Research Project Primus

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1 Theorems and Conjectures

Theorem 1.1. *A natural number* $n > 2$ *is a prime iff* $\prod_{k=1}^{n-1} k \equiv n-1 \pmod{\sum_{k=1}^{n-1} k}$.

Theorem 1.2. *Let* $p \equiv 5 \pmod 6$ *be prime then ,* $2p+1$ *is prime iff* $2p+1 \mid 3^p - 1$.

Theorem 1.3. *Let* p_n *be the nth prime , then*

$$p_n = 1 + \sum_{k=1}^{2\cdot(\lfloor n\ln(n)\rfloor+1)} \left(1 - \left\lfloor \frac{1}{n} \cdot \sum_{j=2}^{k} \left\lceil \frac{3 - \sum_{i=1}^{j} \left\lfloor \frac{\lfloor \frac{j}{i} \rfloor}{\lceil \frac{j}{i} \rceil} \right\rfloor}{j} \right\rceil \right\rfloor \right)$$

Theorem 1.4. *Let* $P_j(x) = 2^{-j} \cdot \left(\left(x - \sqrt{x^2-4}\right)^j + \left(x + \sqrt{x^2-4}\right)^j\right)$ *, where* j *and* x *are nonnegative integers . Let* $N = k \cdot 2^m - 1$ *such that* $m > 2$ *,* $3 \mid k$ *,* $0 < k < 2^m$ *and*

$$\begin{cases} k \equiv 1 \pmod{10} \ with \ m \equiv 2,3 \pmod 4 \\ k \equiv 3 \pmod{10} \ with \ m \equiv 0,3 \pmod 4 \\ k \equiv 7 \pmod{10} \ with \ m \equiv 1,2 \pmod 4 \\ k \equiv 9 \pmod{10} \ with \ m \equiv 0,1 \pmod 4 \end{cases}$$

Let $S_i = S_{i-1}^2 - 2$ *with* $S_0 = P_k(3)$ *, then* N *is prime iff* $S_{m-2} \equiv 0 \pmod N$

Theorem 1.5. *Let* $P_j(x) = 2^{-j} \cdot \left(\left(x - \sqrt{x^2-4}\right)^j + \left(x + \sqrt{x^2-4}\right)^j\right)$ *, where* j *and* x *are nonnegative integers . Let* $N = k \cdot 2^m - 1$ *such that* $m > 2$ *,* $3 \mid k$ *,* $0 < k < 2^m$ *and*

$$\begin{cases} k \equiv 3 \pmod{42} \ with \ m \equiv 0,2 \pmod 3 \\ k \equiv 9 \pmod{42} \ with \ m \equiv 0 \pmod 3 \\ k \equiv 15 \pmod{42} \ with \ m \equiv 1 \pmod 3 \\ k \equiv 27 \pmod{42} \ with \ m \equiv 1,2 \pmod 3 \\ k \equiv 33 \pmod{42} \ with \ m \equiv 0,1 \pmod 3 \\ k \equiv 39 \pmod{42} \ with \ m \equiv 2 \pmod 3 \end{cases}$$

Let $S_i = S_{i-1}^2 - 2$ *with* $S_0 = P_k(5)$ *, then* N *is prime iff* $S_{m-2} \equiv 0 \pmod N$

Theorem 1.6. *Let* $P_j(x) = 2^{-j} \cdot \left(\left(x - \sqrt{x^2-4}\right)^j + \left(x + \sqrt{x^2-4}\right)^j \right)$ *, where* j *and* x *are nonnegative integers . Let* $N = k \cdot 2^m + 1$ *such that* $m > 2$ *,* $0 < k < 2^m$ *and*

$$\begin{cases} k \equiv 1 \pmod{42} \textit{ with } m \equiv 2,4 \pmod{6} \\ k \equiv 5 \pmod{42} \textit{ with } m \equiv 3 \pmod{6} \\ k \equiv 11 \pmod{42} \textit{ with } m \equiv 3,5 \pmod{6} \\ k \equiv 13 \pmod{42} \textit{ with } m \equiv 4 \pmod{6} \\ k \equiv 17 \pmod{42} \textit{ with } m \equiv 5 \pmod{6} \\ k \equiv 19 \pmod{42} \textit{ with } m \equiv 0 \pmod{6} \\ k \equiv 23 \pmod{42} \textit{ with } m \equiv 1,3 \pmod{6} \\ k \equiv 25 \pmod{42} \textit{ with } m \equiv 0,2 \pmod{6} \\ k \equiv 29 \pmod{42} \textit{ with } m \equiv 1,5 \pmod{6} \\ k \equiv 31 \pmod{42} \textit{ with } m \equiv 2 \pmod{6} \\ k \equiv 37 \pmod{42} \textit{ with } m \equiv 0,4 \pmod{6} \\ k \equiv 41 \pmod{42} \textit{ with } m \equiv 1 \pmod{6} \end{cases}$$

Let $S_i = S_{i-1}^2 - 2$ *with* $S_0 = P_k(5)$ *, then* N *is prime iff* $S_{m-2} \equiv 0 \pmod{N}$ *.*

Theorem 1.7. *Let* $P_j(x) = 2^{-j} \cdot \left(\left(x - \sqrt{x^2-4}\right)^j + \left(x + \sqrt{x^2-4}\right)^j \right)$ *, where* j *and* x *are nonnegative integers . Let* $N = k \cdot 2^m + 1$ *such that* $m > 2$ *,* $0 < k < 2^m$ *and*

$$\begin{cases} k \equiv 1 \pmod{6} \textit{ and } k \equiv 1,7 \pmod{10} \textit{ with } m \equiv 0 \pmod{4} \\ k \equiv 5 \pmod{6} \textit{ and } k \equiv 1,3 \pmod{10} \textit{ with } m \equiv 1 \pmod{4} \\ k \equiv 1 \pmod{6} \textit{ and } k \equiv 3,9 \pmod{10} \textit{ with } m \equiv 2 \pmod{4} \\ k \equiv 5 \pmod{6} \textit{ and } k \equiv 7,9 \pmod{10} \textit{ with } m \equiv 3 \pmod{4} \end{cases}$$

Let $S_i = S_{i-1}^2 - 2$ *with* $S_0 = P_k(8)$ *, then* N *is prime iff* $S_{m-2} \equiv 0 \pmod{N}$ *.*

Theorem 1.8. *Let* $N = k \cdot 2^n + 1$ *with* $n > 1$ *,* k *is odd ,* $0 < k < 2^n$ *,* $3 \mid k$ *and*

$$\begin{cases} k \equiv 3 \pmod{30}, & \textit{with } n \equiv 1,2 \pmod{4} \\ k \equiv 9 \pmod{30}, & \textit{with } n \equiv 2,3 \pmod{4} \\ k \equiv 21 \pmod{30}, & \textit{with } n \equiv 0,1 \pmod{4} \\ k \equiv 27 \pmod{30}, & \textit{with } n \equiv 0,3 \pmod{4} \end{cases}$$

then N *is prime iff* $5^{\frac{N-1}{2}} \equiv -1 \pmod{N}$ *.*

Theorem 1.9. *Let* $N = k \cdot 2^n + 1$ *with* $n > 1$ *,* k *is odd ,* $0 < k < 2^n$ *,* $3 \mid k$ *and*

$$\begin{cases} k \equiv 3 \pmod{42}, & \textit{with } n \equiv 2 \pmod{3} \\ k \equiv 9 \pmod{42}, & \textit{with } n \equiv 0,1 \pmod{3} \\ k \equiv 15 \pmod{42}, & \textit{with } n \equiv 1,2 \pmod{3} \\ k \equiv 27 \pmod{42}, & \textit{with } n \equiv 1 \pmod{3} \\ k \equiv 33 \pmod{42}, & \textit{with } n \equiv 0 \pmod{3} \\ k \equiv 39 \pmod{42}, & \textit{with } n \equiv 0,2 \pmod{3} \end{cases}$$

then N *is prime iff* $7^{\frac{N-1}{2}} \equiv -1 \pmod{N}$ *.*

Theorem 1.10. *Let $N = k \cdot 2^n + 1$ with $n > 1$, k is odd , $0 < k < 2^n$, $3 \mid k$ and*

$$\begin{cases} k \equiv 3 \pmod{66}, & \textit{with } n \equiv 1,2,6,8,9 \pmod{10} \\ k \equiv 9 \pmod{66}, & \textit{with } n \equiv 0,1,3,4,8 \pmod{10} \\ k \equiv 15 \pmod{66}, & \textit{with } n \equiv 2,4,5,7,8 \pmod{10} \\ k \equiv 21 \pmod{66}, & \textit{with } n \equiv 1,2,4,5,9 \pmod{10} \\ k \equiv 27 \pmod{66}, & \textit{with } n \equiv 0,2,3,5,6 \pmod{10} \\ k \equiv 39 \pmod{66}, & \textit{with } n \equiv 0,1,5,7,8 \pmod{10} \\ k \equiv 45 \pmod{66}, & \textit{with } n \equiv 0,4,6,7,9 \pmod{10} \\ k \equiv 51 \pmod{66}, & \textit{with } n \equiv 0,2,3,7,9 \pmod{10} \\ k \equiv 57 \pmod{66}, & \textit{with } n \equiv 3,5,6,8,9 \pmod{10} \\ k \equiv 63 \pmod{66}, & \textit{with } n \equiv 1,3,4,6,7 \pmod{10} \end{cases}$$

then N is prime iff $11^{\frac{N-1}{2}} \equiv -1 \pmod{N}$.

Theorem 1.11. *A positive integer n is prime iff $\varphi(n)! \equiv -1 \pmod{n}$*

Theorem 1.12. *For $m \geq 1$ number n greater than one is prime iff*

$$(n^m - 1)! \equiv (n-1)^{\left\lceil \frac{(-1)^{m+1}}{2} \right\rceil} \cdot n^{\frac{n^m - mn + m - 1}{n-1}} \pmod{n^{\frac{n^m - mn + m + n - 2}{n-1}}}$$

Theorem 1.13. *Sequence S_i is defined as $S_i = \begin{cases} 8 & \textit{if } i = 0; \\ (S_{i-1}^2 - 2)^2 - 2 & \textit{otherwise} . \end{cases}$ then , $F_n = 2^{2^n} + 1, (n \geq 2)$ is a prime if and only if F_n divides $S_{2^{n-1}-1}$.*

Theorem 1.14. *Let $p \equiv 1 \pmod{6}$ be prime and let $5 \nmid 4p + 1$, then $4p + 1$ is prime iff $4p + 1 \mid 2^{2p} + 1$.*

Theorem 1.15. *Let $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2 - 4}\right)^m + \left(x + \sqrt{x^2 - 4}\right)^m \right)$, where m and x are nonnegative integers. Let $F_n(b) = b^{2^n} + 1$ such that $n \geq 2$ and b is even number . Let $S_i = P_b(S_{i-1})$ with $S_0 = P_b(6)$, thus If $F_n(b)$ is prime, then $S_{2^n - 1} \equiv 2 \pmod{F_n(b)}$.*

Theorem 1.16. *Let $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2 - 4}\right)^m + \left(x + \sqrt{x^2 - 4}\right)^m \right)$, where m and x are nonnegative integers. Let $E_n(b) = \frac{b^{2^n} + 1}{2}$ such that $n > 1$, b is odd number greater than one . Let $S_i = P_b(S_{i-1})$ with $S_0 = P_b(6)$, thus If $E_n(b)$ is prime, then $S_{2^n - 1} \equiv 6 \pmod{E_n(b)}$.*

Theorem 1.17. *Let $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2 - 4}\right)^m + \left(x + \sqrt{x^2 - 4}\right)^m \right)$.*

Let $N_p(b) = \frac{b^p + 1}{b + 1}$, where p is an odd prime and b is an odd natural number greater than one .

CASE(1). $b \equiv 1, 9 \pmod{12}$, or $b \equiv 3, 7 \pmod{12}$ and $p \equiv 1 \pmod{4}$, or $b \equiv 5 \pmod{12}$ and $p \equiv 1, 7 \pmod{12}$, or $b \equiv 11 \pmod{12}$ and $p \equiv 1, 11 \pmod{12}$.

CASE(2). $b \equiv 3, 7 \pmod{12}$ and $p \equiv 3 \pmod{4}$, or $b \equiv 5 \pmod{12}$ and $p \equiv 5, 11 \pmod{12}$, or $b \equiv 11 \pmod{12}$ and $p \equiv 5, 7 \pmod{12}$.

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_b(4)$. Suppose $N_p(b)$ is prime , then :

- *$S_{p-1} \equiv P_b(4) \pmod{N_p(b)}$ if Case(1) holds ;*
- *$S_{p-1} \equiv P_{b+2}(4) \pmod{N_p(b)}$ if Case(2) holds ;*

Theorem 1.18. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$.

Let $M_p(a) = \frac{a^p-1}{a-1}$ *, where* p *is an odd prime and* a *is an odd natural number greater than one*

CASE(1). $a \equiv 3, 11 \pmod{12}$ *, or* $a \equiv 5, 9 \pmod{12}$ *and* $p \equiv 1 \pmod 4$ *, or* $a \equiv 7 \pmod{12}$ *and* $p \equiv 1, 7 \pmod{12}$ *, or* $a \equiv 1 \pmod{12}$ *and* $p \equiv 1, 11 \pmod{12}$ *.*

CASE(2). $a \equiv 5, 9 \pmod{12}$ *and* $p \equiv 3 \pmod 4$ *, or* $a \equiv 7 \pmod{12}$ *and* $p \equiv 5, 11 \pmod{12}$ *, or* $a \equiv 1 \pmod{12}$ *and* $p \equiv 5, 7 \pmod{12}$ *.*

Let $S_i = P_a(S_{i-1})$ *with* $S_0 = P_a(4)$ *. Suppose* $M_p(a)$ *is prime , then :*

- $S_{p-1} \equiv P_a(4) \pmod{M_p(a)}$ *if Case(1) holds ;*
- $S_{p-1} \equiv P_{a-2}(4) \pmod{M_p(a)}$ *if Case(2) holds ;*

Conjecture 1.1. *Let* $b_n = b_{n-2} + lcm(n-1, b_{n-2})$ *with* $b_1 = 2$ *,* $b_2 = 2$ *and* $n > 2$ *. Let* $a_n = b_{n+2}/b_n - 1$ *, then*

1. Every term of this sequence a_i *is either prime or* 1 *. 2. Every odd prime number is member of this sequence . 3. Every new prime in sequence is a next prime from the largest prime already listed .*

Conjecture 1.2. *Let* $b_n = b_{n-1} + lcm(\lfloor \sqrt{n^3} \rfloor, b_{n-1})$ *with* $b_1 = 2$ *and* $n > 1$ *. Let* $a_n = b_{n+1}/b_n - 1$ *, then*

1. Every term of this sequence a_i *is either prime or* 1 *. 2. Every odd prime of the form* $\lfloor \sqrt{n^3} \rfloor$ *is member of this sequence . 3. Every new prime of the form* $\lfloor \sqrt{n^3} \rfloor$ *in sequence is a next prime from the largest prime already listed .*

Conjecture 1.3. *Let* $b_n = b_{n-1} + lcm(\lfloor \sqrt{2} \cdot n \rfloor, b_{n-1})$ *with* $b_1 = 2$ *and* $n > 1$ *. Let* $a_n = b_{n+1}/b_n - 1$ *, then*

1. Every term of this sequence a_i *is either prime or* 1 *. 2. Every prime of the form* $\lfloor \sqrt{2} \cdot n \rfloor$ *is member of this sequence . 3. Every new prime of the form* $\lfloor \sqrt{2} \cdot n \rfloor$ *in sequence is a next prime from the largest prime already listed .*

Conjecture 1.4. *Let* $b_n = b_{n-1} + lcm(\lfloor \sqrt{3} \cdot n \rfloor, b_{n-1})$ *with* $b_1 = 3$ *and* $n > 1$ *. Let* $a_n = b_{n+1}/b_n - 1$ *, then*

1. Every term of this sequence a_i *is either prime or* 1 *. 2. Every prime of the form* $\lfloor \sqrt{3} \cdot n \rfloor$ *is member of this sequence . 3. Every new prime of the form* $\lfloor \sqrt{3} \cdot n \rfloor$ *in sequence is a next prime from the largest prime already listed .*

Conjecture 1.5. *Let* b *and* n *be a natural numbers ,* $b \geq 2$ *,* $n > 2$ *and* $n \neq 9$ *. Then* n *is prime if and only if* $\sum\limits_{k=1}^{n-1} \left(b^k - 1\right)^{n-1} \equiv n \pmod{\frac{b^n-1}{b-1}}$

Conjecture 1.6. *Let* a *,* b *and* n *be a natural numbers ,* $b > a > 1$ *,* $n > 2$ *and* $n \notin \{4, 9, 25\}$ *. Then* n *is prime iff* $\prod\limits_{k=1}^{n-1} \left(b^k - a\right) \equiv \frac{a^n-1}{a-1} \pmod{\frac{b^n-1}{b-1}}$

Conjecture 1.7. *Let* a *,* b *and* n *be a natural numbers ,* $b > a > 0$ *,* $n > 2$ *and* $n \notin \{4, 9, 25\}$ *. Then* n *is prime iff* $\prod\limits_{k=1}^{n-1} \left(b^k + a\right) \equiv \frac{a^n+1}{a+1} \pmod{\frac{b^n-1}{b-1}}$

Conjecture 1.8. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$*, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n - 1$ *such that* $k > 0$ *,* $3 \nmid k$ *,* $k<2^n$ *,* $b > 0$ *,* b *is even number,* $3 \nmid b$ *and* $n > 2$ *. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{kb/2}(P_{b/2}(4))$ *, then* N *is prime iff* $S_{n-2} \equiv 0 \pmod{N}$ *.*

Conjecture 1.9. *Let* $P_j(x) = 2^{-j} \cdot \left(\left(x - \sqrt{x^2-4}\right)^j + \left(x + \sqrt{x^2-4}\right)^j \right)$*, where* j *and* x *are nonnegative integers. Let* $N = k \cdot 2^m + 1$ *with* k *odd ,* $0<k<2^m$ *and* $m > 2$ *. Let* F_n *be the nth Fibonacci number and let* $S_i = S_{i-1}^2 - 2$ *with* $S_0 = P_k(F_n)$ *, then* N *is prime iff there exists* F_n *for which* $S_{m-2} \equiv 0 \pmod{N}$ *.*

Conjecture 1.10. *Let* $P_j(x) = 2^{-j} \cdot \left(\left(x - \sqrt{x^2-4}\right)^j + \left(x + \sqrt{x^2-4}\right)^j \right)$*, where* j *and* x *are nonnegative integers. Let* $F_m(b) = b^{2^m} + 1$ *with* b *even ,* $b > 0$ *and* $m \geq 2$ *. Let* F_n *be the nth Fibonacci number and let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(P_{b/2}(F_n))$ *, then* $F_m(b)$ *is prime iff there exists* F_n *for which* $S_{m-2} \equiv 0 \pmod{F_m(b)}$ *.*

Conjecture 1.11. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n - b - 1$ *such that* $n > 2$*,* $b \equiv 0, 6 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv P_{(b+2)/2}(6) \pmod{N}$*.*

Conjecture 1.12. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n - b - 1$ *such that* $n > 2$*,* $b \equiv 2, 4 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv -P_{b/2}(6) \pmod{N}$*.*

Conjecture 1.13. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n + b + 1$ *such that* $n > 2$*,* $b \equiv 0, 6 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv P_{b/2}(6) \pmod{N}$*.*

Conjecture 1.14. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n + b + 1$ *such that* $n > 2$*,* $b \equiv 2, 4 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv -P_{(b+2)/2}(6) \pmod{N}$*.*

Conjecture 1.15. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n - b + 1$ *such that* $n > 3$*,* $b \equiv 0, 2 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv P_{b/2}(6) \pmod{N}$*.*

Conjecture 1.16. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n - b + 1$ *such that* $n > 3$*,* $b \equiv 4, 6 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv -P_{(b-2)/2}(6) \pmod{N}$*.*

Conjecture 1.17. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n + b - 1$ *such that* $n > 3$*,* $b \equiv 0, 2 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv P_{(b-2)/2}(6) \pmod{N}$*.*

Conjecture 1.18. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = b^n + b - 1$ *such that* $n > 3$*,* $b \equiv 4, 6 \pmod{8}$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{b/2}(6)$*, thus if* N *is prime, then* $S_{n-1} \equiv -P_{b/2}(6) \pmod{N}$*.*

Conjecture 1.19. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$

Let $N = k \cdot 3^n - 2$ *such that* $n > 3$ *,* $k \equiv 1, 3 \pmod 8$ *and* $k > 0$ *. Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{3k}(6)$ *, thus If* N *is prime then* $S_{n-1} \equiv P_3(6) \pmod N$

Conjecture 1.20. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$

Let $N = k \cdot 3^n - 2$ *such that* $n > 3$ *,* $k \equiv 5, 7 \pmod 8$ *and* $k > 0$ *. Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{3k}(6)$ *, thus If* N *is prime then* $S_{n-1} \equiv P_1(6) \pmod N$

Conjecture 1.21. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot 3^n + 2$ *such that* $n > 2$ *,* $k \equiv 1, 3 \pmod 8$ *and* $k > 0$ *. Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{3k}(6)$ *, thus If* N *is prime then* $S_{n-1} \equiv P_3(6) \pmod N$

Conjecture 1.22. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot 3^n + 2$ *such that* $n > 2$ *,* $k \equiv 5, 7 \pmod 8$ *and* $k > 0$ *. Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{3k}(6)$ *, thus If* N *is prime then* $S_{n-1} \equiv P_1(6) \pmod N$

Conjecture 1.23. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n - c$ *such that* $b \equiv 0 \pmod 2), n > bc, k > 0, c > 0$ *and* $c \equiv 1, 7 \pmod 8$ *Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6)),$ *thus If* N *is prime then* $S_{n-1} \equiv P_{(b/2)\cdot\lceil c/2 \rceil}(6) \pmod N$

Conjecture 1.24. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n - c$ *such that* $b \equiv 0, 4, 8 \pmod{12}, n > bc, k > 0, c > 0$ *and* $c \equiv 3, 5 \pmod 8$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6)),$ *thus If* N *is prime then* $S_{n-1} \equiv P_{(b/2)\cdot\lfloor c/2 \rfloor}(6) \pmod N$

Conjecture 1.25. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n - c$ *such that* $b \equiv 2, 6, 10 \pmod{12}, n > bc, k > 0, c > 0$ *and* $c \equiv 3, 5 \pmod 8$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6)),$ *thus If* N *is prime then* $S_{n-1} \equiv -P_{(b/2)\cdot\lfloor c/2 \rfloor}(6) \pmod N$

Conjecture 1.26. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n + c$ *such that* $b \equiv 0 \pmod 2), n > bc, k > 0, c > 0$ *and* $c \equiv 1, 7 \pmod 8$ *Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6)),$ *thus If* N *is prime then* $S_{n-1} \equiv P_{(b/2)\cdot\lfloor c/2 \rfloor}(6) \pmod N$

Conjecture 1.27. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n + c$ *such that* $b \equiv 0, 4, 8 \pmod{12}, n > bc, k > 0, c > 0$ *and* $c \equiv 3, 5 \pmod 8$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6)),$ *thus If* N *is prime then* $S_{n-1} \equiv P_{(b/2)\cdot\lceil c/2 \rceil}(6) \pmod N$

Conjecture 1.28. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n + c$ *such that* $b \equiv 2, 6, 10 \pmod{12}, n > bc, k > 0, c > 0$ *and* $c \equiv 3, 5 \pmod 8$*. Let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(6)),$ *thus If* N *is prime then* $S_{n-1} \equiv -P_{(b/2)\cdot\lceil c/2 \rceil}(6) \pmod N$

Conjecture 1.29. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = 2 \cdot 3^n - 1$ *such that* $n > 1$ *. Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_3(a)$ *, where* $a = \begin{cases} 6, & \text{if } n \equiv 0 \pmod 2 \\ 8, & \text{if } n \equiv 1 \pmod 2 \end{cases}$ *thus ,* N *is prime iff* $S_{n-1} \equiv a \pmod N$

Conjecture 1.30. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = 8 \cdot 3^n - 1$ *such that* $n > 1$ *. Let* $S_i = P_3(S_{i-1})$ *with* $S_0 = P_{12}(4)$ *thus ,* N *is prime iff* $S_{n-1} \equiv 4 \pmod N$

Conjecture 1.31. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot 6^n - 1$ *such that* $n > 2$ *,* $k > 0$ *,* $k \equiv 2, 5 \pmod 7$ *and* $k < 6^n$ *Let* $S_i = P_6(S_{i-1})$ *with* $S_0 = P_{3k}(P_3(5))$ *, thus* N *is prime iff* $S_{n-2} \equiv 0 \pmod N$

Conjecture 1.32. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot 6^n - 1$ *such that* $n > 2$ *,* $k > 0$ *,* $k \equiv 3, 4 \pmod 5$ *and* $k < 6^n$ *Let* $S_i = P_6(S_{i-1})$ *with* $S_0 = P_{3k}(P_3(3))$ *, thus* N *is prime iff* $S_{n-2} \equiv 0 \pmod N$

Conjecture 1.33. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n - 1$ *such that* $n > 2$ *,* $k < 2^n$ *and*

$$\begin{cases} k \equiv 3 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 0, 3 \pmod 4 \\ k \equiv 3 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 0, 2 \pmod 4 \\ k \equiv 3 \pmod{30} \text{ with } b \equiv 6 \pmod{10} \text{ and } n \equiv 0, 1, 2, 3 \pmod 4 \\ k \equiv 3 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 0, 1 \pmod 4 \end{cases}$$

Let $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(18))$ *, then* N *is prime iff* $S_{n-2} \equiv 0 \pmod N$

Conjecture 1.34. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n - 1$ *such that* $n > 2$ *,* $k < 2^n$ *and*

$$\begin{cases} k \equiv 9 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 0, 1 \pmod 4 \\ k \equiv 9 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 0, 2 \pmod 4 \\ k \equiv 9 \pmod{30} \text{ with } b \equiv 6 \pmod{10} \text{ and } n \equiv 0, 1, 2, 3 \pmod 4 \\ k \equiv 9 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 0, 3 \pmod 4 \end{cases}$$

Let $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(18))$ *, then* N *is prime iff* $S_{n-2} \equiv 0 \pmod N$

Conjecture 1.35. *Let* $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m \right)$ *, where* m *and* x *are nonnegative integers. Let* $N = k \cdot b^n - 1$ *such that* $n > 2$ *,* $k < 2^n$ *and*

$$\begin{cases} k \equiv 21 \pmod{30} \text{ with } b \equiv 2 \pmod{10} \text{ and } n \equiv 2, 3 \pmod 4 \\ k \equiv 21 \pmod{30} \text{ with } b \equiv 4 \pmod{10} \text{ and } n \equiv 1, 3 \pmod 4 \\ k \equiv 21 \pmod{30} \text{ with } b \equiv 8 \pmod{10} \text{ and } n \equiv 1, 2 \pmod 4 \end{cases}$$

Let $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{bk/2}(P_{b/2}(3))$ *, then* N *is prime iff* $S_{n-2} \equiv 0 \pmod N$

Conjecture 1.36. *Let* F_p *be the pth Fibonacci number .If* p *is prime , not* 5 *, and* $M \geq 2$ *then* $M^{F_p} \equiv M^{(p-1)^{(1-(\frac{p}{5}))/2}} \pmod{\frac{M^p-1}{M-1}}$

Conjecture 1.37. *Let* b *and* n *be a natural numbers ,* $b \geq 2$ *,* $n > 1$ *and* $n \notin \{4,8,9\}$ *. Then* n *is prime if and only if* $\sum\limits_{k=1}^{n} \left(b^k+1\right)^{n-1} \equiv n \pmod{\dfrac{b^n-1}{b-1}}$

Conjecture 1.38. *If* q *is the smallest prime greater than* $\prod\limits_{i=1}^{n} C_i + 1$ *, where* $\prod\limits_{i=1}^{n} C_i$ *is the product of the first* n *composite numbers , then* $q - \prod\limits_{i=1}^{n} C_i$ *is prime .*

Conjecture 1.39. *If* q *is the greatest prime less than* $\prod\limits_{i=1}^{n} C_i - 1$ *, where* $\prod\limits_{i=1}^{n} C_i$ *is the product of the first* n *composite numbers , then* $\prod\limits_{i=1}^{n} C_i - q$ *is prime .*

Conjecture 1.40. *Let* n *be an odd number and* $n > 1$ *. Let* $T_n(x)$ *be Chebyshev polynomial of the first kind and let* $P_n(x)$ *be Legendre polynomial , then* n *is a prime number if and only if the following congruences hold simultaneously* • $T_n(3) \equiv 3 \pmod{n}$ • $P_n(3) \equiv 3 \pmod{n}$

Conjecture 1.41. *Let* n *be a natural number greater than two . Let* r *be the smallest odd prime number such that* $r \nmid n$ *and* $n^2 \not\equiv 1 \pmod{r}$ *. Let* $T_n(x)$ *be Chebyshev polynomial of the first kind , then* n *is a prime number if and only if* $T_n(x) \equiv x^n \pmod{x^r - 1, n}$ *.*

Conjecture 1.42. *Let* n *be a natural number greater than two and* $n \neq 5$ *. Let* $T_n(x)$ *be Chebyshev polynomial of the first kind . If there exists an integer* a *,* $1 < a < n$ *, such that* $T_{n-1}(a) \equiv 1 \pmod{n}$ *and for every prime factor* q *of* $n-1$ *,* $T_{(n-1)/q}(a) \not\equiv 1 \pmod{n}$ *then* n *is prime . If no such number* a *exists then* n *is composite .*

Conjecture 1.43. *Let* $P_a(x) = 2^{-a} \cdot \left(\left(x - \sqrt{x^2-4}\right)^a + \left(x + \sqrt{x^2-4}\right)^a\right)$ *. Let* $N = k \cdot b^m \pm 1$ *with* b *an even positive integer ,* $0<k<b^m$ *and* $m > 2$ *. Let* F_n *be the nth Fibonacci number and let* $S_i = P_b(S_{i-1})$ *with* $S_0 = P_{kb/2}(P_{b/2}(F_n))$ *, then* N *is prime iff there exists* F_n *for which* $S_{m-2} \equiv 0 \pmod{N}$ *.*

Conjecture 1.44. *Let* n *be a natural number greater than one . Let* r *be the smallest odd prime number such that* $r \nmid n$ *and* $n^2 \not\equiv 1 \pmod{r}$ *. Let* $L_n(x)$ *be Lucas polynomial , then* n *is a prime number if and only if* $L_n(x) \equiv x^n \pmod{x^r - 1, n}$ *.*

Conjecture 1.45. *Let* b *and* n *be a natural numbers ,* $b \geq 2$ *, then* $\frac{b^n-1}{b-1} \cdot \frac{b^{\sigma(n)}-1}{b-1} \equiv b+1 \pmod{\frac{b^{\varphi(n)}-1}{b-1}}$ *for all primes and no composite with the exception of* 4 *and* 6 *.*

Conjecture 1.46. *Let* b *and* n *be a natural numbers ,* $b \geq 2$ *, then* $\frac{b^{\varphi(n)}-1}{b-1}(b^{\tau(n)}-1) + b \equiv b^{n-1} \pmod{\frac{b^n-1}{b-1}}$ *for all primes and no composite with the exception of* 4 *.*

Conjecture 1.47. *Let* p *be prime number greater than three and let* $T_n(x)$ *be Chebyshev polynomial of the first kind , then* $T_{p-1}(2) \equiv 1 \pmod{p}$ *if and only if* $p \equiv 1, 11 \pmod{12}$ *.*

Conjecture 1.48. *Let* p *be prime number greater than two and let* $T_n(x)$ *be Chebyshev polynomial of the first kind , then* $T_{p-1}(3) \equiv 1 \pmod{p}$ *if and only if* $p \equiv 1, 7 \pmod{8}$ *.*

Conjecture 1.49. *Let p be prime number greater than three and let $T_n(x)$ be Chebyshev polynomial of the first kind , then $T_{p-1}(5) \equiv 1 \pmod{p}$ if and only if $p \equiv 1, 5, 19, 23 \pmod{24}$* .

Conjecture 1.50. *Let n be an odd natural number greater than one , let k be a natural number such that $k \leq n$, then n is prime if and only if :* $\displaystyle\sum_{i=0}^{k-1} i^{n-1} + \sum_{j=0}^{n-k} j^{n-1} \equiv -1 \pmod{n}$

Conjecture 1.51. *Let n be a natural number greater than one and let $T_n(x)$ be Chebyshev polynomial of the first kind , then n is prime if and only if :* $\displaystyle\sum_{k=0}^{n-1} 2T_{n-1}\left(\frac{k}{2}\right) \equiv -1 \pmod{n}$.

Conjecture 1.52. *Let n be a natural number greater than one and let $L_n(x)$ be Lucas polynomial , then n is prime if and only if :* $\displaystyle\sum_{k=0}^{n-1} L_{n-1}(k) \equiv -1 \pmod{n}$.

Conjecture 1.53. *Let p be an odd prime number , let $R_p(3) = \frac{3^p-1}{2}$ and let $S_i = S_{i-1}^3 + 3S_{i-1}$ with $S_0 = 36$, then $R_p(3)$ is prime number iff $S_{p-1} \equiv 36 \pmod{R_p(3)}$* .

Conjecture 1.54. *Let p be an odd prime number greater than three , let $R_p(-3) = \frac{3^p+1}{4}$ and let $S_i = S_{i-1}^3 + 3S_{i-1}$ with $S_0 = 36$, then $R_p(-3)$ is prime number iff $S_{p-1} \equiv 36 \pmod{R_p(-3)}$* .

Conjecture 1.55. *Let $P_n^{(a)}(x) = \left(\frac{1}{2}\right) \cdot \left(\left(x - \sqrt{x^2+a}\right)^n + \left(x + \sqrt{x^2+a}\right)^n\right)$. Given an odd integer n (≥ 3) and integer a coprime to n , n is prime if and only if $P_n^{(a)}(x) \equiv x^n \pmod{n}$ holds* .

Conjecture 1.56. *Let n be an odd natural number greater than one . Let r be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$. Let $P_n(x)$ be Legendre polynomial , then n is a prime number if and only if $P_n(x) \equiv x^n \pmod{x^r - 1, n}$* .

Conjecture 1.57. *Let n be a natural number greater than one and let $F_n(x)$ be Fibonacci polynomial , then n is prime if and only if :* $\displaystyle\sum_{k=0}^{n-1} F_n(k) \equiv -1 \pmod{n}$.

Conjecture 1.58. *Let a_n be the least unused prime greater than 3 such that $(a_n + a_{n-1})/2$ is prime, with $a_0 = 13$, then :*

1. *Every term of this sequence a_i is prime of the form $12k+1$* .
2. *Every prime of the form $12k+1$ is a member of this sequence* .

Conjecture 1.59. *Let m and n be a natural numbers , $m \geq 1$, $n > 2$, $n \neq 9$ and gcd$(m, n) = 1$. Then n is prime if and only if* $\displaystyle\sum_{k=1}^{n-1} \left(2^{mk} - 1\right)^{n-1} \equiv n \pmod{2^n - 1}$

Conjecture 1.60. *Let p, q, r be three consecutive prime numbers such that $p \geq 11$ and $p < q < r$, then $\frac{1}{p^2} < \frac{1}{q^2} + \frac{1}{r^2}$* .

Conjecture 1.61. *Let p and q be consecutive prime numbers such that $p \geq 5$ and $p < q$, then $\left\lfloor \frac{q}{p} - \frac{p}{q} \right\rfloor = 0$.*

Conjecture 1.62. *Let a, n, k be natural numbers greater than 0 . If n is a prime number then $\sum\limits_{d|n} \left(\sigma_k(d) \cdot a^{n/d}\right) \equiv 2a \pmod{n}$*

Conjecture 1.63. *Let $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2-4})^m + (x + \sqrt{x^2-4})^m)$. Let $F_n(b) = b^{2^n} + 1$ where b is an even integer , $3 \nmid b, 5 \nmid b$ and $n \geq 2$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(P_{b/2}(8))$, then $F_n(b)$ is prime iff $S_{2^n-2} \equiv 0 \pmod{F_n(b)}$.*

Conjecture 1.64. *Let $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2-4})^m + (x + \sqrt{x^2-4})^m)$. Let $F_n(b) = b^{2^n} + 1$ where b is an even integer , $3 \nmid b$, $b \equiv 2, 4, 10, 12 \pmod{14}$ and $n \geq 2$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(P_{b/2}(5))$, then $F_n(b)$ is prime iff $S_{2^n-2} \equiv 0 \pmod{F_n(b)}$.*

Conjecture 1.65. *Let $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2-4})^m + (x + \sqrt{x^2-4})^m)$. Let $F_n(b) = b^{2^n} + 1$ where b is an even integer , $5 \nmid b$, $b \equiv 2, 4, 10, 12 \pmod{14}$ and $n \geq 2$. Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{b/2}(P_{b/2}(12))$, then $F_n(b)$ is prime iff $S_{2^n-2} \equiv 0 \pmod{F_n(b)}$.*

Conjecture 1.66. *Let $a_n = 62a_{n-1} - a_{n-2}$ with $a_1 = 8$ and $a_2 = 488$, let $b_n = 482b_{n-1} - b_{n-2}$ with $b_1 = 22$ and $b_2 = 10582$, then each member of the sequences $\{a_n\}$ and $\{b_n\}$ can be used as an initial value for Inkeri's primality test for Fermat numbers .*

Conjecture 1.67. *Let $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2-4}\right)^m + \left(x + \sqrt{x^2-4}\right)^m\right)$*

Let $N = k \cdot b^n - 1$ such that $n > 2$, $k < 2^n$ and

$$\begin{cases} k \equiv 27 \pmod{30} \; with \; b \equiv 2 \pmod{10} \; and \; n \equiv 1, 2 \pmod{4} \\ k \equiv 27 \pmod{30} \; with \; b \equiv 4 \pmod{10} \; and \; n \equiv 1, 3 \pmod{4} \\ k \equiv 27 \pmod{30} \; with \; b \equiv 8 \pmod{10} \; and \; n \equiv 2, 3 \pmod{4} \end{cases}$$

Let $S_i = P_b(S_{i-1})$ with $S_0 = P_{bk/2}(P_{b/2}(3))$, then N is prime iff $S_{n-2} \equiv 0 \pmod{N}$

Conjecture 1.68. *Let n be a natural number greater than two . Let r be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$. Let $H_n(x)$ be Hermite polynomial , then n is either a prime number or Fermat pseudoprime to base 2 if and only if $H_n(x) \equiv 2x^n \pmod{x^r - 1, n}$.*

Conjecture 1.69. *Let n be an odd natural number greater than one . Let r be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$. Let $P_n^{(\alpha,\beta)}(x)$ be Jacobi polynomial such that α , β are natural numbers and $\alpha + \beta < n$, then n is a prime number if and only if $P_n^{(\alpha,\beta)}(x) \equiv x^n \pmod{x^r - 1, n}$.*

Conjecture 1.70. *Let n be an odd natural number greater than one . Let r be the smallest odd prime number such that $r \nmid n$ and $n^2 \not\equiv 1 \pmod{r}$. Let $F_n(x)$ be Fibonacci polynomial , then n is prime if and only if $F_n(2x) \equiv (1 + x^2)^{\frac{n-1}{2}} \pmod{x^r - 1, n}$.*

Conjecture 1.71. *Let $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2-4})^m + (x + \sqrt{x^2-4})^m)$. Let $F_n(b) = b^{2^n} + 1$ where b is an even natural number and $n \geq 2$. Let a be a natural number greater than two such that $\left(\frac{a-2}{F_n(b)}\right) = -1$ and $\left(\frac{a+2}{F_n(b)}\right) = -1$ where $\left(\,\right)$ denotes Jacobi symbol. Let $S_i = P_b(S_{i-1})$ with S_0 equal to the modular $P_{b/2}(P_{b/2}(a))$ mod $F_n(b)$. Then $F_n(b)$ is prime if and only if $S_{2^n-2} \equiv 0 \pmod{F_n(b)}$.*

Conjecture 1.72. *Let* $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2-4})^m + (x + \sqrt{x^2-4})^m)$ *. Let* $N = k \cdot b^n + 1$ *where* k *is positive natural number ,* $k < 2^n$ *,* b *is an even positive natural number and* $n \geq 3$ *. Let* a *be a natural number greater than two such that* $\left(\frac{a-2}{N}\right) = -1$ *and* $\left(\frac{a+2}{N}\right) = -1$ *where* $\left(\,\right)$ *denotes Jacobi symbol. Let* $S_i = P_b(S_{i-1})$ *with* S_0 *equal to the modular* $P_{kb/2}(P_{b/2}(a)) \mod N$*. Then* N *is prime if and only if* $S_{n-2} \equiv 0 \pmod{N}$ *.*

Conjecture 1.73. *Let* $P_m(x) = 2^{-m} \cdot ((x - \sqrt{x^2-4})^m + (x + \sqrt{x^2-4})^m)$ *. Let* $M = k \cdot b^n - 1$ *where* k *is positive natural number ,* $k < 2^n$ *,* b *is an even positive natural number and* $n \geq 3$ *. Let* a *be a natural number greater than two such that* $\left(\frac{a-2}{M}\right) = 1$ *and* $\left(\frac{a+2}{M}\right) = -1$ *where* $\left(\,\right)$ *denotes Jacobi symbol. Let* $S_i = P_b(S_{i-1})$ *with* S_0 *equal to the modular* $P_{kb/2}(P_{b/2}(a)) \mod M$*. Then* M *is prime if and only if* $S_{n-2} \equiv 0 \pmod{M}$ *.*