equiv 0 \pmod{N}$ .*

**Conjecture 1.9.** *Let $P_j(x)=2^{-j}\cdot\left(\left(x-\sqrt{x^2-4}\right)^j+\left(x+\sqrt{x^2-4}\right)^j\right)$, where $j$ and $x$ are nonnegative integers. Let $N=k\cdot 2^m+1$ with $k$ odd , $0<k<2^m$ and $m>2$ . Let $F_n$ be the nth Fibonacci number and let $S_i=S_{i-1}^2-2$ with $S_0=P_k(F_n)$ , then $N$ is prime iff there exists $F_n$ for which $S_{m-2}\equiv 0 \pmod{N}$ .*

**Conjecture 1.10.** *Let $P_j(x)=2^{-j}\cdot\left(\left(x-\sqrt{x^2-4}\right)^j+\left(x+\sqrt{x^2-4}\right)^j\right)$, where $j$ and $x$ are nonnegative integers. Let $F_m(b)=b^{2^m}+1$ with $b$ even , $b>0$ and $m\geq 2$ . Let $F_n$ be the nth Fibonacci number and let $S_i=P_b(S_{i-1})$ with $S_0=P_{b/2}(P_{b/2}(F_n))$ , then $F_m(b)$ is prime iff there exists $F_n$ for which $S_{m-2}\equiv 0 \pmod{F_m(b)}$ .*

**Conjecture 1.11.** *Let $P_m(x)=2^{-m}\cdot\left(\left(x-\sqrt{x^2-4}\right)^m+\left(x+\sqrt{x^2-4}\right)^m\right)$ , where $m$ and $x$ are nonnegative integers. Let $N=b^n-b-1$ such that $n>2$, $b\equiv 0,6 \pmod{8}$. Let $S_i=P_b(S_{i-1})$ with $S_0=P_{b/2}(6)$, thus if $N$ is prime, then $S_{n-1}\equiv P_{(b+2)/2}(6) \pmod{N}$.*

**Conjecture 1.12.** *Let $P_m(x)=2^{-m}\cdot\left(\left(x-\sqrt{x^2-4}\right)^m+\left(x+\sqrt{x^2-4}\right)^m\right)$ , where $m$ and $x$ are nonnegative integers. Let $N=b^n-b-1$ such that $n>2$, $b\equiv 2,4 \pmod{8}$. Let $S_i=P_b(S_{i-1})$ with $S_0=P_{b/2}(6)$, thus if $N$ is prime, then $S_{n-1}\equiv -P_{b/2}(6) \pmod{N}$.*

**Conjecture 1.13.** *Let $P_m(x)=2^{-m}\cdot\left(\left(x-\sqrt{x^2-4}\right)^m+\left(x+\sqrt{x^2-4}\right)^m\right)$ , where $m$ and $x$ are nonnegative integers. Let $N=b^n+b+1$ such that $n>2$, $b\equiv 0,6 \pmod{8}$. Let $S_i=P_b(S_{i-1})$ with $S_0=P_{b/2}(6)$, thus if $N$ is prime, then $S_{n-1}\equiv P_{b/2}(6) \pmod{N}$.*

**Conjecture 1.14.** *Let $P_m(x)=2^{-m}\cdot\left(\left(x-\sqrt{x^2-4}\right)^m+\left(x+\sqrt{x^2-4}\right)^m\right)$ , where $m$ and $x$ are nonnegative integers. Let $N=b^n+b+1$ such that $n>2$, $b\equiv 2,4 \pmod{8}$. Let $S_i=P_b(S_{i-1})$ with $S_0=P_{b/2}(6)$, thus if $N$ is prime, then $S_{n-1}\equiv -P_{(b+2)/2}(6) \pmod{N}$.*

**Conjecture 1.15.** *Let $P_m(x)=2^{-m}\cdot\left(\left(x-\sqrt{x^2-4}\right)^m+\left(x+\sqrt{x^2-4}\right)^m\right)$ , where $m$ and $x$ are nonnegative integers. Let $N=b^n-b+1$ such that $n>3$, $b\equiv 0,2 \pmod{8}$. Let $S_i=P_b(S_{i-1})$ with $S_0=P_{b/2}(6)$, thus if $N$ is prime, then $S_{n-1}\equiv P_{b/2}(6) \pmod{N}$.*

**Conjecture 1.16.** *Let $P_m(x)=2^{-m}\cdot\left(\left(x-\sqrt{x^2-4}\right)^m+\left(x+\sqrt{x^2-4}\right)^m\right)$ , where $m$ and $x$ are nonnegative integers. Let $N=b^n-b+1$ such that $n>3$, $b\equiv 4,6 \pmod{8}$. Let $S_i=P_b(S_{i-1})$ with $S_0=P_{b/2}(6)$, thus if $N$ is prime, then $S_{n-1}\equiv -P_{(b-2)/2}(6) \pmod{N}$.*

**Conjecture 1.17.** *Let $P_m(x)=2^{-m}\cdot\left(\left(x-\sqrt{x^2-4}\right)^m+\left(x+\sqrt{x^2-4}\right)^m\right)$ , where $m$ and $x$ are nonnegative integers. Let $N=b^n+b-1$ such that $n>3$, $b\equiv 0,2 \pmod{8}$. Let $S_i=P_b(S_{i-1})$ with $S_0=P_{b/2}(6)$, thus if $N$ is prime, then $S_{n-1}\equiv P_{(b-2)/2}(6) \pmod{N}$.*

**Conjecture 1.18.** *Let $P_m(x)=2^{-m}\cdot\left(\left(x-\sqrt{x^2-4}\right)^m+\left(x+\sqrt{x^2-4}\right)^m\right)$ , where $m$ and $x$ are nonnegative integers. Let $N=b^n+b-1$ such that $n>3$, $b\equiv 4,6 \pmod{8}$. Let $S_i=P_b(S_{i-1})$ with $S_0=P_{b/2}(6)$, thus if $N$ is prime, then $S_{n-1}\equiv -P_{b/2}(6) \pmod{N}$.*

**Conjecture 1.19.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$

*Let* $N = k \cdot 3^n - 2$ *such that* $n > 3$ , $k \equiv 1, 3 \pmod 8$ *and* $k > 0$ . *Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{3k}(6)$ , *thus If* $N$ *is prime then* $S_{n-1} \equiv P_3(6) \pmod N$

**Conjecture 1.20.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$

*Let* $N = k \cdot 3^n - 2$ *such that* $n > 3$ , $k \equiv 5, 7 \pmod 8$ *and* $k > 0$ . *Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{3k}(6)$ , *thus If* $N$ *is prime then* $S_{n-1} \equiv P_1(6) \pmod N$

**Conjecture 1.21.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = k \cdot 3^n + 2$ *such that* $n > 2$ , $k \equiv 1, 3 \pmod 8$ *and* $k > 0$ . *Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{3k}(6)$ , *thus If* $N$ *is prime then* $S_{n-1} \equiv P_3(6) \pmod N$

**Conjecture 1.22.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = k \cdot 3^n + 2$ *such that* $n > 2$ , $k \equiv 5, 7 \pmod 8$ *and* $k > 0$ . *Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{3k}(6)$ , *thus If* $N$ *is prime then* $S_{n-1} \equiv P_1(6) \pmod N$

**Conjecture 1.23.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = k \cdot b^n - c$ *such that* $b \equiv 0 \pmod 2), n > bc, k > 0, c > 0$ *and* $c \equiv 1, 7 \pmod 8$ *Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6))$, *thus If* $N$ *is prime then* $S_{n-1} \equiv P_{(b/2)\cdot\lceil c/2 \rceil}(6) \pmod N$

**Conjecture 1.24.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = k \cdot b^n - c$ *such that* $b \equiv 0, 4, 8 \pmod{12}, n > bc, k > 0, c > 0$ *and* $c \equiv 3, 5 \pmod 8$. *Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6))$, *thus If* $N$ *is prime then* $S_{n-1} \equiv P_{(b/2)\cdot\lfloor c/2 \rfloor}(6) \pmod N$

**Conjecture 1.25.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = k \cdot b^n - c$ *such that* $b \equiv 2, 6, 10 \pmod{12}, n > bc, k > 0, c > 0$ *and* $c \equiv 3, 5 \pmod 8$. *Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6))$, *thus If* $N$ *is prime then* $S_{n-1} \equiv -P_{(b/2)\cdot\lfloor c/2 \rfloor}(6) \pmod N$

**Conjecture 1.26.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = k \cdot b^n + c$ *such that* $b \equiv 0 \pmod 2), n > bc, k > 0, c > 0$ *and* $c \equiv 1, 7 \pmod 8$ *Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6))$, *thus If* $N$ *is prime then* $S_{n-1} \equiv P_{(b/2)\cdot\lfloor c/2 \rfloor}(6) \pmod N$

**Conjecture 1.27.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = k \cdot b^n + c$ *such that* $b \equiv 0, 4, 8 \pmod{12}, n > bc, k > 0, c > 0$ *and* $c \equiv 3, 5 \pmod 8$. *Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6))$, *thus If* $N$ *is prime then* $S_{n-1} \equiv P_{(b/2)\cdot\lceil c/2 \rceil}(6) \pmod N$

**Conjecture 1.28.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = k \cdot b^n + c$ *such that* $b \equiv 2, 6, 10 \pmod{12}, n > bc, k > 0, c > 0$ *and* $c \equiv 3, 5 \pmod 8$. *Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6))$, *thus If* $N$ *is prime then* $S_{n-1} \equiv -P_{(b/2)\cdot\lceil c/2 \rceil}(6) \pmod N$

**Conjecture 1.29.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = 2 \cdot 3^n - 1$ *such that* $n > 1$ *. Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_3(a)$ *, where* $a = \begin{cases} 6, & \text{if } n \equiv 0 \pmod 2 \\ 8, & \text{if } n \equiv 1 \pmod 2 \end{cases}$ *thus ,* $N$ *is prime iff* $S_{n-1} \equiv a \pmod N$

**Conjecture 1.30.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = 8 \cdot 3^n - 1$ *such that* $n > 1$ *. Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{12}(4)$ *thus ,* $N$ *is prime iff* $S_{n-1} \equiv 4 \pmod N$

**Conjecture 1.31.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = k \cdot 6^n - 1$ *such that* $n > 2$ *,* $k > 0$ *,* $k \equiv 2, 5 \pmod 7$ *and* $k < 6^n$ *Let* $S_i = P_6(S_{i-1})$ *with* $S_0 = P_{3k}(P_3(5))$ *, thus* $N$ *is prime iff* $S_{n-2} \equiv 0 \pmod N$

**Conjecture 1.32.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = k \cdot 6^n - 1$ *such that* $n > 2$ *,* $k > 0$ *,* $k \equiv 3, 4 \pmod 5$ *and* $k < 6^n$ *Let* $S_i = P_6(S_{i-1})$ *with* $S_0 = P_{3k}(P_3(3))$ *, thus* $N$ *is prime iff* $S_{n-2} \equiv 0 \pmod N$

**Conjecture 1.33.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = k \cdot b^n - 1$ *such that* $n > 2$ *,* $k < 2^n$ *and*

$$\begin{cases} k \equiv 3 \pmod{30} \; with \; b \equiv 2 \pmod{10} \; and \; n \equiv 0, 3 \pmod 4 \\ k \equiv 3 \pmod{30} \; with \; b \equiv 4 \pmod{10} \; and \; n \equiv 0, 2 \pmod 4 \\ k \equiv 3 \pmod{30} \; with \; b \equiv 6 \pmod{10} \; and \; n \equiv 0, 1, 2, 3 \pmod 4 \\ k \equiv 3 \pmod{30} \; with \; b \equiv 8 \pmod{10} \; and \; n \equiv 0, 1 \pmod 4 \end{cases}$$

*Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(18))$ *, then* $N$ *is prime iff* $S_{n-2} \equiv 0 \pmod N$

**Conjecture 1.34.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = k \cdot b^n - 1$ *such that* $n > 2$ *,* $k < 2^n$ *and*

$$\begin{cases} k \equiv 9 \pmod{30} \; with \; b \equiv 2 \pmod{10} \; and \; n \equiv 0, 1 \pmod 4 \\ k \equiv 9 \pmod{30} \; with \; b \equiv 4 \pmod{10} \; and \; n \equiv 0, 2 \pmod 4 \\ k \equiv 9 \pmod{30} \; with \; b \equiv 6 \pmod{10} \; and \; n \equiv 0, 1, 2, 3 \pmod 4 \\ k \equiv 9 \pmod{30} \; with \; b \equiv 8 \pmod{10} \; and \; n \equiv 0, 3 \pmod 4 \end{cases}$$

*Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(18))$ *, then* $N$ *is prime iff* $S_{n-2} \equiv 0 \pmod N$

**Conjecture 1.35.** *Let* $P_m(x) = 2^{-m} \cdot \left( \left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* $m$ *and* $x$ *are nonnegative integers. Let* $N = k \cdot b^n - 1$ *such that* $n > 2$ *,* $k < 2^n$ *and*

$$\begin{cases} k \equiv 21 \pmod{30} \; with \; b \equiv 2 \pmod{10} \; and \; n \equiv 2, 3 \pmod 4 \\ k \equiv 21 \pmod{30} \; with \; b \equiv 4 \pmod{10} \; and \; n \equiv 1, 3 \pmod 4 \\ k \equiv 21 \pmod{30} \; with \; b \equiv 8 \pmod{10} \; and \; n \equiv 1, 2 \pmod 4 \end{cases}$$

*Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(3))$ *, then* $N$ *is prime iff* $S_{n-2} \equiv 0 \pmod N$

**Conjecture 1.36.** *Let* $F_p$ *be the pth Fibonacci number .If* $p$ *is prime , not* $5$ *, and* $M \geq 2$ *then* $M^{F_p} \equiv M^{(p-1)^{(1-(\frac{p}{5}))/2}} \pmod{\frac{M^p-1}{M-1}}$

**Conjecture 1.37.** *Let $b$ and $n$ be a natural numbers , $b \geq 2$ , $n > 1$ and $n \notin \{4,8,9\}$ . Then $n$ is prime if and only if* $\displaystyle\sum_{k=1}^{n} \left(b^k+1\right)^{n-1} \equiv n \pmod{\frac{b^n-1}{b-1}}$

**Conjecture 1.38.** *If $q$ is the smallest prime greater than $\displaystyle\prod_{i=1}^{n} C_i + 1$ , where $\displaystyle\prod_{i=1}^{n} C_i$ is the product of the first $n$ composite numbers , then $q - \displaystyle\prod_{i=1}^{n} C_i$ is prime .*

**Conjecture 1.39.** *If $q$ is the greatest prime less than $\displaystyle\prod_{i=1}^{n} C_i - 1$ , where $\displaystyle\prod_{i=1}^{n} C_i$ is the product of the first $n$ composite numbers , then $\displaystyle\prod_{i=1}^{n} C_i - q$ is prime .*

**Conjecture 1.40.** *Let $n$ be an odd number and $n > 1$ . Let $T_n(x)$ be Chebyshev polynomial of the first kind and let $P_n(x)$ be Legendre polynomial , then $n$ is a prime number if and only if the following congruences hold simultaneously • $T_n(3) \equiv 3 \pmod{n}$ • $P_n(3) \equiv 3 \pmod{n}$*

**Conjecture 1.41.** *Let $n$ be a natural number greater than two . Let $r$ be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$ . Let $T_n(x)$ be Chebyshev polynomial of the first kind , then $n$ is a prime number if and only if $T_n(x) \equiv x^n \pmod{x^r-1, n}$ .*

**Conjecture 1.42.** *Let $n$ be a natural number greater than two and $n \neq 5$ . Let $T_n(x)$ be Chebyshev polynomial of the first kind . If there exists an integer $a$ , $1 < a < n$ , such that $T_{n-1}(a) \equiv 1 \pmod{n}$ and for every prime factor $q$ of $n-1$ , $T_{(n-1)/q}(a) \not\equiv 1 \pmod{n}$ then $n$ is prime . If no such number $a$ exists then $n$ is composite .*

**Conjecture 1.43.** *Let $P_a(x) = 2^{-a} \cdot \left(\left(x-\sqrt{x^2-4}\right)^a + \left(x+\sqrt{x^2-4}\right)^a\right)$ . Let $N = k \cdot b^m \pm 1$ with $b$ an even positive integer , $0<k<b^m$ and $m > 2$ . Let $F_n$ be the nth Fibonacci number and let $S_i = P_b(S_{i-1})$ with $S_0 = P_{kb/2}(P_{b/2}(F_n))$ , then $N$ is prime iff there exists $F_n$ for which $S_{m-2} \equiv 0 \pmod{N}$ .*

**Conjecture 1.44.** *Let $n$ be a natural number greater than one . Let $r$ be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$ . Let $L_n(x)$ be Lucas polynomial , then $n$ is a prime number if and only if $L_n(x) \equiv x^n \pmod{x^r-1, n}$ .*

**Conjecture 1.45.** *Let $b$ and $n$ be a natural numbers , $b \geq 2$ , then $\frac{b^n-1}{b-1} \cdot \frac{b^{\sigma(n)}-1}{b-1} \equiv b+1 \pmod{\frac{b^{\varphi(n)}-1}{b-1}}$ for all primes and no composite with the exception of $4$ and $6$ .*

**Conjecture 1.46.** *Let $b$ and $n$ be a natural numbers , $b \geq 2$ , then $\frac{b^{\varphi(n)}-1}{b-1}(b^{\tau(n)}-1) + b \equiv b^{n-1} \pmod{\frac{b^n-1}{b-1}}$ for all primes and no composite with the exception of $4$ .*

**Conjecture 1.47.** *Let $p$ be prime number greater than three and let $T_n(x)$ be Chebyshev polynomial of the first kind , then $T_{p-1}(2) \equiv 1 \pmod{p}$ if and only if $p \equiv 1, 11 \pmod{12}$ .*

**Conjecture 1.48.** *Let $p$ be prime number greater than two and let $T_n(x)$ be Chebyshev polynomial of the first kind , then $T_{p-1}(3) \equiv 1 \pmod{p}$ if and only if $p \equiv 1, 7 \pmod{8}$ .*

**Conjecture 1.49.** *Let $p$ be prime number greater than three and let $T_n(x)$ be Chebyshev polynomial of the first kind , then $T_{p-1}(5) \equiv 1 \pmod{p}$ if and only if $p \equiv 1, 5, 19, 23 \pmod{24}$* .

**Conjecture 1.50.** *Let $n$ be an odd natural number greater than one , let $k$ be a natural number such that $k \leq n$ , then $n$ is prime if and only if :* $\displaystyle\sum_{i=0}^{k-1} i^{n-1} + \sum_{j=0}^{n-k} j^{n-1} \equiv -1 \pmod{n}$

**Conjecture 1.51.** *Let $n$ be a natural number greater than one and let $T_n(x)$ be Chebyshev polynomial of the first kind , then $n$ is prime if and only if :* $\displaystyle\sum_{k=0}^{n-1} 2T_{n-1}\left(\frac{k}{2}\right) \equiv -1 \pmod{n}$ .

**Conjecture 1.52.** *Let $n$ be a natural number greater than one and let $L_n(x)$ be Lucas polynomial , then $n$ is prime if and only if :* $\displaystyle\sum_{k=0}^{n-1} L_{n-1}(k) \equiv -1 \pmod{n}$ .

**Conjecture 1.53.** *Let $p$ be an odd prime number , let $R_p(3) = \frac{3^p-1}{2}$ and let $S_i = S_{i-1}^3 + 3S_{i-1}$ with $S_0 = 36$ , then $R_p(3)$ is prime number iff $S_{p-1} \equiv 36 \pmod{R_p(3)}$* .

**Conjecture 1.54.** *Let $p$ be an odd prime number greater than three , let $R_p(-3) = \frac{3^p+1}{4}$ and let $S_i = S_{i-1}^3 + 3S_{i-1}$ with $S_0 = 36$ , then $R_p(-3)$ is prime number iff $S_{p-1} \equiv 36 \pmod{R_p(-3)}$* .

**Conjecture 1.55.** *Let $P_n^{(a)}(x) = \left(\frac{1}{2}\right) \cdot \left(\left(x - \sqrt{x^2+a}\right)^n + \left(x + \sqrt{x^2+a}\right)^n\right)$ . Given an odd integer $n$ ($\geq 3$) and integer $a$ coprime to $n$ , $n$ is prime if and only if $P_n^{(a)}(x) \equiv x^n \pmod{n}$ holds* .

**Conjecture 1.56.** *Let $n$ be an odd natural number greater than one . Let $r$ be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$ . Let $P_n(x)$ be Legendre polynomial , then $n$ is a prime number if and only if $P_n(x) \equiv x^n \pmod{x^r - 1, n}$* .

**Conjecture 1.57.** *Let $n$ be a natural number greater than one and let $F_n(x)$ be Fibonacci polynomial , then $n$ is prime if and only if :* $\displaystyle\sum_{k=0}^{n-1} F_n(k) \equiv -1 \pmod{n}$ .

**Conjecture 1.58.** *Let $a_n$ be the least unused prime greater than $3$ such that $(a_n + a_{n-1})/2$ is prime, with $a_0 = 13$ , then :*

1. *Every term of this sequence $a_i$ is prime of the form $12k + 1$* .
2. *Every prime of the form $12k + 1$ is a member of this sequence* .

**Conjecture 1.59.** *Let $m$ and $n$ be a natural numbers , $m \geq 1$ , $n > 2$ , $n \neq 9$ and gcd$(m, n) = 1$ . Then $n$ is prime if and only if* $\displaystyle\sum_{k=1}^{n-1} \left(2^{mk} - 1\right)^{n-1} \equiv n \pmod{2^n - 1}$

**Conjecture 1.60.** *Let $p, q, r$ be three consecutive prime numbers such that $p \geq 11$ and $p < q < r$ , then $\frac{1}{p^2} < \frac{1}{q^2} + \frac{1}{r^2}$* .

**Conjecture 1.61.** *Let $p$ and $q$ be consecutive prime numbers such that $p \geq 5$ and $p < q$ , then* $\left\lfloor \frac{q}{p} - \frac{p}{q} \right\rfloor = 0$ .

**Conjecture 1.62.** *Let $a, n, k$ be natural numbers greater than $0$ . If $n$ is a prime number then* $\sum\limits_{d|n} \left(\sigma_k(d) \cdot a^{n/d}\right) \equiv 2a \pmod{n}$

**Conjecture 1.63.** *Let $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2-4})^m + (x + \sqrt{x^2-4})^m)$ . Let $F_n(b) = b^{2^n} + 1$ where $b$ is an even integer , $3 \nmid b, 5 \nmid b$ and $n \geq 2$ . Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(P_{b/2}(8))$ , then $F_n(b)$ is prime iff $S_{2^n-2} \equiv 0 \pmod{F_n(b)}$ .*

**Conjecture 1.64.** *Let $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2-4})^m + (x + \sqrt{x^2-4})^m)$ . Let $F_n(b) = b^{2^n} + 1$ where $b$ is an even integer , $3 \nmid b$ , $b \equiv 2, 4, 10, 12 \pmod{14}$ and $n \geq 2$ . Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(P_{b/2}(5))$ , then $F_n(b)$ is prime iff $S_{2^n-2} \equiv 0 \pmod{F_n(b)}$ .*

**Conjecture 1.65.** *Let $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2-4})^m + (x + \sqrt{x^2-4})^m)$ . Let $F_n(b) = b^{2^n} + 1$ where $b$ is an even integer , $5 \nmid b$ , $b \equiv 2, 4, 10, 12 \pmod{14}$ and $n \geq 2$ . Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(P_{b/2}(12))$ , then $F_n(b)$ is prime iff $S_{2^n-2} \equiv 0 \pmod{F_n(b)}$ .*

**Conjecture 1.66.** *Let $a_n = 62a_{n-1} - a_{n-2}$ with $a_1 = 8$ and $a_2 = 488$ , let $b_n = 482b_{n-1} - b_{n-2}$ with $b_1 = 22$ and $b_2 = 10582$ , then each member of the sequences $\{a_n\}$ and $\{b_n\}$ can be used as an initial value for Inkeri's primality test for Fermat numbers .*

**Conjecture 1.67.** *Let $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$*

*Let $N = k \cdot b^n - 1$ such that $n > 2$ , $k < 2^n$ and*

$$\begin{cases} k \equiv 27 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 1, 2 \pmod{4} \\ k \equiv 27 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 1, 3 \pmod{4} \\ k \equiv 27 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 2, 3 \pmod{4} \end{cases}$$

*Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{bk/2}(P_{b/2}(3))$ , then $N$ is prime iff $S_{n-2} \equiv 0 \pmod{N}$*

**Conjecture 1.68.** *Let $n$ be a natural number greater than two . Let $r$ be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$ . Let $H_n(x)$ be Hermite polynomial , then $n$ is either a prime number or Fermat pseudoprime to base $2$ if and only if $H_n(x) \equiv 2x^n \pmod{x^r - 1, n}$ .*

**Conjecture 1.69.** *Let $n$ be an odd natural number greater than one . Let $r$ be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$ . Let $P_n^{(\alpha,\beta)}(x)$ be Jacobi polynomial such that $\alpha$ , $\beta$ are natural numbers and $\alpha + \beta < n$ , then $n$ is a prime number if and only if $P_n^{(\alpha,\beta)}(x) \equiv x^n \pmod{x^r - 1, n}$ .*

**Conjecture 1.70.** *Let $n$ be an odd natural number greater than one . Let $r$ be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$ . Let $F_n(x)$ be Fibonacci polynomial , then $n$ is prime if and only if $F_n(2x) \equiv (1 + x^2)^{\frac{n-1}{2}} \pmod{x^r - 1, n}$ .*

**Conjecture 1.71.** *Let $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2-4})^m + (x + \sqrt{x^2-4})^m)$ . Let $F_n(b) = b^{2^n} + 1$ where $b$ is an even natural number and $n \geq 2$ . Let $a$ be a natural number greater than two such that $\left(\frac{a-2}{F_n(b)}\right) = -1$ and $\left(\frac{a+2}{F_n(b)}\right) = -1$ where $(\,)$ denotes Jacobi symbol. Let $S_i = P_b(S_{i-1})$ with $S_0$ equal to the modular $P_{b/2}(P_{b/2}(a))$ mod $F_n(b)$. Then $F_n(b)$ is prime if and only if $S_{2^n-2} \equiv 0 \pmod{F_n(b)}$ .*

**Conjecture 1.72.** *Let* $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m)$ *. Let* $N = k \cdot b^n + 1$ *where* $k$ *is positive natural number ,* $k < 2^n$ *,* $b$ *is an even positive natural number and* $n \geq 3$ *. Let* $a$ *be a natural number greater than two such that* $\left(\frac{a-2}{N}\right) = -1$ *and* $\left(\frac{a+2}{N}\right) = -1$ *where* $\left(\right)$ *denotes Jacobi symbol. Let* $S_i = P_b(S_{i-1})$ *with* $S_0$ *equal to the modular* $P_{kb/2}(P_{b/2}(a)) \mod N$*. Then* $N$ *is prime if and only if* $S_{n-2} \equiv 0 \pmod{N}$ *.*

**Conjecture 1.73.** *Let* $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m)$ *. Let* $M = k \cdot b^n - 1$ *where* $k$ *is positive natural number ,* $k < 2^n$ *,* $b$ *is an even positive natural number and* $n \geq 3$ *. Let* $a$ *be a natural number greater than two such that* $\left(\frac{a-2}{M}\right) = 1$ *and* $\left(\frac{a+2}{M}\right) = -1$ *where* $\left(\right)$ *denotes Jacobi symbol. Let* $S_i = P_b(S_{i-1})$ *with* $S_0$ *equal to the modular* $P_{kb/2}(P_{b/2}(a)) \mod M$*. Then* $M$ *is prime if and only if* $S_{n-2} \equiv 0 \pmod{M}$ *.*

**Conjecture 1.74.** *Let* $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m)$ *. Let* $N = k \cdot b^n + 1$ *where* $k$ *is an even positive natural number ,* $k < 2^n$ *,* $b$ *is an odd positive natural number greater than one and* $n \geq 2$ *. Let* $a$ *be a natural number greater than two such that* $\left(\frac{a-2}{N}\right) = -1$ *and* $\left(\frac{a+2}{N}\right) = 1$ *where* $\left(\right)$ *denotes Jacobi symbol. Let* $S_i = P_b(S_{i-1})$ *with* $S_0$ *equal to the modular* $P_{kb/2}(a) \mod N$*. Then, if* $N$ *is prime then* $S_{n-1} \equiv a \pmod{N}$ *.*

**Conjecture 1.75.** *Let* $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m)$ *. Let* $M = k \cdot b^n - 1$ *where* $k$ *is an even positive natural number ,* $k < 2^n$ *,* $b$ *is an odd positive natural number greater than one and* $n \geq 2$ *. Let* $a$ *be a natural number greater than two such that* $\left(\frac{a-2}{M}\right) = 1$ *and* $\left(\frac{a+2}{M}\right) = 1$ *where* $\left(\right)$ *denotes Jacobi symbol. Let* $S_i = P_b(S_{i-1})$ *with* $S_0$ *equal to the modular* $P_{kb/2}(a) \mod M$*. Then, if* $M$ *is prime then* $S_{n-1} \equiv a \pmod{M}$ *.*

**Conjecture 1.76.** *Let* $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m)$ *. Let* $M_p(a) = \frac{a^p - 1}{a - 1}$ *where* $a$ *is a natural number greater than one and* $p$ *is an odd prime number. Let* $c$ *be a natural number greater than two such that* $\left(\frac{c-2}{M_p(a)}\right) = \left(\frac{c+2}{M_p(a)}\right) = 1$ *where* $\left(\right)$ *denotes Jacobi symbol. Let* $S_i = P_a(S_{i-1})$ *with* $S_0 = P_a(c)$*. Then , if* $M_p(a)$ *is prime then* $S_{p-1} \equiv P_a(c) \pmod{M_p(a)}$ *.*

**Conjecture 1.77.** *Let* $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m)$ *. Let* $N_p(b) = \frac{b^p + 1}{b + 1}$ *where* $b$ *is a natural number greater than one and* $p$ *is an odd prime number. Let* $c$ *be a natural number greater than two such that* $\left(\frac{c-2}{N_p(b)}\right) = \left(\frac{c+2}{N_p(b)}\right) = 1$ *where* $\left(\right)$ *denotes Jacobi symbol. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_b(c)$*. Then , if* $N_p(b)$ *is prime then* $S_{p-1} \equiv P_b(c) \pmod{N_p(b)}$ *.*

**Conjecture 1.78.** *Let* $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m)$ *. Let* $M = k \cdot b^n - c$ *where* $k, b, n, c$ *are natural numbers such that* $k > 0$ *,* $b > 1$ *,* $n > 1$ *and* $c > 0$*. Let* $a$ *be a natural number greater than two such that* $\left(\frac{a-2}{M}\right) = -1$ *and* $\left(\frac{a+2}{M}\right) = 1$ *where* $\left(\right)$ *denotes Jacobi symbol. Let* $S_i = P_b(S_{i-1})$ *with* $S_0$ *equal to the modular* $P_{kb}(a) \mod M$*. Then, if* $M$ *is prime then* $S_{n-1} \equiv P_{c-1}(a) \pmod{M}$ *.*

**Conjecture 1.79.** *Let* $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2 - 4})^m + (x + \sqrt{x^2 - 4})^m)$ *. Let* $N = k \cdot b^n + c$ *where* $k, b, n, c$ *are natural numbers such that* $k > 0$ *,* $b > 1$ *,* $n > 1$ *and* $c > 0$ *. Let* $a$ *be a natural number greater than two such that* $\left(\frac{a-2}{N}\right) = 1$ *and* $\left(\frac{a+2}{N}\right) = 1$ *where* $\left(\right)$ *denotes Jacobi symbol. Let* $S_i = P_b(S_{i-1})$ *with* $S_0$ *equal to the modular* $P_{kb}(a) \mod N$*. Then, if* $N$ *is prime then* $S_{n-1} \equiv P_{c-1}(a) \pmod{N}$ *.*

**Conjecture 1.80.** *Let* $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2-4})^m + (x + \sqrt{x^2-4})^m)$ *. Let* $M_p(a) = \frac{a^p-1}{a-1}$ *where* $a$ *is a natural number greater than one and* $p$ *is an odd prime number . Let* $c$ *be a natural number greater than two such that* $\left(\frac{c-2}{M_p(a)}\right) = -1$ *and* $\left(\frac{c+2}{M_p(a)}\right) = 1$ *where* $(\,)$ *denotes Jacobi symbol. Let* $S_i = P_a(S_{i-1})$ *with* $S_0 = P_a(c)$*. Then , if* $M_p(a)$ *is prime then* $S_{p-1} \equiv P_{a-2}(c) \pmod{M_p(a)}$ *.*

**Conjecture 1.81.** *Let* $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2-4})^m + (x + \sqrt{x^2-4})^m)$ *. Let* $N_p(b) = \frac{b^p+1}{b+1}$ *where* $b$ *is a natural number greater than one and* $p$ *is an odd prime number . Let* $c$ *be a natural number greater than two such that* $\left(\frac{c-2}{N_p(b)}\right) = -1$ *and* $\left(\frac{c+2}{N_p(b)}\right) = 1$ *where* $(\,)$ *denotes Jacobi symbol. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_b(c)$*. Then , if* $N_p(b)$ *is prime then* $S_{p-1} \equiv P_{b+2}(c) \pmod{N_p(b)}$ *.*

**Conjecture 1.82.** *Let* $n_1, n_2, \ldots, n_k$ *be a sequence of* $k$ *consecutive odd composite numbers . Let* $\operatorname{gpf}(n_i)$ *be the greatest prime factor of* $n_i$ *. Then , all* $\operatorname{gpf}(n_i)$ *,* $1 \le i \le k$ *are mutually different .*